PROBLEMS

2.1 What is the probability when throwing a die three times of getting a four in any of the throws?

Answer: The probability of getting a four in one throw is: $f(4) = \frac{1}{6}$. Using the binomial distribution, the probability to have one 4 in 3 throws is:

$$\text{P(only one 4)} = \frac{3!}{(3-1)!\,1!}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{2} = \frac{75}{216} = 0.3472$$

The same result can be obtained by enumerating all possible events.

Throw	1	2	3
Outcome	4	No – 4	No – 4
	No – 4	4	No – 4
	No – 4	No – 4	4

To throw only one 4:

$$\text{P(only and 4)} = \frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6}+\frac{5}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}+\frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} = \frac{75}{216}$$

Note: The probability to have at least one four is larger

P(at least one 4) = P(one 4) + P(two 4's) + P(three 4's) =

$$\frac{75}{216}+\frac{3!}{(3-2)!\,2!}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{1}+\frac{3!}{(3-3)!\,3!}\left(\frac{1}{6}\right)^{3}\left(\frac{5}{6}\right)^{2} = \frac{91}{216} = 0.4213$$

Using possible outcomes, one should extend the table given above, as follows:

4	4	No – 4	$\rightarrow$	Prob. $= \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6}$	$\frac{15}{16}$
4	No – 4	4	$\rightarrow$	Prob. $= \frac{1}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$	$\frac{5}{16}$
No – 4	4	4	$\rightarrow$	Prob. $= \frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$	$\frac{5}{16}$
4	4	4	$\rightarrow$	$\frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6}$	$\frac{1}{216}$

$$\text{Total Probability} = \frac{75}{216}+3\times\frac{5}{216}+\frac{1}{216} = \frac{91}{216}$$

2.2 What is the probability when drawing one card from each of three decks of cards that all three cards will be diamonds?

Answer: The probability of drawing a diamond from one deck is $\frac{13}{52}$. The probability that this event happens three times is (using Equation 2.15):

$$\text{P}(3-\text{diamonds}) = \frac{13}{52}\cdot\frac{13}{52}\cdot\frac{13}{52} = (0.25)^3 = 0.0156$$

2.3 A box contains 2000 computer cards. If five faulty cards are expected to be found in the box, what is the probability of finding two faulty cards in a sample of 250?

Answer: The probability of finding a faulty card is P(faulty card) = $\frac{5}{2000} = 0.0025$. The probability of finding two faulty cards in a sample of 250 is (using Binomial distribution, Equation 2.45):

$$\frac{250!}{(250-2)!\,2!}(0.0025)^2(1-0.0025)^{250-2} = \frac{249\times 250}{2}(0.0025)^2(0.9975)^{248}$$
$$= 31125 \times 6.25\times 10^{-6} \times (0.5375) = 0.104$$

2.4 Calculate the average and the standard deviation of the probability density function $f(x) = 1/(b-a)$ when: $a \le \times \le b$. (This probability distribution function is used for the calculation to round off errors.)
Answer: Use Equation 2.27, or in this case Equation 2.26, for $\bar{x}$. Use Equation 2.33 for r^2:

$$\bar{x} = \int_a^b x \frac{1}{b-a} dx = \frac{b+a}{2}$$

$$\sigma^2 = \int_a^b x(x-\bar{x})^2 \frac{1}{b-a} dx = \frac{(b-a)^2}{12}, \quad \sigma = \frac{b-a}{\sqrt{12}}$$

2.5 The energy distribution of thermal (slow) neutrons in a light-wave reactor follows very closely the Maxwell-Boltzmann distribution:

$$N(E)dE = A\sqrt{E}e^{-E/kT}dE$$

Where:
N(E) dE = number of neutrons with kinetic energy between *E* and *E + dE*
k = Boltzmann constant = 1.380662 × 10^{-23}J/° K
T = temperature, *K*
A = constant
Show that:

(a) The mode of this distribution is: $E = \frac{1}{2}kT$

(b) The mean is: $\bar{E} = \frac{3}{2}kT$

Answer:
(a) Use Equation 2.24

$$\frac{dN(E)}{dE} = 0 = A\frac{1}{2}E^{\frac{1}{2}}e^{\frac{-E}{kT}} - AE^{\frac{1}{2}}\frac{1}{kT}e^{\frac{-E}{kT}} = Ae^{\frac{-E}{kT}}\left(\frac{1}{\sqrt[2]{E}} - \frac{\sqrt{E}}{kT}\right)$$

