CHAPTER 2

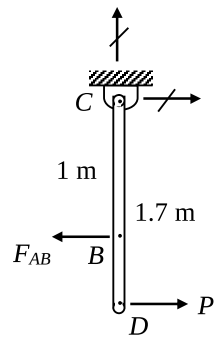
SOLUTION (2.1)

Free Body: *CD*

$$\sum M_C = 0: \quad F_{AB} = 1.7P$$

and

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}}$$

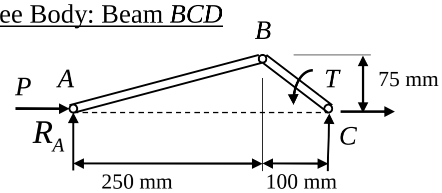


Substitute the numerical values:

$$50(10^6) = \frac{1.7P}{500(10^{-6})}, \qquad P = 14.71 \text{ kN}$$ ◄

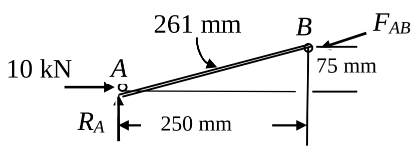
SOLUTION (2.2)

(*a*) Free Body: Beam *BCD*



$$\sum M_C = 0: \quad -R_A(0.35) + T = 0, \qquad T = 0.35R_A \qquad \textbf{(a)}$$

Free Body: Rod *AB*



$$AB = \sqrt{250^2 + 75^2} = 261 \text{ mm}$$

$$\sum M_B = 0: \quad 10(0.075) - R_A(0.25) = 0 \quad R_A = 3 \text{ kN}$$

Equation ***(a)***:

$$T = 0.35(3) = 1.05 \text{ kN} \cdot \text{m}$$ ◄

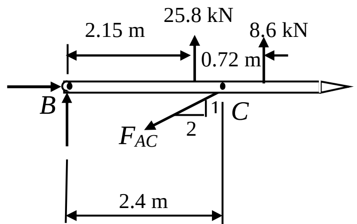
(*b*)

$$\sum F_x = 0: \quad 10 - \frac{250}{261} F_{AB} = 0 \quad F_{AB} = 10.44 \text{ kN}$$

Thus,

$$\sigma_{AB} = \frac{10.44(10^3)}{0.5} = 20.88 \text{ kN}$$ ◄

SOLUTION (2.3)



Load resultants are shown in the figure (above). Hence

$$\sum M_B = 0: \quad 25.8(2.15) + 8.6(2.15 + 0.72) - F_{AC}\frac{1}{\sqrt{5}}(2.4) = 0$$

or

$$F_{AC} = 74.68\,\text{kN}$$

Thus,

$$A_{AC} = \frac{74.68(10^3)}{80\times10^6} = 933.5(10^{-6})\,\text{m}^2 = 933.5\ \text{mm}^2 = 908.4\ \text{mm}^2 \quad ◂$$

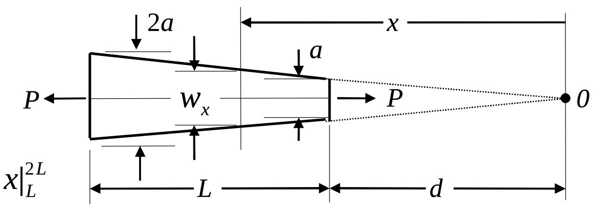
SOLUTION (2.4)

From geometry $\frac{d+L}{2a} = \frac{d}{a}$; $d=L$

$$\frac{a}{L} = \frac{w_x}{x}; \quad w_x = \frac{a}{L}x$$

and $A_x = w_x t = \frac{at}{L}x$

Thus,



$$\delta = \int_L^{2L}\frac{Pdx}{A_x E} = \frac{PL}{atE}\int_L^{2L}\frac{dx}{x} = \frac{PL}{atE}\ln x\Big|_L^{2L}$$

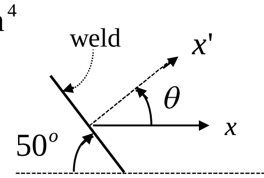
$$= \frac{PL}{atE}\ln 2 \quad ◂$$

SOLUTION (2.5)

$$J = \frac{\pi}{2}(75^4 - 65^4) = 21.661\times10^{-6}\ \text{m}^4$$

c=0.075 m $\quad \theta = 40^o$

$$\tau_{xy} = \frac{Tr}{J}, \quad \sigma_x = \sigma_y = 0$$



Using Eq. (1.11a);

$$\sigma_x{}' = 0 + 0 + \tau_{xy}\sin 2\theta$$

or

$$200(10^6) = \frac{T(0.075)(0.985)}{21.661(10^{-6})}; \quad T = 58.65\ \text{kN}\cdot\text{m} \quad ◂$$

SOLUTION (2.6)

$$J_b = \frac{\pi}{2}(0.03^4 - 0.02^4) = 102.102(10^{-8})\ \text{m}^4$$

Statics: $\quad T_s + T_b = 1\ \text{kN}\cdot\text{m} \qquad$ (***a***)

Geometry:

$$\phi_c = \frac{T_b b}{\frac{\pi}{2}(0.03^4 - 0.02^4)(42\times10^9)} = \frac{T_s b}{\frac{\pi}{2}(0.02^4)(80\times10^9)}$$

or

$$T_b = 2.1328T_s \qquad (\boldsymbol{b})$$

(CONT.)

