

## CHAPTER 2

### Problem 2.1

$$\mathbf{a)} \quad e^{-2t} \frac{dy}{dt} = y^{-1}(1 - e^{-2t}); \quad y(0) = 0$$

$$y dy = e^{2t}(1 - e^{-2t}) dt \Rightarrow \frac{1}{2} y^2 \Big|_0^y = \int_0^t (e^{2t} - 1) dt$$

$$\frac{1}{2} y^2 = \left( \frac{1}{2} e^{2t} - t \right) \Big|_0^t$$

$$y = \sqrt{e^{2t} - 2t - 1}$$

$$\mathbf{b)} \quad \frac{dy}{dt} = y \cos(t) + y; \quad y(0) = 2$$

$$\int_2^y \frac{dy}{y} = \int_0^t (\cos(t) + 1) dt \Rightarrow \ln(y) \Big|_2^y = (\sin(t) + t) \Big|_0^t$$

$$\ln\left(\frac{y}{2}\right) = \sin(t) + t$$

$$y = 2 \exp[\sin(t) + t]$$

$$\mathbf{c)} \quad \frac{dy}{dt} = \frac{t+2}{y}; \quad y(0) = 2$$

$$\int_2^y y dy = \int_0^t (t+2) dt \Rightarrow \frac{1}{2} y^2 \Big|_2^y = \frac{1}{2} t^2 + 2t \Big|_0^t$$

$$y^2 - 4 = t^2 + 4t$$

$$y = \sqrt{t^2 + 4t + 4}$$

$$\mathbf{d)} \quad \frac{d}{dt} \left[ t \frac{dy}{dt} \right] = 2t; \quad y(1) = 1; \quad y'(1) = 3$$

$$\text{Let } u = t \frac{dy}{dt} \text{ with } u(1) = 3$$

$$\frac{du}{dt} = 2t \Rightarrow \int_3^u du = \int_1^t 2t dt \Rightarrow u = t^2 + 2$$

$$u = t^2 + 2 = t \frac{dy}{dt} \Rightarrow \int_1^y dy = \int_1^t (t + 2t^{-1}) dt$$

$$y - 1 = \frac{t^2}{2} + 2 \ln(t) - \frac{1}{2}$$

$$y = \frac{t^2}{2} + 2 \ln(t) + \frac{1}{2}$$

e)  $\frac{d^2y}{dt^2} = 32e^{-4t}$   $y(0) = 1$  ;  $y'(0) = 0$

$$\frac{d}{dt} \left[ \frac{dy}{dt} \right] = 32e^{-4t}$$

Let  $u = \frac{dy}{dt}$

Then,  $\frac{du}{dt} = 32e^{-4t}$ ;  $u(0) = 0$

$$\int_0^u du = \int_0^t 32e^{-4t} dt \Rightarrow u = -\frac{32}{4} e^{-4t} \Big|_0^t = -\frac{32}{4} e^{-4t} + \frac{32}{4}$$

$$u = \frac{dy}{dt} = 8 - 8e^{-4t}$$

$$\int_1^y dy = \int_0^t (8 - 8e^{-4t}) dt \Rightarrow y - 1 = 8t + 2e^{-4t} \Big|_0^t = 8t + 2e^{-4t} - 2$$

$$y = 8t + 2e^{-4t} - 1$$

f)  $\frac{1}{t^2} \frac{dy}{dt} = y$ ;  $y(0) = 1$

$$\int_1^y \frac{dy}{y} = \int_0^t t^2 dt \Rightarrow \ln(y) \Big|_1^y = \frac{1}{3} t^3 \Big|_0^t \Rightarrow \ln(y) = \frac{1}{3} t^3$$

$$y = e^{\frac{t^3}{3}}$$

g)  $\frac{dy}{dt} = -y^2 e^{2t}$ ;  $y(0) = 1$

$$\int_1^4 \frac{dy}{y^2} = -\int_0^t e^{2t} dt \Rightarrow -\frac{1}{y} \Big|_1^y = -\frac{1}{2} e^{2t} \Big|_0^t \Rightarrow -\frac{1}{y} + 1 = -\frac{1}{2} e^{2t} + \frac{1}{2}$$

