<u>CHAPTER 2 HOMEWORK</u>

2.1) Give at least four advantages of miniaturization in machine design.

<u>less material used in production, multifunctionality, compactness, lighter weight, improved strength-to-weight ratio, easier storage (both for manufacturers and consumers)</u>

2.2) True or false: Miniaturization improves the factor of safety of a product.

<u>False</u>—while this is true in certain cases, it is not always true.

2.3) Scaling laws are: (a) general engineering guidelines for miniaturization, (b) useful for estimating how the characteristics of something will vary with changes in characteristic dimension, (c) helpful estimates of a device's performance at the nanoscale (d) accurate predictors of physical characteristics at the macro-, micro- and nanoscales.

<u>B</u>—useful for estimating how the characteristics of something will vary with changes in characteristic dimension

2.4) Scaling laws derive from: (a) design engineers' experience, (b) market demands, (c) the laws of physics, (d) the material used.

<u>C</u>—the laws of physics

2.5) The characteristic dimension is: (a) the dimension in which an object is largest in a three-dimensional representation of that object, (b) metric units, (c) a representative measurement of something for comparison purposes, (d) a variable used to determine the surface-to-volume ratio of an object.

<u>C</u>—a representative measurement of something for comparison purposes

2.6) True or false: the characteristic dimension, *D*, of an object is the average of that object's width, height and length.

<u>False</u>—while width, height or length can be used for *D*, this metric is not typically an average

2.7) Based on the scaling laws, how many times greater is the strength-to-weight ratio of a nanotube (*D*=10 nm) than the leg of a flea (*D*=100 μm)? Than the leg of an elephant (*D*=2 m)?

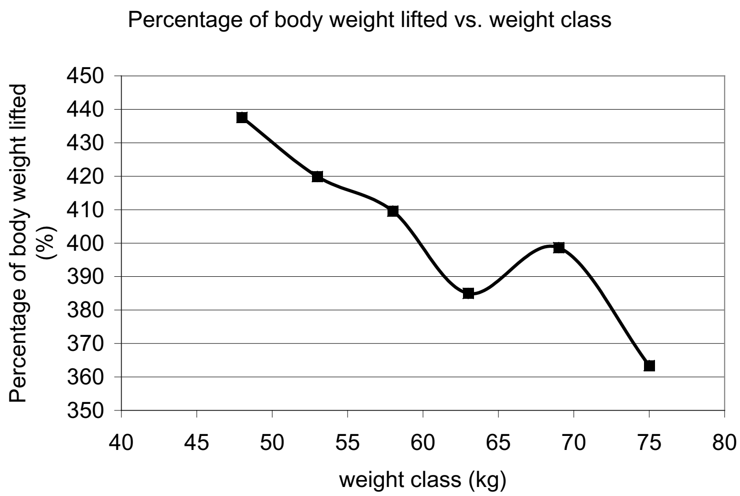
For the flea, S/W=1/100E-6=10000; for the nanotube, 1/10E-9=1E8; for the elephant, 1/2= 0.5. <u>So the strength-to-weight ratio of the nanotube is 10,000 times better than the flea's leg and 2E8 times better than the elephant's leg</u>.

2.8) Data from the 2004 Olympic Games in Athens, Greece are provided in Table 2.2. Plot the percentage of body weight lifted versus weight class. Is the same trend evident in the women's weightlifting event as the men's event shown in Figure 2.4? In what weight class is there a discrepancy and how might this be explained?

<u>ANSWER</u>

Convert lbs to kg, then divide lift weight by weight class for percentage of body weight lifted.

WOMEN'S GOLD MEDAL WINNERS			
Weight class (kg)	Lift (lbs)	**Lift (kg)**	**PERCENTAGE**
48	463.05	**210**	**438**
53	490.61	**223**	**420**
58	523.69	**238**	**410**
63	534.71	**243**	**385**
69	606.38	**275**	**399**
75	600.86	**273**	**363**



<u>There is a discrepancy in the 69 kg weight class that might be explained by a weak showing in that particular category during that Olympic games. Data from world *records*, where each category reflects the best showing ever, more closely follow the trend.</u>

2.9) Derive the mass-to-volume ratio as a function of characteristic dimension. Explain the result.

Both mass and volume are proportional to L^3, so the ratio is 1—the mass to volume ratio is therefore independent of scale.

2.10) The micro-mirror shown in Figure 2.8 is used for redirecting light in an optical communication system. The torque needed to spin it on the y axis is directly proportional to its mass moment of inertia, I_{yy}:

$$I_{yy} = \frac{1}{12}\rho h t w^3$$

Here ρ is the density of the mirror material.

a) Derive the scaling law for I_{yy}.

b) If the mirror's dimensions can be reduced to one third original size, what is the corresponding percent reduction in the torque required to turn the mirror?

ANSWER

A) $I_{yy} = \frac{1}{12}\rho h t w^3 \propto \frac{1}{12}\rho (L)(L)(L)^3 \propto L^5$

B) $Torque \propto I_{yy} \propto L^5 = \left(\frac{1}{3}\right)^5 = 0.004$ **or 0.4% of the original torque is needed, corresponding to a 99.6% *reduction* in torque.**

2.11) What is the resistance of a cubic micrometer of copper (ρ = 17.2 × 10^{-9} Ωm)?