Or:

$$E_{\text{mode}} = \frac{1}{2}kT$$

(b) Use Equation 2.27

$$\bar{E} = \frac{\int_0^\infty EA\sqrt{Ee}^{E/kT}dE}{\int_0^\infty A\sqrt{Ee}^{-E/kT}dE} = \frac{3}{2}kT$$

2.6 If the average for a large number of counting measurements is 15, what is the probability that a single measurement will produce the result 20?
Answer: Use Poisson statistics, Equation 2.50:

$$P(20) = \frac{15^{20}}{20!}e^{-15} = \frac{3.325\times 10^{23}\times 3.159\times 10^{-7}}{2.433\times 10^{18}} = 4.18\times 10^{-2} \text{ or } 4.18\%$$

2.7 For the binomial distribution, prove:

$$(a)\sum_{n=0}^{N} p_N^n = 1 \quad (b)\,\overline{n} = pN \quad (c)\,\sigma^2 = m(1-p)$$

Answer:

(a)

$$\sum_{n=0}^{N} p_N^n = 1 \quad \sum_{N=0}^{N} \frac{N!}{(N-n)!\,n!} p^n (1-p)^{N-n} = 1$$

Because the binomial distribution represents the nth term of the binomial expansion $(x+y)^N$. The nth term is:

$$\frac{N(N-(n-2))(N-(n-1))}{n!} x^{N-n} \cdot y^n = \text{(multiply and divide by (N-n)!)} = \frac{N!}{(N-n)!\,n!} \cdot x^{N-n} \cdot y^n$$

In the present case: $x=p$, $y = 1\text{-}p$

$$\sum_{n=0}^{N} p_N^{(n)} = (p+(1-p))^N = 1^N = 1$$

(b)

$$\overline{n} = \sum_{n=0}^{N} np_N^{(n)} = \sum_n n \frac{N!}{(N-n)!\,n!} p^n (1-p)^{N-n} = Np \sum_{n=0}^{N} \frac{(N-1)^y}{(n-n)!\,(n-1)!} = Np(p+1-p)^{N-1} = Np$$

(c)

$$\sigma^2 = \sum_{n=0}^{N} (m-n)^2 p_N^{(n)} = m^2 \sum_n p_N^{(n)} - 2m \sum_n np_N^{(n)} + \sum_n n^2 p_N^{(n)} = m^2 - 2m \cdot m + \sum_n n^2 p_N^{(n)} = \overline{n^2} - m^2$$

$$\overline{n^2} = \sum_n n^2 p_N^{(n)} = \sum (n(n-1)+n) p_N^{(n)} = \sum n(n-1) p_N^{(n)} + \sum_n np_N^{(n)}$$

$$= \sum_n n(n-1) \frac{N!}{(N-n)!\,n!} p^n (1-p)^{N-n} + m = N(N-1)p^2 \sum_n \frac{(n-2)!}{(N-n)!(n-2)!} p^{n-2} (1-p)^{N-2} + m$$

$$= N(N-1)p^2 (p+1-p)^{N-2} + m = N(N-1)p^2 + m$$

Combining terms

$$\sigma^2 = N(N-1)p^2 + m - m^2 = N^2p^2 - Np^2 + Np - N^2p^2 = Np(1-p) = m(1-p)$$

$$\sigma = \sqrt{m(1-p)}$$

2.8 For the Poisson distribution, prove

$$(a)\sum_{n=0}^{\infty} P_n = 1 \quad (b)\,\overline{x} = m \quad (c)\,\sigma^2 = m$$

Answer:

(a) $\sum_{n=0}^{\infty} Pn = \sum_n \frac{m^n}{n!} e^{-m} = e^{-m} \sum_n \frac{m^n}{n!} = e^{-m} e^{-m} = 1$

(b) $m = \sum_n nPn = \sum_n n \frac{m^n}{n!} e^{-m} = e^{-m} \sum_n \frac{m^n}{(n-1)!} = me^{-m} e^m = m$

(c) $\sigma^2 = \sum_n (m-n)^2 Pn = m^2 \sum P_n - 2m \sum nP_n + \sum n^2 P_n$

$$= m^2 - 2m^2 + \overline{n^2} = \overline{n^2} - m^2$$

$$\overline{n^2} = \sum_n n^2 P_n = \sum_n (n(n-1)+n) P_n = \sum_n n(n-1) \frac{m^n}{n!} e^{-m} + \sum_n nP_n$$

$$= m^2 e^{-m} \sum_n \frac{m^{n-2}}{(n-2)!} + m - m^2 + m$$

Combining terms:

$$\sigma^2 = m^2 + m - m^2 = m$$
$$\sigma = \sqrt{m}$$

2.9 For the normal distribution, show

$$\text{(a)} \quad \int_{-\infty}^{\infty} P(x)dx = 1 \quad \text{(b)} \ \bar{x} = m \quad \text{(c) the variance is } \sigma^2$$