$$\frac{1}{y} = \frac{1}{2}(e^{2t} + 1) \Rightarrow y = \frac{2}{e^{2t} + 1}$$

**h)**  $\frac{dy}{dt} - (2t + 1)y = 0; \quad y(0) = 2$

$$\frac{dy}{dt} = y(2t + 1) \Rightarrow \int_2^y \frac{dy}{y} = \int_0^t (2t + 1) dt \Rightarrow \ln(y) \Big|_2^y = (t^2 + t) \Big|_0^t$$

$$\ln\left(\frac{y}{2}\right) = t^2 + t \Rightarrow y = 2e^{(t^2+t)}$$

**i)**  $\frac{dy}{dt} + 4ty^2 = 0; \quad y(0) = 1$

$$\int_1^y \frac{dy}{y^2} = -\int_0^t 4t dt \Rightarrow -\frac{1}{y} \Big|_1^y = -2t^2 \Big|_0^t$$

$$y = \frac{1}{1 + 2t^2}$$

**j)**  $\frac{d^2y}{dt^2} = \cos\left(\frac{t}{2}\right); \quad y(0) = 0; \quad y'(0) = 1$

Let  $u = \frac{dy}{dt}$ , then

$$\frac{du}{dt} = \cos\left(\frac{t}{2}\right); \quad u(0) = 1$$

$$\int_1^u du = \int_0^t \cos\left(\frac{t}{2}\right) dt$$

$$u = 1 + 2 \sin\left(\frac{t}{2}\right)$$

and

$$\frac{dy}{dt} = 1 + 2 \sin\left(\frac{t}{2}\right); \quad y(0) = 0$$

$$\int_0^y dy = \int_0^t \left(1 + 2 \sin\left(\frac{t}{2}\right)\right) dt$$

$$y = \left( t - 4 \cos\left(\frac{t}{2}\right) \right) \Big|_0^t = t - 4 \cos\left(\frac{t}{2}\right) + 4$$

$$y = 4 + t - 4 \cos\left(\frac{t}{2}\right)$$

### Problem 2.2

a)  $\frac{du}{dy} = y^2 - 1 \lll = \text{linear}$

$$du = (y^2 - 1)dy$$

$$u = \frac{1}{3}y^3 - y + C$$

b)  $\frac{dT}{dt} = -0.0002(T - 5) \lll = \text{linear}$

$$\frac{dT}{T-5} = -0.0002dt \implies \ln(T - 5) = -0.0002t + C_1$$

$$T = 5 + C e^{-0.0002t} \text{ where } C = e^{C_1}$$

c)  $\frac{dy(x)}{dx} = e^{2y} \lll = \text{nonlinear}$

$$e^{-2y} dy = dx \implies -\frac{1}{2}e^{-2y} = x + C$$

$$y = -\frac{1}{2} \ln(C - 2x)$$

d)  $\frac{du}{dt} = u^2 \cos(\pi t) \quad u(0) = -\frac{1}{2} \lll = \text{nonlinear}$

$$\int_{-\frac{1}{2}}^u \frac{du}{u^2} = \int_0^t \cos(\pi t) dt \implies \frac{1}{u} \Big|_{-\frac{1}{2}}^u = \frac{1}{\pi} \sin(\pi t) \Big|_0^t$$

$$u = \frac{1}{-\frac{1}{\pi} \sin(\pi t) - 2}$$

e)  $\frac{dy}{dt} = \left( \frac{y^2 + 4y - 5}{y} \right) t \lll = \text{nonlinear}$

$$\int \frac{y}{y^2 + 4y - 5} dy = \int t dt$$

Use Partial Fraction to simplify the first integrand.

$$\frac{y}{y^2 + 4y - 5} = \frac{A_1}{y-1} + \frac{A_2}{y+5}$$

and from here  $A_1 = \frac{1}{6}$  and  $A_2 = \frac{5}{6}$ . Therefore,

$$\int \frac{1/6}{y-1} dy + \int \frac{5/6}{y+5} dy = \int t dt$$

$$\frac{1}{6} \ln(y-1) + \frac{5}{6} \ln(y+5) = \frac{1}{2} t^2 + C$$

f)  $\frac{du}{dt} = u^3 + 6u^2 + 11u + 6$  <<== nonlinear

$$\int \frac{1}{u^3 + 6u^2 + 11u + 6} du = \int dt$$

Use Partial Fractions to simplify the first integrand.