(2.6 CONT.)

From Eqs (***a***) and (***b***):

$$T_b = 680.8\ \text{N}\cdot\text{m} \quad T_s = 319.2\ \text{N}\cdot\text{m}$$

It is required that, $\phi_A = \phi_C + \phi_{AC}$:

$$0.01 = \tfrac{1}{J_b E_b}[680.8b + 1000(0.5 - b)] = \tfrac{500-319.2b}{102.102(10^{-8})(42\times10^{9})}$$

Solving, b=0.223 m = 223 mm ◂

SOLUTION (2.7)

State of pure shear; $\sigma_1 = -\sigma_2 = \tau$

$$\varepsilon_{\max} = \tfrac{1}{E}(\sigma_1 - \nu\sigma_2) = \tfrac{\tau}{E}(1+\nu)$$

Thus,

$$\tau = \tfrac{E\varepsilon_{\max}}{1+\nu} = \tfrac{120(10^3)(1900)}{1+0.33} = 171.4\ \text{MPa}$$

We have

$$J = \tfrac{Tr}{\tau} = \tfrac{150(0.015)}{171.4(10^6)} = 13.127(10^9)\ \text{m}^4$$

Hence

$$J = \tfrac{\pi}{32}(30^4 - d^4) = 13.172(10^3)\ \text{mm}^4$$

Solving, $d = 28.68\ \text{mm}$ ◂

SOLUTION (2.8)

(*a*) $\tau_{\max} = \frac{3V}{2A} = \frac{3}{2}\frac{pL/2}{bh} = \frac{3}{4}\frac{pL}{bh}$ (***a***)

$\sigma_{\max} = \frac{Mc}{I} = \frac{(pL^2/8)(h/2)}{bh^3/12} = \frac{3}{4}\frac{pL^2}{bh^2}$ (***b***)

Thus,

$$\tau_{\max}/\sigma_{\max} = h/L \qquad (\boldsymbol{c})$$

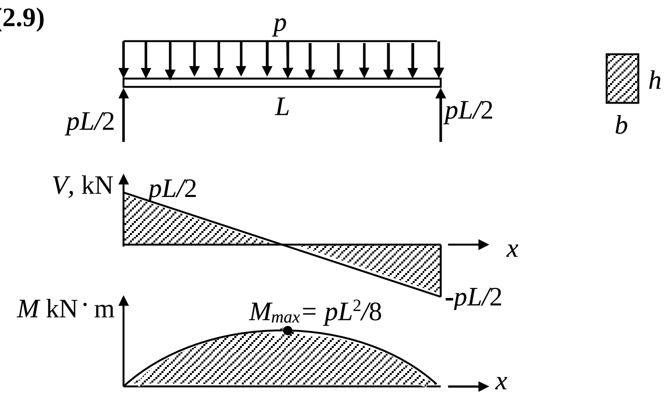
Equation ***(c)***:

$$L = h\frac{\sigma_{all}}{\tau_{all}} = 0.15(\tfrac{9}{1.5}) = 0.9\ \text{m}$$ ◂

(*b*) Equation ***(a)***:

$$p_{all} = \tfrac{4}{3}\tfrac{bh}{L}\tau_{all} = \tfrac{4}{3}\tfrac{0.05\times0.15}{0.9}(1.5\times10^6) = 16.67\ \text{kN/m}$$ ◂

SOLUTION (2.9)



(CONT.)

(2.9 CONT.)

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{pL/2}{bh} = \frac{3}{4}\frac{pL}{bh} \qquad (a)$$

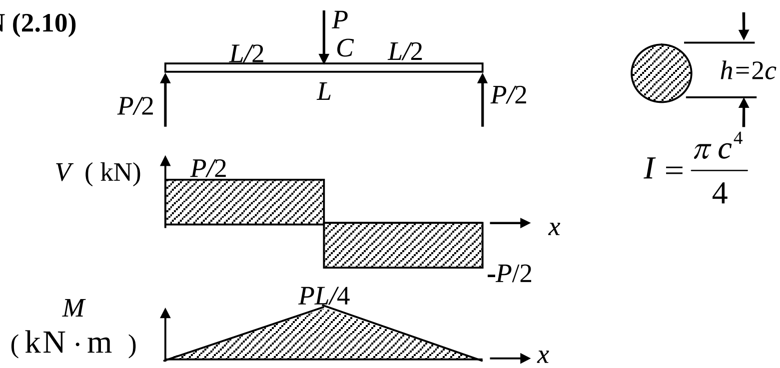
$$\sigma_{\max} = \frac{Mc}{I} = \frac{pL^2/8(h/2)}{bh^3/12} = \frac{3}{4}\frac{pL^2}{bh^2} \qquad (b)$$

Thus,

$$\tau_{\max}/\sigma_{\max} = h/L \qquad (c) \quad ◂$$

For example, if $L = 10h$, the above ratio is $1/10$.