Cubic micrometer: $R = \frac{(17.2\times10^{-9})(1\times10^{-6})}{(1\times10^{-6})(1\times10^{-6})} =$ **0.017 Ω**

2.12) How much more or less resistance does a cubic micrometer of copper have versus a cubic millimeter?

$R \propto \frac{1}{D}$. **Since a cubic millimeter has *D* 1000 times larger, its resistance is <u>1000 times less</u> than that of the smaller block.**

2.13) In designing an electrostatic actuator made from parallel plates, you can either double the plates' areas or halve the distance separating them. Which provides a greater improvement in the electrostatic force?

$F_e = -\frac{1}{2}\frac{\varepsilon_o \varepsilon_r A V^2}{d^2}$ **Doubling the area doubles the force, while halving the separation distance quadruples the force—<u>so changing the distance is the better option</u>.**

2.14) Consider two parallel wires 100 μm long, each carrying 20 μA of current, separated by 1 μm.

a) What is the electromagnetic force between these wires?

b) If the orientation is right, the electromagnetic force created by these two wires can be enough to lift one of the wires. The force of gravity opposing that motion, $F_g = mg$, where m is mass and g=9.8 m/s^2. If the wire is made of copper (8.96 g/mL) with a diameter of 2 μm, what is the force of gravity holding it down?

c) How many times greater or smaller is this force than the electromagnetic force being used to lift the wire?

d) If the characteristic dimension of the wires was allowed to increase, and with it the current through the wires as governed by Equation 2.21, would the electromagnetic force ever overcome the gravitational force?

ANSWER

A) Refer to BACK-OF-THE-ENVELOPE 2.5

$$F_{mag} = \frac{(100\times10^{-6}\,\text{m})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20\times10^{-6}\,\text{A})^2}{2\pi(1\times10^{-6}\,\text{m})} = \underline{\mathbf{8\times10^{-15}\ N}}$$

B) The volume of the wire is $\pi(1E\text{-}6)^2(100E\text{-}6) = 3.14E\text{-}16\ m^3$

The density converted to kg/m^3 is 8960 kg/m^3

$F_g = mg = (8960\ kg/m^3)(\ 3.14E\text{-}16\ m^3)(9.8\ m/s^2) =$ **<u>2.8E-11 N</u>**

C) <u>3,400 times greater than the magnetic force</u>.

D) <u>No</u>. The magnetic force scales with D^2 while the gravitational force scales with D^3 so as the characteristic dimension increases, the ratio between the two forces (F_{mag}/F_g) decreases linearly—never allowing the magnetic force to "catch up" and equal the gravitational force.

2.15) True or false: The smallest spot we can illuminate with a given lens depends on the wavelength of light we use.

<u>True</u>

2.16) Consider a compact disc on which the pits used to store the digital information measure 850 nm long by 500 nm wide. Visible light has the following wavelengths: 380-420 nm for violet; 420-440 nm for indigo; 440-500 nm for blue; 500-520 nm for cyan; 520-565 nm for green; 565-590 nm for yellow; 590-625 nm for orange; 625-740 nm for red. Which of these colors, if any, could be used for building the pits on the disc?

violet, indigo, blue

2.17) Light passes through a lens with a refractive index of 1.6 and front and back surfaces with radii of 11 cm and 15 cm, respectively. (a) What is the focal length of the lens? (b) What would be the focal length of an identically shaped lens 100 times larger?

ANSWER

A) $\frac{1}{f} = (1.6-1)\left(\frac{1}{0.11} - \frac{1}{0.15}\right)$=**1.5 m**

B) Focal length is directly proportional to *D*, so a lens 100X larger would have a focal length 100 times longer = 150 m.

2.18) The calorie is a unit of energy defined as the amount of energy needed to raise 1 g of water by 1°C.

a) How many calories are required to bring a pot of water at 1°C to a boil? The pot is full to the brim, with diameter 20 cm and depth 20 cm. The density of water is 1000 kg/m^3.

b) If we consider *D* for the pot to be 20 cm, approximately how much more energy is needed to heat a hot tub with *D*=2 m? How many calories is that?

c) If energy costs 10 cents per kilowatt hour, how much does it cost to heat this hot tub?

d) How does the price (in dollars) of heating the hot tub scale with the tub's characteristic dimension?

e) What percentage of the cost of heating the tub would be saved by reducing the tub's characteristic dimension by 33%?

ANSWER

A) Volume of the water = $\pi(0.10\text{m})^2(0.20) = 0.0063\ \text{m}^3$

Mass of the water = $(0.0063\ \text{m}^3)(1000\ \text{kg/m}^3) = 6.3$ kg

Calories required = (99°C)(6300g) = **623700 calories**

B) $E_{th} \propto D^3$, so with D 10 times larger, the hot tub requires 10^3 =**1000 times as much energy:** (623700)(1000) = **623700000 calories**

C) 623700000 calories = 725 kilowatt hours. At 10 cents per kilowatt hour, that's **$72.50** to heat the tub.