$$\frac{1}{u^3 + 6u^2 + 11u + 6} = \frac{A_1}{u+1} + \frac{A_2}{u+2} + \frac{A_3}{u+3}$$

and from here  $A_1 = \frac{1}{2}$ ;  $A_2 = -1$ ; and  $A_3 = \frac{1}{2}$ . Therefore,

$$\int \frac{1/2}{u+1} du - \int \frac{1}{u+2} du + \int \frac{1/2}{u+3} du = \int dt$$

$$\frac{1}{2} \ln(u+1) - \ln(u+2) + \frac{1}{2} \ln(u+3) = t + C$$

### Problem 2.3

$$-\frac{10C^s}{12 + C^s} = \frac{dC^s}{dt}$$

Using Separation of Variables,

$$\int_{13.33}^{C^S} \frac{12+C^S}{10C^S} dC^S = 1.2 \int_{13.33}^{C^S} \frac{1}{C^S} dC^S + 0.1 \int_{13.33}^{C^S} dC^S = -\int_0^t dt$$

and after evaluating the integrals

$$t = 4.44 - 1.2 \ln C^S - 0.1 C^S$$

### Problem 2.4

$$\tau \frac{dy(t)}{dt} + y(t) = Kx(t)$$

$$\int_{y(0)}^{y(t)} \frac{dy(t)}{Kx(t) - y(t)} = \frac{1}{\tau} \int_0^t dt \Rightarrow \ln[Kx(t) - y(t)] \Big|_{y(0)}^{y(t)} = -\frac{t}{\tau}$$

$$\ln[Kx(t) - y(t)] - \ln[Kx(t) - y(0)] = -\frac{t}{\tau}$$

$$\frac{Kx(t) - y(t)}{Kx(t) - y(0)} = e^{-\frac{t}{\tau}}$$

$$y(t) = Kx(t) - [Kx(t) - y(0)]e^{-\frac{t}{\tau}}$$

and using  $x(t) = x(0) + D$ ,

$$y(t) = K(x(0) + D) - [K(x(0) + D) - y(0)]e^{-\frac{t}{\tau}}$$

and with  $y(0) = K x(0)$ ,

$$y(t) = y(0) + KD - [y(0) + KD - y(0)]e^{-\frac{t}{\tau}}$$

$$y(t) = y(0) + KD \left(1 - e^{-\frac{t}{\tau}}\right)$$

### Problem 2.5

$$2 \frac{dv}{dt} + 4v = 16u(t) \quad \text{with } v(0) = 0$$

$$\int_0^v \frac{dv}{16-4v} = \int_0^t \frac{1}{2} dt \Rightarrow -\frac{1}{4} \ln(16-4v) \Big|_0^v = \frac{1}{2} t$$

$$\ln(16 - 4v) - \ln(16) = -2t \Rightarrow \ln\left(\frac{16-4v}{16}\right) = -2t$$

$$v = 4(1 - e^{-2t})$$

### Problem 2.6

$$i_1 = 1.43 \times 10^{-4} v_s + 0.286 i_2 \quad (1)$$

$$2 \times 10^{-3} \frac{di_2}{dt} + 21428 i_2 = 0.286 v_s \quad (2)$$

From (2):  $9.3 \times 10^{-8} \frac{di_2}{dt} + i_2 = 1.33 \times 10^{-5} v_s$

where  $\tau = 9.3 \times 10^{-8} \text{ sec}$ , and  $K = 1.33 \times 10^{-5} \frac{V}{A}$

From Problem 2-5,  $i_2 = i_2(0) + KD \left(1 - e^{-\frac{t}{\tau}}\right)$

$$i_2 = 2.66 \times 10^{-4} + 0.000267 \left(1 - e^{-\frac{t}{9.3 \times 10^{-8}}}\right)$$

From (1):

$$i_1 = 57.2 \times 10^{-4} + 0.76 \times 10^{-4} + 7.64 \times 10^{-5} \left(1 - e^{-\frac{t}{9.3 \times 10^{-8}}}\right)$$

$$i_1 = 57.96 \times 10^{-4} + 0.76 \times 10^{-4} \left(1 - e^{-\frac{t}{9.3 \times 10^{-8}}}\right)$$