SOLUTION (2.10)



From Table B.6:

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{P/2}{\pi c^2} = \frac{2}{3}\frac{P}{\pi c^2}$$

Also

$$\sigma_{\max} = \frac{Mc}{I} = \frac{4M}{\pi c^3} = \frac{PL}{\pi c^3}$$

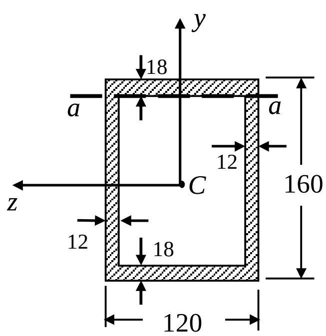
Thus, $\quad \tau_{\max}/\sigma_{\max} = 2c/3L = h/3L$ ◂

For example, if $L = 10h$, the above quotient is $1/30$.

SOLUTION (2.11)

$$I = \frac{1}{12}(120)(160)^3 - \frac{1}{12}(96)(124)^3$$

$$= 25.7(10^6)\ \text{mm}^4$$



(*a*) Maximum shear stress (at N.A.):

$$Q = (120)(80)(40) - (96)(62)(31)$$

$$= 199.5(10^3)\ mm^3$$

$$\tau_{\max} = \frac{VQ}{Ib} = \frac{250(10^3)(199.5\times 10^{-6})}{25.7(10^{-6})(0.24)}$$

$$= 8.09\,\text{MPa}$$ ◂

(CONT.)

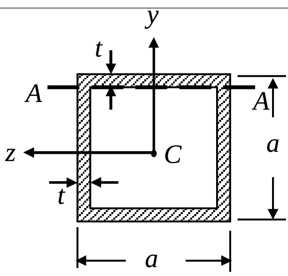
(2.11 CONT.)

(*b*) Minimum shear stress (at section *a-a*):

$$Q = (120)(18)(80-9) = 153.36(10^3)\ \text{mm}^3$$

$$\tau_{\min} = \frac{VQ}{Ib} = \frac{250(10^3)(153.36\times 10^{-6})}{25.7(10^{-6})(0.24)} = 6.22\,\text{MPa}$$

SOLUTION (2.12)



$$I_z = \frac{a^4}{12} - \frac{1}{12}(a-2t)^4$$

$$= \frac{200^4}{12} - \frac{1}{12}(170)^4 = 63.73(10)^6\,\text{mm}^4$$

(*a*) Maximum shear stress (at N.A.).

$$Q = a(\frac{a}{2})(\frac{a}{4}) - (a-2t)(\frac{a}{2}-t)(\frac{a}{2}-t)(\frac{1}{2})$$

$$= (200)(100)(50) - (170)(85)(85)(\frac{1}{2})$$

$$= 385{,}875\,\text{mm}^3$$

$$\tau_{\max} = \frac{VQ}{Ib} = \frac{120(10^3)(385{,}875)}{(63.73)(10^6)(2\times 15)} = 24.22\,\text{MPa}$$

(*b*) Minimum shear stress (at section *A-A*).

$$Q = at(\frac{a}{2} - \frac{t}{2}) = (200)(15)(92.5) = 88{,}425\,\text{mm}^3$$

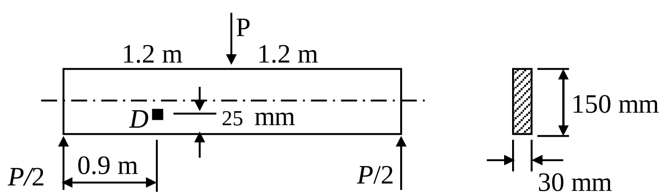
$$\tau_{\max} = \frac{VQ}{Ib} = \frac{120(10^3)(88{,}425)}{(63.73(10^6)(2\times 15)} = 5.55\,\text{MPa}$$

SOLUTION (2.13)

We have

$$I = \frac{1}{12}(30)(150)^3 = 8.44(10^{-6})\ \text{m}^4$$

$$Q_D = (0.03)(0.05)(0.0625) = 93.75(10^{-6})\ \text{m}^3$$



(CONT.)

(2.13 CONT.)

$$M_D = 0.45P \qquad V_D = 0.5P$$

$$\sigma_D = \frac{Mc}{I} = \frac{0.45P(0.025)}{8.44\times10^{-6}} = 1333P$$

$$\tau_D = \frac{VQ}{Ib} = \frac{0.5P(93.75\times10^{-6})}{8.44\times10^{-6}(0.03)} = 185.13P$$

Equation (1.13):

$$(\sigma_1)_D = 15\times10^6 = \frac{1333P}{2} + P\sqrt{(\frac{1333}{2})^2 + (185.13)^2}$$

$$= 666.5P + 691.7P = 1358.2P$$

or

$$P_{all} = 11.04 \text{ kN}$$ ◂

SOLUTION (2.14)

$$EIw^{IV} = p = \tfrac{p_o}{L^2}(L^2 - x^2), \qquad EIw''' = \tfrac{p_o}{L^2}(L^2 x - \tfrac{x^3}{3}) + c_1$$