Answer:

(a)

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m))^2}{2\sigma^2}} dx = 1$$

(Look up tables; integral is of the form $\int_0^{\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}$)

(b)

$$\bar{x} = \int_{-\infty}^{\infty} \frac{x}{\sqrt{2\pi}\sigma} e^{\frac{-(x-m)^2}{2\sigma^2}} dx$$

Change variable to $y = x - m$

$$\bar{x} = \int_{-\infty}^{\infty} \frac{(y+m)}{\sqrt{2\pi}\sigma} e \frac{-y}{2\sigma^2} \ dy = m \int_{-\infty}^{\infty} \frac{e^{\frac{-y^2}{2\sigma 2}} dx}{\sqrt{2\pi}\sigma} + \int_{-\infty}^{\infty} \frac{ye^{\frac{-y^2}{2\sigma 2}}}{\sqrt{2\pi}\sigma} dy$$

$$= m + zero(add\ function) = m$$

(c)

$$\sigma^2 = \int_{-\infty}^{\infty} (x-m)^2 P(x)dx = \int_{-\infty}^{\infty} x^2 P(x)dx - 2m \int_{-\infty}^{\infty} xP(x)dx + m^2 \int_{-\infty}^{\infty} P(x)$$
$$= \overline{x^2} - 2m^2 + m^2 = \overline{x^2} - m^2$$

$$\overline{x^2} = \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}} dx \qquad \text{set} \quad \frac{(x-m)}{\sqrt{2}\sigma} = y\,, \ x = \sqrt{2}\sigma y + m$$

$$= \int_{-\infty}^{\infty} \left(\sqrt{2}\sigma y + m\right)^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-y^2} \sqrt{2}\sigma dy = \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} y^2 e^{-y^2} dy + \frac{2\sqrt{2}\sigma}{\sqrt{\pi}} \int_{-\infty}^{\infty} ye^{-y^2} dy + \frac{m^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} 2 \frac{\sqrt{\pi}}{4} + (0) + \frac{m^2}{\sqrt{\pi}} 2 \frac{1}{2} \sqrt{\pi} = \sigma^2 + m^2$$

Combining terms:

$$\sigma^2 = \sigma^2 + m^2 - m^2 = \sigma^2$$

2.10 If n_1, n_2,...,n_N are mutually uncorrelated random variables with a common variance σ^2, show that

$$\overline{\left(n_i - \bar{n}\right)^2} = \frac{N-1}{N} \sigma^2$$

Answer:

Consider the expression $n - \bar{n}$ as being of the form $\sum_{j=1}^{N} a_j n_j$

(Equation 2.36a) with:

$$a_j = \frac{1}{N}\Big|i \neq j\,,\; a_i = 1 - \frac{1}{N}\,,$$

Using Equation 2.41:

$$\overline{(n_i - \bar{n})^2} = \sum_{j=1}^{N} a_j{}^2 \sigma_i{}^2 = \sigma^2\left[\sum_{i=1}^{N-1}\frac{1}{N^2} + \left(1-\frac{1}{N}\right)^2\right] = \sigma^2\left[\frac{1}{N^2}(N-1) + \left(1-\frac{1}{N}\right)^2\right] = \frac{N-1}{N}\sigma^2$$

2.11 Show that in a series of N measurements, the result R that minimizes the quantity

$$Q = \sum_{i=1}^{N}(R-n_i)^2$$

is $R = \bar{n}$, where $\bar{n}$ is given by Equation 2.31.

Answer: To find the minimum of Q, solve $\frac{\partial \alpha}{\partial R} = 0$.

$$\frac{\partial Q}{\partial R} = 2\sum_{i=1}^{N}(R-n_i) = 0 \rightarrow \sum R = \sum n_i\,, NR = \sum_{i=1}^{N} n_i$$

$$R = \frac{1}{N}\sum_{i}^{N} n_i$$

2.12 Prove Equation 2.62 using tables of the error function. The table of error functions can be found http://www-dsp.elet.polimi.it/fondstiol/comperrfnc.pdf.

