2.1 Fill in the details leading to Eq. (2.17) and use it to find the weighted residuals formulation of Example 2.1

Solution:

a)

$$\int_0^L \phi(x)\left[-K\frac{d^2T}{dx^2}-Q\right]dx=0 \qquad (Eq.2.13)$$

Integration by parts: $\int_0^L u\,dv = uv\big|_0^L - \int_0^L v\,du$. Set $u=\phi(x)$ and $dv=-K\frac{d^2T}{dx^2}$, then

$v=-K\frac{dT}{dx}$ and $du=\frac{d\phi}{dx}dx$ and we have

$$\int_0^L \phi(x)\left[-K\frac{d^2T}{dx^2}-Q\right]dx=\phi(x)\left(-K\frac{dT}{dx}\right)_0^L-\int_o^L\left(-K\frac{dT}{dx}\right)\frac{d\phi}{dx}dx$$
$$=\int_o^L K\frac{d\phi}{dx}\frac{dT}{dx}dx-K\phi\frac{dT}{dx}\bigg|_0^L$$

Substitute into Eq. 2.13 and we get

$$\int_o^L K\frac{d\phi}{dx}\frac{dT}{dx}dx-\int_o^L\phi Q\,dx-K\phi\frac{dT}{dx}\bigg|_0^L=0 \quad (Eq.2.17)$$

b)

$$T(0)=T_0 \qquad and \qquad -K\frac{dT}{dx}\bigg|_{x=L}=h(T-T_\infty)$$

Replacing in Eq. 2.17

$$\int_o^L K\frac{d\phi}{dx}\frac{dT}{dx}dx-\int_o^L\phi Q\,dx-\phi(L)h(T(L)-T_\infty)=0$$