Boundary Condition:

$$w'''(L) = 0; \qquad c_1 = -\tfrac{2}{3}p_o L$$

$$EIw'' = \tfrac{p_o}{L^2}(L^2\tfrac{x^2}{2} - \tfrac{x^4}{12}) + c_1 x + c_2$$

Boundary Conditions:

$$EIw''(L) = 0; \qquad c_2 = \tfrac{p_o L^2}{4}$$

$$EIw'' = \tfrac{p_o}{12L^2}(3L^4 - 8L^3 x + 6L^2x^2 - x^4)$$

$$EIw' = \tfrac{p_o}{12L^2}(3L^4 x - 8L^3\tfrac{x^2}{2} + 6L^2\tfrac{x^3}{3} - \tfrac{x^5}{5}) + c_3$$

Boundary Condition:

$$EIw'(0) = 0; \qquad c_3 = 0.$$

$$EIw = \tfrac{p_0}{12L^2}(3L^4\tfrac{x^2}{2} - 4L^3\tfrac{x^3}{3} + 2L^2\tfrac{x^4}{4} - \tfrac{x^6}{30}) + c_4$$

Boundary Condition:

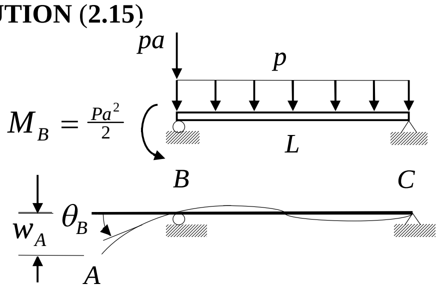
$$w(0) = 0; \qquad c_4 = 0$$

Thus, $$w = \tfrac{p_o x^2}{360EIL^2}(45L^4 - 40L^3x + 15L^2x^2 - x^4)$$ ◂

At $x=L$;

$$w_B = \tfrac{19p_oL^4}{360EI} \qquad \theta_B = w_B' = \tfrac{p_oL^3}{15EI}$$ ◂

SOLUTION (2.15)



Refer to Table B.7 (Case 5 and 7):

$$\theta_B = \tfrac{-pL^3}{24EI} + \tfrac{M_BL}{3EI} = \tfrac{pL(4a^2+L^2)}{24EI}$$

Deflection w_1 of A due to only θ_B:

$$w_1 = \theta_B a = \tfrac{paL(4a^2-L^2)}{24EI}$$

(CONT.)

(2.15 CONT.)

Table B.7 (Case 2 with b=0), cantilever:

$$w_2 = \frac{pL^3}{24EI}(4L - L) = \frac{pL^4}{8EI}$$

Total deflection

$$w_A = w_1 + w_2 = \frac{pa}{24EI}(3L^4 + 4a^2L - L^3) \quad ◂$$

SOLUTION (2.16)

$$EIw^{IV} = p_0 \sin\tfrac{\pi x}{L}; \quad EIw''' = -p_o(\tfrac{L}{\pi})\cos + c_1$$
$$EIw'' = -p_o(\tfrac{L}{\pi})^2 \sin\tfrac{\pi x}{L} + c_1 x + c_2$$

Boundary Conditions:

$$w''(0) = 0, \quad c_2 = 0; \qquad w''(L) = 0, \quad c_1 = 0$$
$$EIw' = p_o(\tfrac{L}{\pi})^3 \cos\tfrac{\pi x}{L} + c_3$$
$$EIw = p_o(\tfrac{L}{\pi})^4 \sin\tfrac{\pi x}{L} + c_3 x + c_4$$

Boundary Conditions:

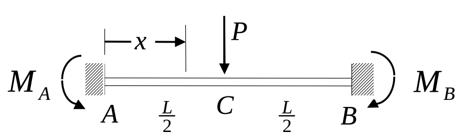
$$w(0) = 0, \quad c_4 = 0; \qquad w(L) = 0, \quad c_3 = 0$$

Thus

$$w = \frac{p_o L^4}{\pi^4 EI}\sin\tfrac{\pi x}{L} \quad ◂$$

Slope at x=0: $\theta_A = w'_1(0) = \frac{p_o L^3}{\pi^3 EI} = -\theta_B$ ◂

SOLUTION (2.17)



Symmetry

$$M_A = -M_B$$
$$R_A = R_B = \tfrac{P}{2} \uparrow \quad ◂$$

Segment AC

$$EIw^{IV} = 0, \qquad EIw''' = c_1, \qquad EIw'' = c_1 x + c_2$$

$$EIw' = \tfrac{1}{2}c_1 x^2 + c_2 x + c_3$$
$$EIw = \tfrac{1}{6}c_1 x^3 + \tfrac{1}{2}c_2 x^2 + c_3 x + c_4 \qquad (\boldsymbol{a})$$

We have

$$EIw'''(0) = c_1 = -V = -\tfrac{P}{2}$$
$$EIw''(0) = c_2 = -(-M_A): \quad c_2 = M_A$$
$$w'(0) = 0: \quad c_3 = 0, \qquad w'(\tfrac{L}{2}) = 0: \quad M_A = -M_B = \tfrac{PL}{8}$$
$$w(0) = 0: \quad c_4 = 0$$

Equation ($\boldsymbol{a}$) is thus

$$w = \frac{Px^2}{48EI}(3L - 4x) \quad ◂$$

SOLUTION (2.18)