Answer: Equation 2.62 can be written as:

$$A = \int_{m-\sigma}^{m+\sigma} G(x)dx = \int_{m-\sigma}^{m+\sigma}\frac{dx}{\sqrt{2\pi}\sigma} e^{\frac{(x-m)^2}{2\sigma^2}}$$

Call: $\frac{x-m}{\sigma} = y,\; x = \sigma y + m$

$$A = \int_{-F}^{1}\frac{\sigma dy}{\sqrt{2\pi}\sigma} e^{\frac{-y^2}{2}} = \frac{2}{\sqrt{2\pi}}\int_{0}^{1} e^{\frac{-y^2}{2_{dy}}}$$

In tables, one finds the integral:

$$\int_0^t \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy = \frac{1}{2} erf\left(\frac{t}{\sqrt{2}}\right)$$

$$2\int_0^1 \frac{dy}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy = 2 \times 0.3413 = 0.6826 \approx 0.683$$

2.13 As part of a quality control experiment, the lengths of 10 nuclear fuel rods have been measured with the following results in meters:

2.60	2.62	2.65	2.58	2.61
2.62	2.59	2.59	2.60	2.63

What is the average length? What is the standard deviation of this series of measurements?

Answer: Average length:

$$\bar{\ell} = \sum_{i=1}^{10}\frac{\ell i}{10} = 2.609\ m$$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{10}\left(\ell_i - \bar{\ell}\right)^2 = \frac{1}{9}(0.00409) = 0.000454, \sigma = 0.021\ m$$

2.14 The average number of calls in a 911 switchboard is 4 calls/hour. What is the probability to receive 6 calls in the next hour?

Answer:

$$p_n = \left(\frac{m^n}{n!}\right)e^{-m} = \left(\frac{4^6}{6!}\right)e^{-4} = 0.10 = 10\%$$

2.15 At a uranium pellet fabrication plant the average pellet density is 17×10^3 kg/m^3 with a standard deviation equal to 10^3 kg/m^3. What is the probability that a given pellet has a density less than 14×10^3 kg/m^3?

Answer: We have to find the probability that the density $\rho < 14\times10^3 \frac{kg}{m^3}$ with $\bar{\rho} = 17\times10^3 \frac{kg}{m^3}$ and $\sigma = 10^3 \frac{kg}{m^3}$ (See section 2.14).

$$\rho = \bar{\rho} - (17-14)\times10^3 = 17\times10^3 - 3\times10^3 = \bar{\rho} - 3\sigma$$

$$P(\rho < \bar{\rho} - 3\sigma) = G(t > 3) = \int_3^\infty \frac{1}{\sqrt{2\pi}} e\frac{-t^2}{2} dt \quad \text{(Equation 2.78)}$$

From table 2.2:

$$P(\rho < \bar{\rho} - 3\sigma) = 0.0013 = 0.13\%$$

2.16 A radioactive sample was counted once and gave 500 counts in 1 min. The corresponding number for the background is 480 counts. Is the sample radioactive or not? What should one report based on this measurement alone?

Answer: The answer depends on the standard error of the net counting rate using Equation 2.93:

$$r = g - b = 500 - 480 = 20 \frac{c}{min}$$

Using Equation 2.96:

$$\sigma_r = \sqrt{\frac{500}{1^2} + \frac{480}{1^2}} = 31.3 \frac{c}{min}$$

$$\frac{\sigma_r}{r} = \frac{31.3}{20} = 156\% \text{ error.}$$

Based on this measurement alone, one cannot tell whether or not the sample is radioactive.
Note: The counting rate is so low that dead time need not be considered.

2.17 A radioactive sample gave 750 counts in 5 min. When the sample was removed, the sealer recorded 1000 counts in 10 min. What is the net counting rate and its standard percent error?

Answer: Use Equation 2.93:

$$r = \frac{750}{5} - \frac{1000}{10} = 150 - 100 = 50 \frac{c}{min}$$

And Equation 2.96:

$$\sigma_r = \sqrt{\frac{750}{25} + \frac{1000}{100}} = \sqrt{30+10} = 6.32 \frac{c}{min}$$

$$\frac{\sigma_r}{r} = \frac{6.3}{50} = 0.126 = 13\% \quad \text{standard error}$$

(Again, dead time need not be considered)