D) $E_{th} \propto D^3$, therefore the cost of the energy scales as $Cost \propto 0.10D^3$

E) Reducing D by 33% means reducing the cost by 71.3%. As an example, if the characteristic dimension was 100, the cost would be $0.1(100)^3$ =$100,000. If a third smaller, at 66, the cost would be $28,749. The difference in price is $71,250, or about **71.3%** or the original cost.

2.19) A can (355 ml) of Coke has 140 "food calories" (1 food calorie = 1kcal). How many equivalently sized cans worth of water could be brought to a boil using the energy in a single Coke? Assume the water is initially at room temperature (20°C). The density of water is 1000 kg/m^3.

ANSWER

Mass of water boiled = calories/ΔT = 140000 cal/80°C = 1750 g

$(1.75\text{kg})/(1000\ \text{kg/m}^3) = 0.00175\ \text{m}^3$

1 can = 355 ml = 3.55E-4 m^3

# of cans worth of water = 0.00175 m^3/3.55E-4 m^3 = **4.93 or about 5 cans**

2.20) True or false: At the microscale, reducing a pipe's radius causes an increase in the pressure drop per unit length.

True

2.21) A crude oil pipe's radius is reduced by 5 percent. What is the corresponding percentage change in the pressure drop per unit length?

Let D=1. 5% reduction would make D=0.95

$1/(0.95^2) = 1.11$ or 111%

So the change in pressure drop per unit length is 111-100 = 11% increase.

2.22) A pipe's radius can each be thought of as a characteristic dimension, *D.* What is the surface to volume ratio (S/V) of the inside of the pipe as a function of *D*?

Surface= $2\pi rL=2\pi DL$

Volume= $\pi r^2L= \pi D^2L$

S/V = 2/D

2.23) True or false: The number of heartbeats during an animal's lifetime tends to be approximately one billion beats, no matter its size.

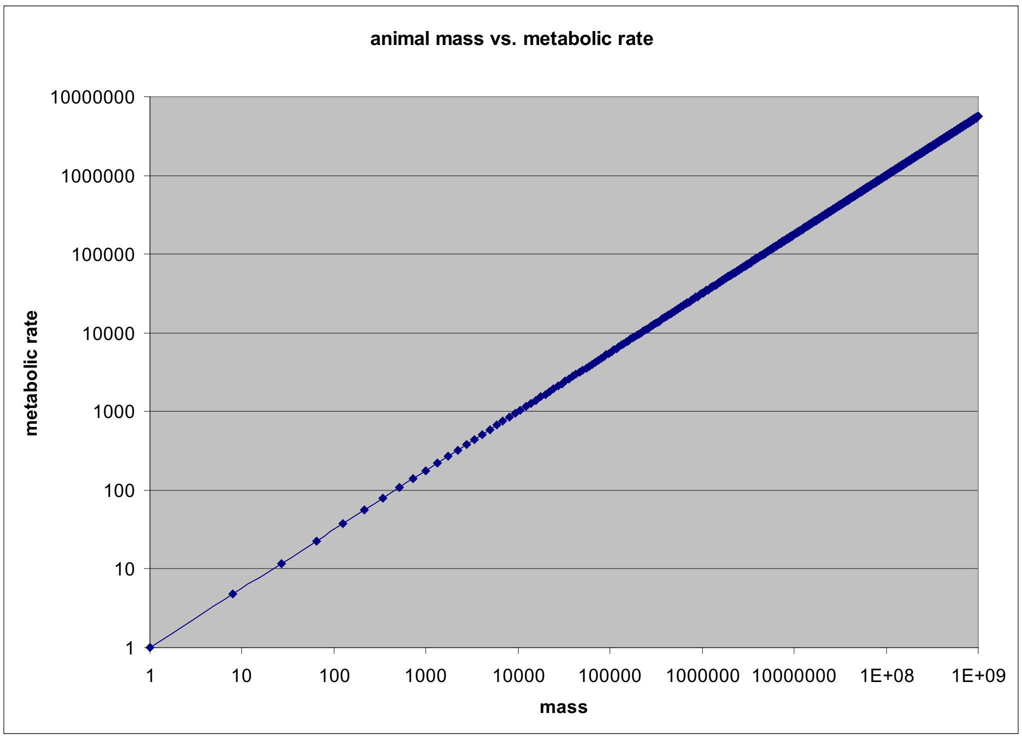
True

2.24) Plot mass versus metabolic rate (each as a function of characteristic dimension $1 \leq D \leq 1000$) on a log-log scale.

a) Is the relationship linear or exponential on this graph?

b) How much faster is the metabolic rate of an animal 10,000 times more massive than another?

ANSWER



A) linear

B) 1,000 times faster

SHORT ANSWER

2.25) Name two examples of products you use that have been miniaturized and say how these changes were improvements.

2.26) Write a paragraph supporting or refuting the idea that the scaling laws are truly *laws*.