### Problem 2.7

$$0.3768X \left(1 - \frac{X}{18.91}\right) = \frac{dX}{dt}$$

Using Separation of Variables,

$$\frac{dX}{X \left(1 - \frac{X}{18.91}\right)} = 0.3768 dt$$

$$\frac{dX}{X(18.91 - X)} = 0.02 dt$$

Using Partial Fractions Expansion,

$$\frac{1}{X(18.91-X)} = \frac{A_1}{X} + \frac{A_2}{18.91-X}$$
$$\frac{1}{X(18.91-X)} = \frac{(A_2 - A_1)X + 18.91A_1}{X(18.91-X)}$$

Equating equal terms yields,

$$A_1 = A_2 = 0.053$$

Then,

$$0.053 \left[ \int_{9.52}^X \frac{dX}{X} + \int_{9.52}^X \frac{dX}{18.91-X} \right] = 0.02 \int_0^t dt$$
$$\ln \left[ \frac{0.986X}{18.91-X} \right] = 0.377t$$
$$X = \frac{18.91e^{0.377t}}{0.986 + e^{0.377t}}$$

### Problem 2.8

$$6.915C_v \sqrt{1000 - P(t)} = 1.08 \frac{dP(t)}{dt}$$

Using Separation of Variables

$$\frac{dP(t)}{\sqrt{1000 - P(t)}} = \frac{6.915C_v}{1.08} dt$$

And using the information given in the statement

$$\int_{100}^{500} \frac{dP(t)}{\sqrt{1000 - P(t)}} = \frac{6.915C_v}{1.08} \int_0^4 dt$$
$$-2\sqrt{1000 - P(t)} \Big|_{100}^{500} = \frac{6.915C_v}{1.08} t \Big|_0^4$$

And solving for  $C_v$ ,

$$C_v = 0.596$$

### Problem 2.9

$$\frac{dC^A}{dt} = -85.28(C^A)^2(0.132 + 0.5C^A)$$

Using Separation of Variables,

$$\frac{dC^A}{(C^A)^2(0.132 + 0.5C^A)} = -85.28 dt$$

Using partial fractions expansion we factor the term on the left side

$$, \frac{1}{(C^A)^2(0.132 + 0.5C^A)} = \frac{A_1}{(C^A)^2} + \frac{A_2}{C^A} + \frac{A_3}{0.132 + 0.5C^A}$$

From here

$$\frac{1}{(C^A)^2(0.132 + 0.5C^A)} = \frac{(A_3 + 0.5A_2)(C^A)^2 + (0.5A_1 + 0.132A_2)C^A + 0.132A_1}{(C^A)^2(0.132 + 0.5C^A)}$$

and equating equal numerator coefficients  $A_1 = 7.58$ ;  $A_2 = -28.71$ ;  $A_3 = 14.36$

Then,

$$7.58 \int_{1.842}^{C^A} \frac{dC^A}{(C^A)^2} - 28.71 \int_{1.842}^{C^A} \frac{dC^A}{C^A} + 14.36 \int_{1.842}^{C^A} \frac{dC^A}{0.132 + 0.5C^A} = -85.28 \int_0^t dt$$

And after evaluating the integrals,

$$t = 0.18 h$$

### Problem 2.10

$$12.5 \frac{dx_3^{NaOH}}{dt} + x_3^{NaOH} = 0.73 x_1^{NaOH} \quad \text{with } x_3^{NaOH}(0) = 0.55 \text{ and } x_1^{NaOH} = 0.75 - 0.08 u(t)$$

$$12.5 \frac{dx_3^{NaOH}}{dt} = 0.73 x_1^{NaOH} - x_3^{NaOH} = 0.73(0.67) - x_3^{NaOH}$$

or

$$12.5 \frac{dx_3^{NaOH}}{dt} = [0.4891 - x_3^{NaOH}](1)$$

Using Separation of Variables

$$\int_{0.55}^{x_3^{NaOH}} \frac{dx_3^{NaOH}}{0.4891 - x_3^{NaOH}} = \frac{1}{12.5} \int_0^t dt \Rightarrow \ln(0.4891 - x_3^{NaOH}) \Big|_{0.55}^{x_3^{NaOH}} = -\frac{t}{12.5}$$

$$\frac{0.4891 - x_3^{NaOH}}{-0.0609} = e^{-\frac{t}{12.5}} \Rightarrow x_3^{NaOH} = 0.4891 + 0.0609e^{-\frac{t}{12.5}}$$

and

$$x_3^{NaOH} = 0.4891 + 0.0609e^{-\frac{t}{12.5}} = 0.55 - 0.0609(1 - e^{-\frac{t}{12.5}})$$