2.18 Calculate the average net counting rate and its standard error from the data given below:

G	$t_G(min)$	B	$t_{B(min)}$
355	5	120	10
355	5	130	10
355	5	132	10

Answer: Use Equation 2.97:

$$\bar{r} = \frac{1}{3}\left(\frac{355+385+365}{5} - \frac{120+130+132}{10}\right) = \frac{1}{3}(221 - 38.2) = 60.9 \approx 61 \frac{c}{min}$$

$$\sigma_r \text{ (Equation 2.98)} \rightarrow \sigma_r = \frac{1}{3}\sqrt{\frac{1105}{25}+\frac{382}{100}} = \frac{6.93}{3} = 2.3\frac{c}{min}$$

$$\frac{\sigma_r}{r} = \frac{2.3}{61} = 0.038 \approx 4\% \text{ standard error}$$

2.19 A counting experiment has to be performed in 5 min. The approximate gross and background rates are 200 counts/min. and 50 counts/min., respectively.

(a) Determine the optimum gross and background count times.

(b) Based on the times obtained in (a), what is the standard percent error of the net counting rate?

Answer:

(a) From Equation 2.100:

$$\frac{t_B}{t_G} = \sqrt{\frac{50}{200}} = 0.5,$$

Since:

$$t_G + t_B = 5,\ 0.5\, t_G + t_B = 5,\ t_G = 333 \text{ min.},\ t_B = 1.67 \text{ min.}$$

$$r = 220 - 50 = 150\ \frac{c}{min}$$

(b)

$$\sigma_r = \sqrt{\frac{g}{t_G}+\frac{b}{t_B}} = \sqrt{\frac{220}{3.33}+\frac{50}{1.67}} = 9.49\frac{c}{min}$$

$$\frac{\sigma_r}{r} = \frac{9.49}{150} = 0.063 = 6.3\%$$

2.20 The strength of a radioactive source was measured with a 2% standard error by taking a gross count for time t min and a background for time $2t$ min. Calculate the time t if it is given that the background is 300 counts/min and the gross count 45,000 counts/min.

Answer: Since no information is given about dead time, consider it negligible (e.g. detector is scintillator). Using Equations 2.93 and 2.96:

$$\frac{\sigma_r}{r} = 0.02 = \frac{\sqrt{\frac{G}{t^2}+\frac{B}{2t^2}}}{\frac{G}{t}-\frac{B}{2t}} = \frac{\sqrt{\frac{g}{t}+\frac{b}{2t}}}{45000-300} = \frac{\sqrt{45000+\frac{300}{2}}}{45000-300}$$

$$894\sqrt{t} = 212.48,\ t = 0.056 \text{ min} = 3.39s$$

Check:

$$\sigma_r = \sqrt{45000+\frac{300}{2}} \times \sqrt{\frac{1}{t}} = 898\frac{c}{min},\ \frac{\sigma_r}{r} = \frac{898}{44700} = 0.02 = 2\%$$

2.21 The strength of radioactive source is to be measured with a detector that has a background of 120 ± 8 counts/min. The approximate gross counting rate is 360 counts/min. How long should one count if the net counting rate is to be measured with an error of 2%?

Answer: Use Equation 2.102:

$$t_G = \frac{360}{(360-120)^2\left(\frac{2}{100}\right)^2 - 8^2} = \frac{360}{23-64} < 0$$

There is no way to get 2 % accuracy under these conditions. The error in background alone is 3.3% $\left(\frac{8}{240}\right)$

With the data given, the best one can do is:

$$(360-120)^2 \times \left(\frac{x^2}{100^2}\right) - 8^2$$

Which gives:

$$x = \sqrt{\frac{64}{5.76}} = 3.3\,\%$$

2.22 The buckling B^2 of a cylindrical reactor is given by:

$$B^2 = \left(\frac{2.405}{R}\right)^2 = \left(\frac{\pi}{H}\right)^2$$

Where: R = reactor radius and H = reactor height

If the radius changes by 2% and the height by 8%, by what percent will B^2 change? Take R = 1 m, H = 2 m.

Answer:

$$\Delta B^2 = \sqrt{\left(\frac{\partial B^2}{\partial R}\right)^2 (\Delta R)^2 + \left(\frac{\partial B^2}{\partial H}\right)^2 (\Delta H)^2} = \sqrt{4\left(\frac{2.405^2}{R^3}\right)^2 (\Delta R)^2 + 4\left(\frac{\pi^2}{H^3}\right)^2 (\Delta H)^2}$$

$$= \sqrt{\frac{4\times 2.405^4}{R^4}\left(\frac{\Delta R}{R}\right)^2 + \frac{4\pi^4}{H^4} - \left(\frac{\Delta H}{H}\right)^2} = \sqrt{\frac{4\times 2.405^4}{1^4}(0.02)^2 + \frac{4\pi^4}{2^4}(0.08)^2}$$

$$= \sqrt{0.05+0.156} = 0.454\,m^{-2}\text{, } B^2 = 8.25\,m^{-2}\text{, } \frac{\Delta B^2}{B^2} = \frac{0.45}{8.25} = 0.054$$