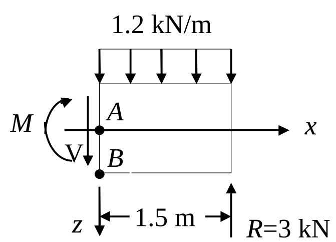
We have $250/2 = 125$ MPa . Equation (2.30*b*) gives the limiting value of pressure for the tangential stress as

$$p = \frac{\sigma_{all} t}{r} = \frac{125\times10^6(0.005)}{0.2} = 3.125 \text{ MPa} \qquad ◄$$

Note that, the axial stress formula, Eq. (2.38a) requires

$$p = 2\frac{\sigma_{all} t}{r} = 6.25 \text{ MPa}$$

SOLUTION (2.19)



$$V = 3 - 1.5(1.2) = 1.2 \text{ kN}$$

$$M = 3(1.5) - \tfrac{1}{2}(1.2)(1.5)^2 = 3.15 \text{ kN}\cdot\text{m}$$

Point *A*

$$\sigma = \frac{Mc}{I} = 0, \qquad \sigma_\theta = \frac{pr}{t} = \frac{4(10^4)(0.5)}{0.005} = 4 \text{ MPa}$$

$$\sigma_x = \sigma_a = 2 \text{ MPa}$$

Table B-4;

$$\tau = \frac{VQ}{Ib} = \frac{12000(\pi r t)(2r/\pi)}{\pi r^3 t(2t)} = 153 \text{ kPa}$$

Thus $\quad \tau_{\max} = [(\frac{2-4}{2})^2 + (0.153)^2]^{1/2} = 1.012 \text{ MPa} \qquad ◄$

$$\theta_s = \tfrac{1}{2}\tan^{-1}\frac{2-4}{2(0.153)} = -40.65^o$$

Point *B*;

$$\sigma = \frac{Mc}{I} = \frac{3150(0.5)}{\pi(0.5)^3(0.005)} = 802.1 \text{ kPa}, \qquad \tau = \frac{VQ}{Ib} = 0$$

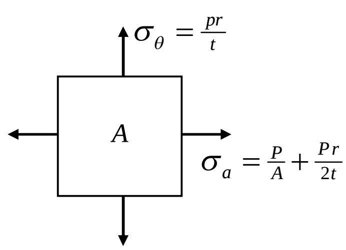
$$\sigma_1 = \sigma_\theta = 4 \text{ MPa}, \qquad \sigma_2 = 2 + 0.802 = 2.802 \text{ MPa}$$

and

$$\tau_{\max} = \tfrac{1}{2}(4 - 2.802) = 599 \text{ kPa}, \qquad \theta_s = 45^o \qquad ◄$$

SOLUTION (2.20)

(*a*)



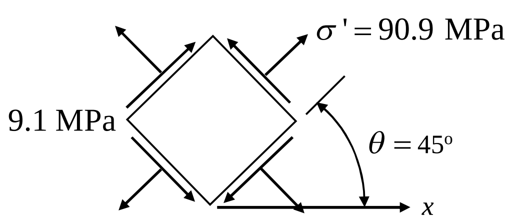
$$A = 2\pi r t$$

$$= 2\pi(250)(10) = 15,708 \text{ mm}^2$$

$$\sigma_\theta = \frac{4(10^6)(0.25)}{0.01} = 100 \text{ MPa} \qquad ◄$$

$$\sigma_x = \frac{500(10^3)}{15,708(10^{-6})} + 50 = 81.8 \text{ MPa}$$

(*b*) $\tau_{\max} = \tfrac{1}{2}[100 - 81.8] = 9.1$ MPa $\qquad ◄$

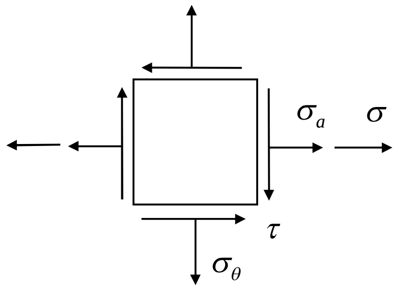


$$\sigma' = \tfrac{1}{2}(\sigma_\theta + \sigma_x) = 90.9 \text{ MPa}$$

SOLUTION (2.21)

At a point on circumference, we have

$$\sigma_a = \frac{pr}{2t} = \frac{4(100)}{2(8)} = 25 \text{ MPa}, \quad \sigma_\theta = 50 \text{ MPa}$$



$$\sigma = \frac{P}{2\pi r t} = \frac{50(10^3)}{2\pi(100)(8)10^{-6}} = 9.947 \text{ MPa}$$

$$\tau = \frac{Tr}{J} = \frac{-30(10^3)(0.1)}{2\pi(0.1)^3(0.008)} = -59.68 \text{ MPa}$$

Thus

$$\sigma_{1,2} = \frac{34.95+50}{2} \pm [(\frac{34.95-50}{2})^2 + 59.68^2]^{1/2}$$

$$= 42.48 \pm 60.12$$

or

$$\sigma_1 = 102.6 \text{ MPa}, \quad \sigma_2 = -17.64 \text{ MPa}$$

(*a*) $|\sigma_1| \le \sigma_u;$ $\quad |102.6| < 240$ $\quad \therefore$ no failure ◂