### Problem 2.11

a)

$$m \frac{dv}{dt} = \sum F_y = F_g + F_d + F_T = -mg - kv + 2u(t)$$

$$0.05 \frac{dv}{dt} = -0.05(9.8) - 0.01v + 2u(t)$$

$$0.05 \frac{dv}{dt} + 0.01v = -0.49 + 2u(t) \quad \text{with } v_y(0) = 0 \text{ m/s}$$

Using Separation of Variables

$$\int_0^{v_y} \frac{1}{1.51 - 0.01v_y} dv_y = \int_0^t 20 dt$$

and

$$v_y(t) = 151(1 - e^{-0.2t}) \rightarrow v_y(2) = 151(1 - e^{-0.4}) = 49.8 \text{ m/s}$$

b)

$$m \frac{dv}{dt} = \sum F_y = F_g + F_d = -mg - kv$$

$$0.05 \frac{dv}{dt} + 0.01v = -0.49$$

Using Separation of Variables

$$\int_{49.8}^0 \frac{1}{-0.49 - 0.01v_y} dv_y = \int_2^t 20 dt$$
$$-\frac{1}{0.01} \ln(-0.49 - 0.01v_y) \Big|_{49.8}^0 = 20(t - 2)$$

and using algebra

$$t = 2 - 5 \ln \left( \frac{-0.49}{-0.49 - 0.01(49.8)} \right) = 5.5 \text{ s}$$

Thus, after the thrusts stop, the rocket continues upward for 3.5 seconds more.

### Problem 2.12

a)

$$m \frac{dv}{dt} = -kv \quad \text{with} \quad v(0) = 50 \text{ m/s}$$

$$\frac{dx}{dt} = v \quad \text{with} \quad x(0) = 0 \text{ m}$$

b)

$$\frac{dv}{dt} = -\frac{100}{500} v = -0.2v \Rightarrow \int_{50}^v \frac{dv}{v} = -0.2 \int_0^t dt \Rightarrow \ln v \Big|_{50}^v = -0.2t$$

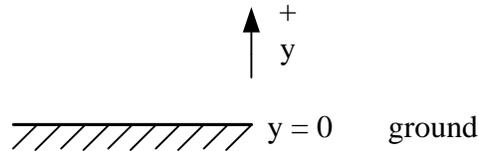
$$v = 50e^{-0.2t} \quad \text{and for} \quad v = 1 \text{ m/s}, \quad t = 19.5 \text{ s}$$

c)

$$\int_0^x dx = \int_0^{19.5} v dt = 50 \int_0^{19.5} e^{-0.2t} dt$$
$$x = -\frac{50}{0.2} e^{-0.2t} \Big|_0^{19.5} = 245 \text{ m}$$

## Problem 2.13

### Going Up



$$\sum F_y = ma_y = m \frac{dv_y}{dt} \quad (1) \text{ 1 eq 2 unk } [\sum F_y, v_y]$$

Assuming no air resistance:  $\sum F_y = F_g = -mg \quad (2) \text{ 2 eq 2 unk}$

Substituting (2) into (1) and rearranging,

$$\frac{dv_y}{dt} = -g \quad (3) \quad \text{with } v_y(0) = 35 \text{ m/s}$$

Maximum height is when  $v_y = 0 \frac{m}{s}$

From (3)

$$\int_{35}^0 dv_y = -g \int_0^t dt \Rightarrow v_y|_{35}^0 = -gt|_0^t \Rightarrow 0 - 35 = -gt$$
$$t = 3.57 \text{ sec}$$

We can also obtain an expression for  $v_y$

$$\int_{35}^{v_y} dv_y = -g \int_0^t dt$$
$$v_y = 35 - gt = 35 - 9.8t$$

For the maximum height,

$$\frac{dy}{dt} = v_y \Rightarrow \int_0^y dy = \int_0^t v_y dt = \int_0^t (35 - 9.8t) dt$$
$$y = 35t - 4.9t^2$$

Then, at  $t = 3.57 \text{ s}$   $y_{max} = 35(3.57) - 4.9(3.57)^2$

$$y_{max} = 62.5 \text{ m}$$

### Going Down

$$\sum F_y = ma_y = m \frac{dv_y}{dt} \quad (4) \text{ 1 eq., 2 unk } [\sum F_y, v_y]$$