2.23 Using Chauvenet's criterion, should any of the sealer readings listed below be rejected?

115	121	103	151
121	105	75	103
105	107	100	108
113	110	101	97
110	109	103	101

Answer: N=20, $1-\frac{1}{40} = 0.975$. A reading should be rejected if it deviates from the average by more than the 97.5% error. By interpolation from Table 2.4, number of standard deviation is 2.23.

$$\bar{n} = \frac{\sum ni}{N} = \frac{2158}{20} = 107.9 \approx 108 \qquad \sigma = \sqrt{\frac{\sum (ni-\bar{n})^2}{19}} = \sqrt{\frac{3741}{19}} = 14$$

Since $2.23\times 14 = 31.2$, the data 75 and 151 should be rejected.

Note: To obtain number of σ's exactly, one may use tables of error function (http://www-dsp.elet.polimi.it/fondstiol/comperrfnc.pdf). Proceed as follows:

$$\frac{1}{2N} = \frac{1}{40} = 0.025,\ \frac{1}{2}\left(\frac{1}{2N}\right) = 0.0125$$

Area listed in table:

$$0.5 - 0.0125 = 0.4875,\ \text{number of } \sigma' s = 2.24$$

2.24 As a quality control test in a nuclear fuel fabrication plant, the diameter of 10 fuel pellets has been measured with the following results (in mm): 9.50, 9.80, 9.75, 9.82, 9.93, 9.79, 9.81, 9.65, 9.99, and 9.57. Calculate

(a) the average diameter
(b) the standard deviation of this set of measurements
(c) the standard error of the average diameter
(d) should any of the results be rejected based on the Chauvenet criterion?

Answer:

(a)

$$d_{av} = \left(\frac{9.50+9.80+9.75+9.82+9.93+9.79+9.81+9.65+9.99+9.57}{10}\right) = 9.76\text{ mm}$$

(b)

$$\sigma = \sqrt{\frac{\sum_1^{10}(\text{di}-\text{dav})^2}{\text{N}-1}} = \sqrt{\frac{0.205}{9}} = 0.15\text{ mm}$$

(c)

$$\sigma_{dav} = \frac{\sigma}{\sqrt{10}} = 0.048 \text{ mm},$$

$$\frac{\sigma_{dav}}{d_{av}} = \frac{0.048}{9.761} = 0.049 = 0.5\%$$

(d) Since N = 10, check whether any measurements lie away from average by more than the 95% error (see section 2.16) which is 1.96 σ = 0.294 mm. No measurements should be rejected based on that criterion. (Check the largest value first, 9.99 – 9.76 = 0.230 < 0.294).

2.25 Using the data of Problem 2.13, what is the value of accepted length x_a if the confidence limit is 99.4%?

Answer: From Problem 2.13, $\bar{\ell} = 2.609$ m and σ = 0.21 m. If the confidence limit is 99.4%, ℓ (x_a) should not deviate from $\bar{\ell}$ by more than 2.5 σ's (see Table 2.2), or:

$$\ell < \bar{\ell} + 2.5\,\sigma = 2.609 + 2.5 * 0.021 = 2.661 \text{ m}$$

2.26 An environmental sample has been collected for determination of ^{210}Po content. The sample is chemically separated and counted in an instrument with the following results 60 days after sampling.

Chemical Yield	80%
Counting Efficiency	20%
Sample Counts (gross)	20 counts
Sample Count Time	30 minutes
Background Counts	10 counts
Background Count Time	30 minutes
Half-life of Po-210	138 days

(a) What was the sample ^{210}Po net counting rate at the time of the sampling?
(b) What is the standard error of value determined in part (a)?
(c) The lower limit of detection (LLD) at the 95% confidence level has been defined as: LLD = $1.645(2\sqrt{2})S_b$ where S_b is the standard deviation. Calculate the LLD for this determination.
(d) Does the activity level of this sample exceed the LLD for this determination?

