2.8   Let us consider once more Problem 2.3 above. Assume that $u(x) = a_1 x^2$ and $W_1(x) = x^2$, but this time use Eq. (2.11) and proceed as in Problem 2.2 in order to obtain a solution. Show that this solution is identical to that obtained in Problem 2.3(b)(ii), and hence, the process of integration by parts does not introduce any changes.

Solution:

$$\int_0^1 \left[W_1 \left(\frac{d^2 u}{dx^2} - 1 \right) \right] dx = \int_0^1 x^2 (2a - 1) dx = 0 \quad , \quad \left(\frac{2}{3} x^3 a_1 - \frac{x^3}{3} \right)\Big|_0^1 = 0 \quad , \quad \frac{2}{3} a - \frac{1}{3} = 0 \quad , \quad a_1 = 1/2 \quad , \quad u(x) = \frac{x^2}{2}$$