

Solutions to Introduction to Probability With Texas Hold'em Examples by Frederic Paik Schoenberg.

1.2. By Axiom 2,  $P(\text{Wasicka wins}) = 1 - 60/89 - 11/89 = 18/89$ .

1.4. Answers may vary, but this is most likely a Bayesian interpretation of probability because one will not be in the same situation, with the same number of chips, against the same opponents, multiple times.

1.5.  $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$ .

1.6. Let  $B_1 = A_2$ , and for  $i = 2, 3, \dots$ , let  $B_i = A_i A_1^c A_2^c \dots A_{i-1}^c$ .

Observe that  $(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n) = (B_1 \text{ or } B_2 \text{ or } \dots \text{ or } B_n)$ , because for any element  $w$  in  $A_1$  or  $\dots$  or  $A_n$ , there must be a first  $A_i$  that it is in, and in this case  $w$  is in  $B_i$ . Also, notice that  $B_i$  are mutually exclusive.

So,  $P(A_1 \text{ or } \dots \text{ or } A_n) = P(B_1 \text{ or } B_2 \text{ or } \dots \text{ or } B_n) = P(B_1) + P(B_2) + \dots + P(B_n)$ .

For each  $i$ ,  $B_i$  is a subset of  $A_i$ , so  $A_i B_i = B_i$ . And,  $A_i = A_i B_i \cup A_i B_i^c = B_i \cup A_i B_i^c$ , and  $B_i$  and  $A_i B_i^c$  are mutually exclusive, so

$P(A_1) + P(A_2) + \dots + P(A_n) = P(B_1) + P(A_1 B_1^c) + P(B_2) + P(A_2 B_2^c) + \dots + P(B_n) + P(A_n B_n^c)$

$\geq P(B_1) + P(B_2) + \dots + P(B_n)$ , since the other probabilities  $P(A_i B_i^c)$  must be non-negative by Axiom 1,

$= P(A_1 \text{ or } A_2 \text{ or } \dots \text{ or } A_n)$ .

2.1. Each player is equally likely to win the pot, and ignoring split pots, someone must win the pot, so the probability of player A winning the pot is simply  $1/5$ .

2.2.  $P(\text{AK, AQ, AK, A10, or a pocket pair}) = (4 \times 4 \times 4 + 13 \times C(4,2))/C(52,2) \sim 10.71\%$ .

2.4.  $P(\text{flop a straight or royal flush}) = 4 \times 10 / C(52,5) = 0.00154\%$ .

2.6.  $P(\text{flop a flush}) = 4 C(13,5) / C(52,5) \sim 0.198\%$ .

2.8.  $P(\text{make a flush}) = (3 \times C(13,5) + C(11,5) + C(11,4) \times 39 + C(11,3) \times C(39,2)) / C(50,5) = 6.58\%$

2.10.  $P(\text{rainbow flop}) = C(4,3) \times 13 \times 13 \times 13 / C(52,3) \sim 39.76\%$ . Alternatively, you could find this as  $1 \times 39/51 \times 26/50 \sim 39.76\%$ .

2.12.  $C(12,2) / C(52,2) = 4.98\%$ .

2.14.  $P(2 \text{ face cards but not a pocket pair}) = (C(12,2) - 3 \times C(4,2)) / C(52,2) \sim 3.62\%$ .

$$2.16. (2 \cdot C(48,2) + 12 \cdot C(4,3)) / C(50,3) = 11.8\%.$$

2.18. QQQ87. You have QQ and the board is Q8732.

2.20. Even with AA and AAX on the board, someone can possibly make a straight flush. So, the only possibilities are straight flushes where you have

5A of a 5 high straight flush = 1 possibility,

62, 63, 64, or 65 of a 6 high straight flush = 4 possibilities,

..., T6, T7, T8, or T9 on a T high straight flush = 4 possibilities,

or Jx or Tx on a J high straight flush = 7 possibilities,

or any part but 89 on a Q high straight flush = 9 possibilities,

or any part on a K high straight flush = 10 possibilities,

or any part on an A high straight flush = 10 possibilities,

=  $4 \cdot (1 + 20 + 7 + 9 + 10 + 10) / C(52,5) / C(5,2) = 1$  in 113989.5, or about 1 in 113990.

$$3.2. a) O_{AB'} = P\{(AB)'\} / P(AB) = \{1 - P(A)P(B)\} / \{P(A)P(B)\} = 1 / \{P(A)P(B)\} - 1.$$

$$b) O_{A'} = P(A') / P(A) = 1 / P(A) - 1, \text{ so } 1 / P(A) = O_{A'} + 1. \text{ And, } 1 / P(B) = O_{B'} + 1.$$

$$\text{So, } O_{AB'} = 1 / \{P(A)P(B)\} - 1$$

$$= [1 / P(A)] [1 / P(B)] - 1$$

$$= [1 + O_{A'}] [1 + O_{B'}] - 1.$$

Note that there might be slightly different correct answers. For instance, in 3.2a, the correct answer can also be written as  $\{1 - P(A)P(B)\} / \{P(A)P(B)\}$ .

3.4. To see that  $P(A_i | B)$  satisfies the three axioms,

(i)  $P(A_i | B) = P(A_i B) / P(B) \geq 0$ , since both  $P(A_i B)$  and  $P(B)$  must be  $\geq 0$  by Axiom 1.

(ii)  $A_i B$  and  $A_i^c B$  are mutually exclusive, and  $A_i B \cup A_i^c B = B$ , so by Axiom 3,  $P(A_i B) + P(A_i^c B) = P(A_i B \cup A_i^c B) = P(B)$ . Thus,  $P(A_i | B) + P(A_i^c | B) = P(A_i B) / P(B) + P(A_i^c B) / P(B) = (P(A_i B) + P(A_i^c B)) / P(B) = P(B) / P(B) = 1$ .

(iii) If  $A_i$  are mutually exclusive, then so are  $A_i B$ , so  $P(A_1 \text{ or } \dots \text{ or } A_n | B) = P((A_1 \text{ or } \dots \text{ or } A_n) B) / P(B) = P(A_1 B \text{ or } \dots \text{ or } A_n B) / P(B) = (P(A_1 B) + \dots + P(A_n B)) / P(B)$ , by Axiom 3,  $= P(A_1 | B) + \dots + P(A_n | B)$ .

3.6.  $A$  = both face cards,  $B$  = pair.

$$P(A) = C(12,2) / C(52,2) = 11/221.$$

$$P(B) = 3/51.$$

$$\text{So, } P(A)P(B) = (11/221) \times (3/51) \sim 0.293\%.$$

$$P(AB) = P(\text{you have a pair of face cards}) = 3 \times C(4,2) / C(52,2) \sim 1.36\%.$$

So,  $A$  and  $B$  are not independent.

This one seems weird at first, but it makes sense if you think about it. Given that you have two face cards, there are fewer possibilities for the numbers on your two cards so it's a lot more likely that you have a pair.