Answer:

(a) Let A_n, A_g, A_b and S_n, S_g, S_b, represent net, gross and background count rates and standard deviations at the time of analysis: A_c, and S_c represent net activity and standard deviation, corrected for chemical yield and counting efficiency at the time of sampling. The net count rate at time of analysis is:

$$A_n = A_g - A_b = \frac{20\ counts}{30\ min} - \frac{10\ counts}{30\ min} = 0.33\ min^{-1}$$

which must be converted for radiological decay, counting efficiency, and chemical yield to determine the disintegration rate at the time of sampling. Corrections for radiological decay are made by using the decay constant λ (determined from the half-life; $\lambda = \ln 2/t_{1/2}$) and the time t from sampling to analysis. A overall efficiency term Y, can be defined as the product of individual efficiencies (such as the chemical yield and counting efficiency) and applied to the laboratory result. The chemical yield is defined, as the amount if the end-product recovered in a chemical process, is sometimes expressed as a percentage of expected recovery to indicate the efficiency of the process. If a correction factor C is defined as:

$$C = \frac{\exp(\lambda\, t)}{Y} = \frac{\exp[\frac{(\ln 2)}{(138\ d)}\ (60\ d)]}{(0.2)(0.8)} = 8.45$$

Then the rate in disintegrations per minute at the time of sampling is:

$$A_C = A_n\, C = (0.33\ min^{-1})(8.45) = 2.8\ min^{-1}$$

(b) The distribution of results when observing radioactive decay is described by Poisson statistics when the counting interval is short compared with the half-life. The standard deviation of the distribution is the square root of the mean for a particular sampling interval. In this case, observations are made of the number of background or gross counts, n_b or n_g, registered in a corresponding time interval, t_b or t_g. The time intervals are assumed to have no associated measurement error, and the standard deviation in the observed number of counts n is $\sqrt{n}$. The standard deviation associated with a count rate, therefore is

$S = \frac{\sqrt{n}}{t}$. The standard deviation of the sum or the difference of two measurements is the square root of the sum of the squares of the standard deviations associated with the measurements. The standard deviation of the net count rate obtained in the laboratory is:

$$S_{n=}\sqrt{S_g^2 + S_B^2} = [\frac{n_g}{t_g^2} + \frac{n_b}{t_b^2}]^{1/2}$$

$$[\frac{20}{(30\,min^2)} + \frac{10}{(30\,min^2)}]^{1/2} = 0.18\,min^{-1}$$

The relative uncertainty in A_c is the same as that in A_n. Their associated standard deviations, S_c and S_n. differ by the correction factor C which accounts for yield, chemical efficiency and radioactive decay. The standard deviation in the original activity in the sample is:

$$S_n \;=\; S_n\, C = (0.18\,min^{-1})\ (8.45) = 1.5\ min^{-1}$$

(c) The lower limit of detection for the counting portion of the analysis is:

$$LLD = 1.645(s\sqrt{2}) = \;S_b \;= 1.645(s\sqrt{2})[\frac{n_b}{t_b^2}]^{1/2}$$

$$\sim 4.653[\frac{10}{(30\,\mathrm{min})^2}]^{1/2} = 0.49\ min^{-1}$$

Correcting for yield, efficiency, and radioactive decay as in part)b), gives an LLD for the determination of the original activity of 4.1 min^{-1}

(d) The activity level of the sample does not exceed the LLD since A_n = 0.33 min^{-1} which is less than 0.49 min^{-1}

2.27 Prove that for radioactivity measurements the value of MDA is given by the equation MDA = k^2 + 2CDL, if $k_\alpha = k_\beta = k$. Hint: when n = MDA, the variance $\sigma^2 = MDA + \sigma_0^2$.

Answer: Starting with Equation 2.104:

$$MDA = CDL + k_\beta \sigma_D = CDL + k_\beta\sqrt{MDA + \sigma_D^2} = CDL + k_\beta\sqrt{MDA + \frac{CDL^2}{k_\alpha^2}}$$

$$(MDA - CDL)^2 = k_{\beta^2}\left(MDA + \frac{CDL^2}{k_\alpha^2}\right)$$

(Equation 2.103: $CDL = k_\alpha\ \sigma_0$)

Now use: $k_\alpha \;= k_\beta$:

$$MDA^2 + CDL^2 - 2(MDA)(CDL) = k_\alpha^2 MDA + CDL^2$$

$$MDA(MDA - 2\times CDL - k_\alpha^2) = 0$$

$$MDA = k_\alpha^2 + 2(CDL)$$

2.28 A sample was counted for 5 min and gave 2250 counts; the background, also recorded for 5 min, gave 2050 counts. Is this sample radioactive? Assume confidence limits of both 95% and 90%.