(*b*) $\frac{\sigma_1}{\sigma_u} = \frac{\sigma_2}{\sigma_{uc}} = 1;$ $\quad \frac{102.6}{240} - \frac{-17.64}{600} = 1$

or

0.428+0.029 < 1 $\quad \therefore$ no failure ◂

SOLUTION (2.22)

$$\sigma_{1,2} = \frac{-100-50}{2} \pm [(\frac{-100+50}{2})^2 + 30^2]^{1/2}$$

or

$$\sigma_2 = -36 \text{ MPa}, \quad \sigma_3 = -114 \text{ MPa}, \quad \sigma_1 = 60 \text{ MPa}$$

(*a*) $\quad n = \left|\frac{\sigma_u}{\sigma_1}\right| = \left|\frac{150}{60}\right| = 2.5$

or $\quad n = \left|\frac{150}{114}\right| = 1.32$ ◂

(*b*) $\quad \frac{60}{150} - \frac{-114}{600} = \frac{1}{n}; \quad 0.4 + 0.19 = \frac{1}{n}$

Solving *n*=1.7 ◂

SOLUTION (2.23)

We have σ_{all} = 250/2.2 =113.6 MPa. From Eq. (2.38b) we find that the limiting value of pressure

$$p = \frac{\sigma_{all}\, t}{r} = \frac{113.6(10^6)(0.0036)}{0.9} = 454.4 \text{ kPa}$$

for circumferential stress. The axial stress is thus

$$p = 2\frac{\sigma_{all}\, t}{r} = 908.8 \text{ kPa}$$

Comment: The *gage pressure* may not exceed 454.4 kPa.

SOLUTION (2.24)

The tangential, axial, and radial streses are:

$$\sigma_\theta = \frac{pr}{t} = 25\,p \qquad \sigma_a = \frac{pr}{2t} = 12.5\,p \qquad \sigma_z = 0$$

(*a*) Using Equation 2.42a, we have:

$$25\,p = \frac{1}{1.8}(260)(10^6), \qquad p = 5.78 \text{ MPa}$$

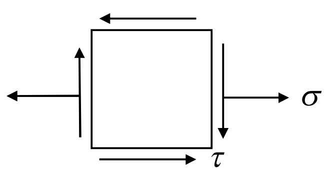
(*b*) From Equation:

$$p(25^2 - 25 \text{ x } 12.5 + 12.5^2)^{1/2} = \frac{260}{1.8}, \qquad p = 6.67 \text{ MPa}$$

Comment: The allowable value of the maximum pressure is limited to 5.78 MPa.

SOLUTION (2.25)

At a point on the surface of the shaft, we have



$$J = \tfrac{\pi}{32}(75)^4 = 3.106(10^6) \text{ mm}^4$$

$$A = \tfrac{\pi}{4}(75)^2 = 4.418(10^3) \text{ mm}^2$$

$$\sigma = \tfrac{P}{A} = \frac{40(10^3)}{4.418(10^{-3})} = 9.054 \text{ MPa}$$

$$\tau = \tfrac{Tr}{J} = \frac{6(10^3)(0.0375)}{3.106(10^{-6})} = 72.44 \text{ MPa}$$

Thus

$$\sigma_{1,2} = \tfrac{9.054}{2} \pm [(\tfrac{9.054}{2})^2 + 72.44^2]^{1/2} = 4.527 \pm 72.581$$

$$\sigma_1 = 77.11 \text{ MPa}, \qquad \sigma_2 = -68.05 \text{ MPa}$$

(*a*) $\tfrac{250}{n} = [(77.11)^2 - (77.11)(-68.05) + (-68.05)^2]^{1/2}$

or n=1.99 ◂

(*b*) $|\sigma_1 - \sigma_2| = \tfrac{\sigma_{yp}}{n}; \qquad |77.11 + 68.05| = \tfrac{250}{n}$

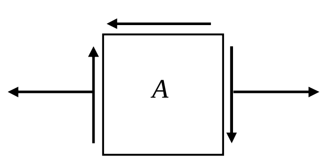
or n=1.72 ◂

SOLUTION (2.26)

At the fixed end A(see Fig. P2.26):

$$T = 1.2R \qquad M_z = 1.6R \qquad V_y = -R$$

The effect of V_y may be neglected. Thus, at a point *A* on the *top* of the bar at the fixed end:



$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(1.6R)}{\pi(0.06)^3} = 7.545R$$

$$\tau = -\frac{16T}{\pi d^3} = -\frac{16(1.2R)}{\pi(0.06)^3} = -2.829R$$

(CONT.)

(2.26 CONT.)