Assuming no air resistance,

$$\sum F_y = F_g = -mg \quad (5) \text{ 2 eq., 2 unk.}$$

$$\frac{dy}{dt} = v_y \quad (6) \text{ 3 eq 3 unk } [y]$$

Substituting (5) into (4) and rearranging,

$$\frac{dv_y}{dt} = -g \implies \int_0^{v_y} dv_y = -g \int_0^t dt \implies v_y = -gt \quad (7)$$

Using (7) in (6):

$$\frac{dy}{dt} = -gt \implies \int_{62.5}^y dy = -g \int_0^t t dt \implies y = 62.5 - 4.9t^2 \quad (8)$$

For  $t, y = 0$

$$t = 3.57 \text{ sec}$$

### **Problem 2.14**

$$\sum F_y = ma_y = m \frac{dv_y}{dt} \quad (1) \text{ 1 eq., 2 unk } [\sum F_y, v_y]$$

Considering air resistance (drag force due to air)

$$\sum F_y = F_g + F_d = -mg - 1.0|v_y|v_y \quad (2) \text{ 2 eq., 2 unk.}$$

Eqs (1) and (2) constitute the model for the velocity. For the position,

$$\frac{dy}{dt} = v_y \quad (3) \text{ 3 eq 3 unk } [y]$$

Units of 1 are  $\frac{N}{m^2/s^2}$ .

### Problem 2.15

$$\sum F_y = ma_y = m \frac{dv_y}{dt} \quad (1) \text{ 1 eq., 3 unk } [\sum F_y, m, v_y]$$

Considering air resistance

$$\sum F_y = F_g + F_d = -mg - 1.0v_y \quad (2) \text{ 2 eq., 3 unk}$$

$$\frac{dy}{dt} = v_y \quad (3) \text{ 3 eq., 4 unk } [y]$$

We need one more equation; at this time there is still one degree of freedom. We don't really have any more equations but, there is a specification  $y(0) = 30 \text{ m}$  and after 3 sec,  $y(3) = 0 \text{ m}$ , that we may use. So:

$$y(3) = 0 \text{ m} \quad (4) \text{ 4 eq., 4 unk}$$

Substituting (2) into (1):

$$m \frac{dv_y}{dt} = -mg - 1.0v_y = -1(mg + v_y)$$

and using Separation of Variables,

$$\int_0^{v_y} \frac{dv_y}{mg+v_y} = -\frac{1}{m} \int dt \Rightarrow \ln(mg + v_y) \Big|_0^{v_y} = -\frac{t}{m}$$

$$\ln(mg + v_y) - \ln(mg) = -\frac{t}{m} \Rightarrow \ln\left(\frac{mg+v_y}{mg}\right) = -\frac{t}{m}$$

$$v_y = mg \left( e^{-\frac{t}{m}} - 1 \right) \quad (5)$$

Substituting (5) into (3):  $\frac{dy}{dt} = mg \left( e^{-\frac{t}{m}} - 1 \right)$

$$\int_{30}^0 dy = mg \int_0^3 \left( e^{-\frac{t}{m}} - 1 \right) dt \Rightarrow 0 - 30 = \left( -m^2 g e^{-\frac{t}{m}} - mgt \right) \Big|_0^3$$

$$-30 = -m^2 g e^{-\frac{3}{m}} + m^2 g - 3mg$$

By trial and error:

$$m \approx 2.3 \text{ kg}$$

**Problem 2.16**

$$\rho A \frac{dh}{dt} + C_v \sqrt{h} = w_1 + w_2$$

When both inlet flows are shut-off,  $w_1 = w_2 = 0$ . So,

$$\rho A \frac{dh}{dt} + C_v \sqrt{h} = 0 \Rightarrow \int_{3.24}^h \frac{dh}{\sqrt{h}} = -\frac{C_v}{\rho A} \int_0^t dt \Rightarrow 2\sqrt{h}|_{3.24}^h = -\frac{C_v}{\rho A} t|_0^t$$

$$h = (1.8 - 0.571t)^2$$

Drain tank:  $h = 0$

$$t = 3.15 \text{ min}$$

**Problem 2.17**

a) For the first part, assume there is no chute that is, no drag force. In that case the model is

$$m \frac{dv}{dt} = -F_{brake} \Rightarrow \frac{dv}{dt} = -F_{brake} / m = -16200 / 900 = -18 \text{ m/s}^2$$

