

2.12 Show that if the expressions for ϕ_i given by Eq. (2.20) are used in Eq. (2.19), then $a_i = T_i$.

Solution:

Eq. 2.19 is $T(x) = \phi_i(x)a_i + \phi_{i+1}(x)a_{i+1}$ $x_i \leq x \leq x_{i+1}$ where

$$\phi_i(x) = \frac{x_{i+1} - x}{x_{i+1} - x_i}, \text{ and } \phi_{i+1}(x) = \frac{x - x_i}{x_{i+1} - x_i}$$

Substituting $T(x) = \left[\frac{x_{i+1} - x}{x_{i+1} - x_i} \right] a_i + \left[\frac{x - x_i}{x_{i+1} - x_i} \right] a_{i+1}$ and evaluating at the nodes

$$T(x_i) = a_i = T_i \text{ and } T(x_{i+1}) = a_{i+1} = T_{i+1}.$$