Then

$$\sigma_{1,2} = \frac{7.545R}{2} \pm \sqrt{(\frac{7.545R}{2})^2 + (-2.829R)^2}$$

$$\sigma_1 = 8.487 \times 10^4 R \qquad \sigma_2 = -0.943 \times 10^4 R \qquad \tau_{\max} = 4.715 \times 10^4 R$$

(a) $\dfrac{130 \times 10^6}{1.9} = 4.715 \times 10^4 R, \qquad R = 1.45\,\text{kN}$ ◄

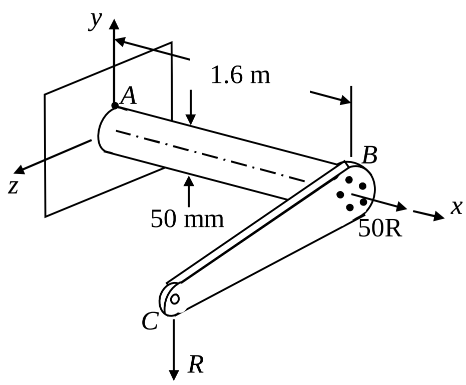
(b) $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = (\sigma_{yp}/n_s)^2$

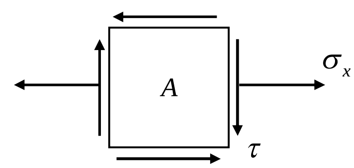
$$[8.487^2 - 8.487(-0.943) + (-0.943)^2]^{1/2}(10^4)R = 240 \times 10^6 / 1.9$$

or

$$R = 1.404\,\text{kN}$$ ◄

SOLUTION (2.27)





At the fixed end:

$$T = 1.2R \qquad M_z = 1.6R$$

$$P_x = 50R \qquad V_y = -R$$

The effect of V_y may be neglected. Therefore, at point A:

$$\sigma_x' = \frac{P_x}{A} = \frac{50R}{\frac{\pi}{4}(0.06)^2} = 1.768 \times 10^4 R$$

$$\sigma_x'' = \frac{32}{\pi d^3} = \frac{32(1.6R)}{\pi(0.06)^3} = 7.545 \times 10^4 R$$

$$\tau = -\frac{16T}{\pi d^3} = -\frac{16(1.2R)}{\pi(0.06)^3} = -2.829 \times 10^4 R$$

$$\sigma_{1,2} = 10^4 R[\frac{9.313}{2} \pm \sqrt{(\frac{9.313}{2})^2 + (-2.829)^2}\,]$$

$$\sigma_1 = 10.106 \times 10^4 R \qquad \sigma_2 = -0.793 \times 10^4 R \qquad \tau_{\max} = 5.449 \times 10^4 R$$

(a) $\dfrac{130 \times 10^6}{1.9} = 5.449 \times 10^4 R, \qquad R = 1.26\,\text{kN}$ ◄

(CONT.)

(2.27 CONT.)

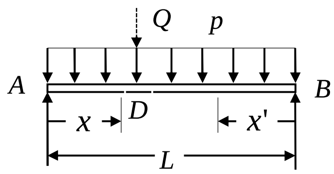
(*b*) $[\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} = \dfrac{\sigma_{yp}}{2}$

$$10^4 R[(10.106)^2 - (10.106)(-0.793) + (-0.793)^2]^{1/2} = 240\times10^6/1.9$$

Solving,

$$R = 1.2 \text{ kN}$$ ◂

SOLUTION (2.28)



Using equations of statics;

$$R_A = \tfrac{pL}{2} + \tfrac{Qb}{L}, \qquad R_B = \tfrac{pL}{2} + \tfrac{Qa}{L}$$

Then

$$M_{AD} = R_A x + \tfrac{px^2}{2}, \quad M_{BD} = R_B x' + \tfrac{px'^2}{2};$$

$$\tfrac{\partial M_{AD}}{\partial Q} = \tfrac{bx}{L}, \qquad \tfrac{\partial M_{BD}}{\partial Q} = \tfrac{ax'}{L}$$

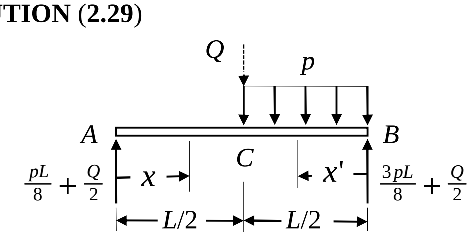
Applying Eq. (2.57):

$$w_D = \tfrac{p}{2EIL}\left[\int_0^a (Lx - x^2)(bx)dx + \int_0^b (Lx' - x'^2)(ax')dx'\right]$$

Integrating, we have

$$w_D = \tfrac{pab}{24EIL}[4L(a^2+b^2) - 3(a^3+b^3)]$$ ◂

SOLUTION (2.29)



Segment *AC*

$$M_1 = (\tfrac{pL}{8} + \tfrac{Q}{2})x, \qquad \tfrac{\partial M_1}{\partial Q} = \tfrac{x}{2}$$

Segment BC

$$M_2 = (\tfrac{3pL}{8} + \tfrac{Q}{2})x' - \tfrac{px'^2}{2}, \qquad \tfrac{\partial M_2}{\partial Q} = \tfrac{x'}{2}$$

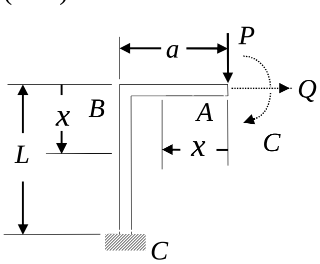
Let Q=0, Thus, Eq. (2.57):

$$EIw_C = \int_0^{L/2} \tfrac{pLx}{8}\tfrac{x}{2}dx + \int_0^{L/2}(\tfrac{3pL}{8}x' - \tfrac{px'^3}{2})\tfrac{x'}{2}dx' = \tfrac{5}{16}\tfrac{pL^4}{48}$$

or $$w_C = \tfrac{5}{768}\tfrac{pL^4}{EI} \downarrow$$ ◂

SOLUTION (2.30)

(*a*)



We have

$$M_{AB} = Px \qquad M_{BC} = Pa$$

$$\delta_v = \tfrac{1}{EI}\int M_i \tfrac{\partial M_i}{\partial P}dx$$

$$= \tfrac{1}{EI}\left[\int_0^a (Px)(x)dx + \int_0^L (Pa)(a)dx\right]$$

$$= \tfrac{1}{EI}(\tfrac{a^3}{3} + a^2L) = \tfrac{Pa^2}{3EI}(a + 3L)$$ ◂

(CONT.)