Integrating once:

$$v = v_0 - 18t \quad (1)$$

Integrating again:

$$x = x_0 + v_0 t - 9t^2 \quad (2)$$

To find the stopping distance, set  $v_0 = 120$ ,  $v = 0$ , and  $x_0 = 0$ . Equation 1 is rearranged to:

$$t = (v_0 - v) / 18 = 120 / 18 = 6.6666 \text{ sec}$$

This is the stopping time in the absence of a parachute. Substituting for  $t$ ,  $v_0$  and  $x_0$  in Equation 2 yields the stopping distance:

$$x = x_0 + v_0 t - 9 t^2 = 0 + 120 * (6.6666) - 9 * (6.6666)^2 = 400 \text{ m}$$

This is twice the desired stopping distance so a parachute is needed.

**b)** Now the model is

$$m \frac{dv}{dt} = -F_{brake} - \frac{1}{2} \rho C_D A v^2 \quad v(0) = 120$$

$$\frac{dx}{dt} = v \quad x(0) = 0$$

The goal is to find the chute area A that will result in a velocity of zero when the position is 200 m. Then the chute diameter d can be found from  $A = \pi d^2 / 4$ . This problem can be solved analytically but it is somewhat tedious. Better yet is to solve it using simulation, Chapter 11.

### Problem 2.18

a) No parachute

$$m \frac{dv_y}{dt} = F_d = -mg \quad \text{with} \quad v_y(0) = 0 \text{ m/s} \quad (1)$$

1 equation, 1 unknown [ $v_y$ ]

$$v_y = \frac{dy}{dt} \quad \text{with} \quad y(0) = y_{initial} \text{ m} \quad (2)$$

2 equations, 2 unknowns [ $y$ ]

b) After 10 seconds

From Equation 1, using Antidifferentiation,

$$\int_0^{v_y} dv_y = -9.8 \int_0^t dt \Rightarrow v_y = -9.8t \text{ m/s}$$

and at  $t = 10 \text{ s}$ ,  $v_y|_{t=10} = -98 \text{ m/s}$ . From Equation 2,

$$\int_{y_{initial}}^y dy = \int_0^t v_y dt \Rightarrow y = y_{initial} - 9.8 \int_0^t t dt = y_{initial} - 4.9t^2$$

and at  $t = 10s$ , the distance covered is  $y|_{t=10} = y_{initial} - 490m$ .

c) Open parachute

$$m \frac{dv_y}{dt} = \Sigma F = -mg - P v_y \quad \text{with} \quad v_y(0) = v_y|_{t=10} = -98m/s \quad (3)$$

1 equation, 1 unknown [ $v_y$ ]

$$v_y = \frac{dy}{dt} \quad \text{with} \quad y(0) = y|_{t=10} = y_{initial} - 490m \quad (4)$$

2 equations, 2 unknowns [ $y$ ]

From Equation 3, and using Separation of Variables,

$$\int_{-98}^{v_y} \frac{dv_y}{mg + P v_y} = -\frac{1}{m} \int_{10}^t dt \Rightarrow \frac{1}{P} \ln \left( \frac{mg + P v_y}{mg - 98P} \right) = -\frac{1}{m} (t-10) \Rightarrow v_y = \frac{1}{P} \left[ (mg - 98P) e^{-\frac{P}{m}(t-10)} - mg \right]$$

$$v_y = \frac{1}{200} \left[ (980 - 19600) e^{-2(t-10)} - 980 \right] = -93.1 e^{-2(t-10)} - 4.9 \quad (5)$$

From Equations 4 and 5,

$$\int_{y_{initial}-490}^y dy = \int_{10}^t v_y dt = \int_{10}^t (-93.1 e^{-2(t-10)} - 4.9) dt$$

$$y = y_{initial} - 490 + \left[ 46.55 e^{-2(t-10)} - 4.9t \right] \Big|_{10}^t = y_{initial} - 487.55 + \left[ 46.55 e^{-2(t-10)} - 4.9t \right] \quad (6)$$

From Equation 5 we obtain the time it takes to reach a velocity of -5 m/s. This time is from the moment the person jumps from the building and equals 13.42 s.