Answer:

(a) 95% confidence limit:

$$\text{k=1.645, G=2250, B=2050, } \sigma_B = \frac{\sqrt{2050}}{5} = 9\frac{c}{min}$$

Using Equation 2.106:

$$MDA = 1.645^2 + 4.053 s_b = 44.8 \approx 45$$

$$n = \frac{2250}{5} - \frac{2050}{5} = 40\frac{c}{min}$$

40<45, not radioactive with 95% confidence level

(b) 90% confidence limit k=1.285

$$MDA = 1.285^2 + 2\sqrt{2}\times 1.285\times 9 = 1.95 + 3.63\sigma_B = 34.3$$

40>34.2, yes it is, with 90% confidence limit

2.29 Determine the dead time of a detector based on the following data obtained with the two-source method:

g_2 = 14,000 counts/min $\qquad$ g_{12} = 26,000 counts/min

g_2 = 15,000 counts/min $\qquad$ b = 50 counts/min

Answer: Use Equation 2.110

$$(14000\times15000\times26000+14000\times15000\times50-14000\times26000\times50-15000\times26000\times50)\tau^2-$$

$$-2(14000\times15000-26000\times50)\tau+14000+15000-26000-50=0$$

$$5.43\times10^{12}\tau^2-4.174\times10^{8}\tau+2.95\times10^{3}=0$$

$$\tau=\frac{4.174\times10^{8}=\sqrt{(4.174\times10^{8})^2-4\times5.43\times10^{12}\times2.75\times10^{3}}}{2\times5.43\times10^{12}}$$

$\tau_1 = 6.9\times10^{-5}$ min or 1.15×10^{-6} sec or 1.15 μ sec
$\tau_2 = 7.87\times10^{-6}$ min or 0.13×10^{-6} sec or 0.13 μ sec
(τ_2 is the correct one since $\tau_1 g >> 1$)

Note: One could use the simpler Equation 2.111, since $b << g_i$ which gives the same results.

2.30 If the dead time of a detector is 100 μs, what is the observed counting rate if the loss of counts due to dead time is equal to 5%?

Answer:

$$g\tau=0.05\to g=\frac{0.05}{100\times10^{-6}}=500\frac{c}{s}$$

Proof:

$$\frac{n-g}{n}=0.05=f \quad \text{but} \quad n=\frac{g}{1-gc_t}\to\frac{g}{n}=1-g\tau$$

$$1-\frac{g}{n}=0.05=f \quad \frac{g}{n}=0.95$$

$$\frac{g}{n}=1-f \qquad\qquad 1-f=0.95=1-g\tau\to f=g\tau$$

$$g\tau=1-0.95=0.05$$

$$g=\frac{f}{\tau}$$

2.31 Calculate the true net activity and its standard percent error for a sample that gave 70,000 counts in 2 min. The dead time of the detector is 200 μs. The background is known to be 100 ± 1 counts/min.

Answer: Use Equation 2.113:

$$r=\frac{\frac{70000}{2}}{1-\frac{70000}{2\times10}\times200\times10^{-6}}-100=39523\frac{c}{min}$$

For σ_r use Equation 2.114:

$$\sigma_r=\sqrt{\left(\frac{1}{1-\frac{70000}{2\times60}\times200\times10^{-6}}\right)^4\frac{70000}{2^2}+1^2}=\sqrt{28743+1}=169.5\frac{c}{min}$$

$$\frac{\sigma_r}{r}=\frac{170}{39523}=4.3\times10^{-3}=0.4\%$$

2.32 Calculate the true net activity and its standard error based on the following data:

G = 100,000 counts $\qquad$ obtained in 10 min
B = 10,000 counts $\qquad$ obtained in 100 min

The dead time of the detector is 150 μs.

Answer: Use Equation 2.113:

$$r = \frac{\frac{100000}{10}}{1-\frac{100000}{10\times 60}\times 150\times 10^{-6}} - \frac{10000}{100} = 10256 - 100 = 10156\,\frac{c}{min}$$

$$\left(b\tau \approx \frac{100}{60}\times 150\times 10^{-6} = 2.5\times 10^{-4}\right)$$

For σ_r, use Equation 2.114:

$$\sigma_r = \sqrt{\left(\frac{1}{1-g\tau}\right)^4 \frac{100000}{10^2} + \frac{10000}{100^2}} = \sqrt{1106+1} = 33\,\frac{c}{min}$$

$$\frac{\sigma_r}{r} = \frac{33}{10156} = 3.2\times 10^{-3} = 0.3\%$$