2.3 Obtain expression (2.30).

Solution:

The weighted residuals form for the second term is

$$\int_0^L \phi(x)\left[-K\frac{d^2T}{dx^2}-Q\right]dx=0$$

After integration by parts on the first term

$$\int_o^L K\frac{d\phi}{dx}\frac{dT}{dx}dx-\int_o^L \phi Q\,dx+\phi\left(-K\frac{dT}{dx}\right)\Bigg|_{L/2}^{L}=0$$

Note: The boundary terms are set to zero if no fluxes are prescribed.

The functions $\phi_i(x)$ for element 2 are

$$\phi_2(x)=\frac{2}{L}(L-x)$$

$$\phi_3(x)=\frac{2}{L}(x-L/2)$$

Let $$T(x)=\frac{2}{L}(L-x)a_2+\frac{2}{L}(x-L/2)a_3 \quad (Eq.\ 2.19)$$

Using Eq. 2.28

$$\int_{L/2}^{L}\begin{bmatrix}-\frac{2}{L}\\ \frac{2}{L}\end{bmatrix}\begin{bmatrix}-\frac{2}{L} & \frac{2}{L}\end{bmatrix}dx\begin{Bmatrix}a_2\\ a_3\end{Bmatrix}-Q\int_{L/2}^{L}\begin{Bmatrix}\frac{2}{L}(L-x)\\ \frac{2}{L}(x-L/2)\end{Bmatrix}dx-\begin{Bmatrix}0\\0\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

Or

$$\frac{4K}{L^2}\int_{L/2}^{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}dx\begin{Bmatrix}a_2\\ a_3\end{Bmatrix}-\frac{2Q}{L}\int_{L/2}^{L}\begin{Bmatrix}(L-x)\\ (x-L/2)\end{Bmatrix}dx-\begin{Bmatrix}0\\0\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$

Integrating we have

$$\frac{2K}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}a_2\\ a_3\end{Bmatrix}-\frac{QL}{4}\int_{L/2}^{L}\begin{Bmatrix}1\\1\end{Bmatrix}dx-\begin{Bmatrix}0\\0\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$$