(2.30 CONT.)

(*b*) Add Q at A. Hence,

$$\delta_H = \tfrac{1}{EI}[\int_0^a M_{AB}\tfrac{\partial M_{AB}}{\partial Q}dx + \int_0^L M_{BC}\tfrac{\partial M_{BC}}{\partial Q}dx,$$

where $M_{AB} = Px, \quad M_{BC} = Pa + Qx$

Let Q=0, Then

$$\delta_H = 0 + \int_0^L (Pa)(x)dx = \tfrac{PaL^2}{2EI}$$

(*c*) Add C at A. We have

$$\theta = \tfrac{1}{EI}\int M_i \tfrac{\partial M_i}{\partial C}dx \qquad \text{where} \quad M_{AB} = Px + C, \qquad M_{BC} = Pa + C$$

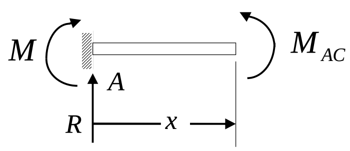
$$\theta_A = \tfrac{1}{EI}[\int_0^a (Px + C)dx + \int_0^L (Pa + C)dx]$$

For C=0:

$$\theta_A = \tfrac{P}{EI}(\tfrac{a^2}{2} + aL) = \tfrac{Pa}{2EI}(a + 2L)$$

SOLUTION (2.31)

(*a*)



$$M_{AC} = Rx + M, \quad M_{BC} = Rx + M - P(x - \tfrac{L}{2})$$

$$\theta = \tfrac{1}{EI}\int M_i \tfrac{\partial M_i}{\partial M}dx, \quad w = \tfrac{1}{EI}\int M_i \tfrac{\partial M_i}{\partial R}dx$$

We have

$$\theta_A = \tfrac{1}{EI}\{\int_0^{L/2}(Rx + M)dx + \int_{L/2}^{L}[Rx + M - P(x - \tfrac{L}{2})]dx\}$$

$$w_A = \tfrac{1}{EI}\{\int_0^{L/2}(Rx + M)xdx + \int_{L/2}^{L}[Rx + M - P(x - \tfrac{L}{2})]xdx\} \qquad (\boldsymbol{a})$$

Boundary conditions are $\theta(0) = 0$ and $w(0) = 0$. Thus, after integrating Eqs.(***a***):

$$\theta_A = \tfrac{RL^2}{2EI} + \tfrac{ML}{EI} - \tfrac{PL^2}{8EI} = 0$$

$$w_A = \tfrac{PL^3}{3EI} + \tfrac{ML^2}{2EI} - \tfrac{5PL^3}{48EI} = 0$$

From which

$$RL^2 + 2ML = \tfrac{PL^2}{4}$$

$$RL^3 + \tfrac{3}{2}ML^2 = \tfrac{5}{16}PL^3$$

Solving

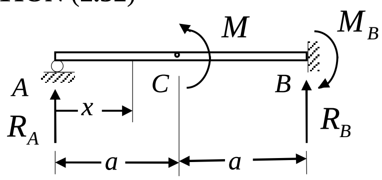
$$M = -\tfrac{PL}{8} = \tfrac{PL}{8} \circlearrowleft \qquad R = \tfrac{P}{2}\uparrow$$

(*c*)

$$w_C = \tfrac{1}{EI}[\int_0^{L/2}(Rx + M)xdx] = \tfrac{1}{EI}[\int_0^{L/2}(\tfrac{P}{2}x - \tfrac{PL}{8})xdx]$$

$$= \tfrac{PL^3}{192EI} = w_{\max}\downarrow$$

SOLUTION (2.32)



$$M_{AC} = R_A x, \qquad M_{BC} = R_A x - M$$

$$w_A = \frac{1}{EI}\int M_i \frac{\partial M_i}{\partial R_A}\, dx$$

$$w_A = \frac{1}{EI}\left[\int_0^a (R_A x)(x)dx + \int_a^{2a}(R_A x - M)(x)dx\right]$$

$$= \frac{8}{3}\frac{R_A a^3}{EI} - \frac{3}{2}\frac{Ma^2}{EI} = 0$$

from which $R_A = \frac{9}{16}\frac{M}{a}$ ↑ ◂

Statics:

$\sum F_y = 0$: $\qquad R_B = -R_A$ ↓ ◂

$\sum M_B = 0$: $\qquad M_B - M + R_A(2a) = 0$

or

$M_B = -\frac{M}{8} = \frac{M}{8}$ ↺ ◂