2.13 Find the weak formulation of Eq. (2.1) with the boundary condition

$$-K\frac{dT}{dx}\bigg|_{x=0} = h(T - T_\infty) \text{ and } -K\frac{dT}{dx}\bigg|_{x=1} = q.$$

Solution:

The general form is given in Eq. 2.17 $\int_0^L \frac{d\phi}{dx}\frac{dT}{dx}dx - \int_0^L \phi Q\, dx - K\phi\frac{dT}{dx}\bigg|_o^L = 0$

Substituting the flux boundary conditions we get

$$\int_0^L \frac{d\phi}{dx}\frac{dT}{dx}dx - \int_0^L \phi Q\, dx + \phi(L)q - \phi(0)h(T(0) - T_\infty) = 0$$