C H A P T E R 2
Polynomial and Rational Functions

CHAPTER 2
Polynomial and Rational Functions

Section 2.1 Quadratic Functions and Models

1. polynomial

2. nonnegative integer; real

3. quadratic; parabola

4. negative; maximum

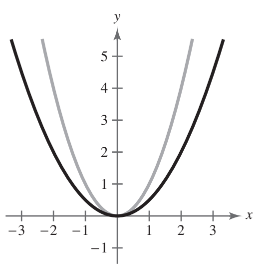
5. $f(x) = x^2 - 2$ opens upward and has vertex $(0, -2)$. Matches graph (b).

6. $f(x) = (x + 1)^2 - 2$ opens upward and has vertex $(-1, -2)$. Matches graph (a).

7. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (c).

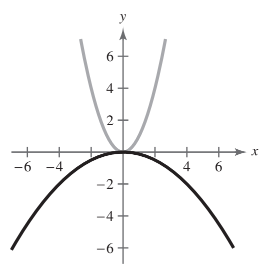
8. $f(x) = 4 - (x - 2)^2 = -(x - 2)^2 + 4$ opens downward and has vertex $(2, 4)$. Matches graph (d).

9. (a) $y = \frac{1}{2}x^2$



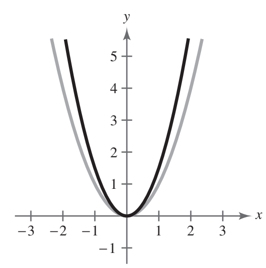
Vertical shrink

(b) $y = -\frac{1}{8}x^2$



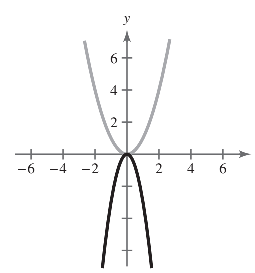
Vertical shrink and a reflection in the x-axis

(c) $y = \frac{3}{2}x^2$



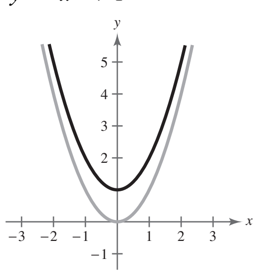
Vertical stretch

(d) $y = -3x^2$



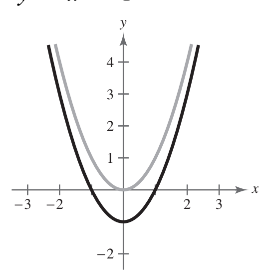
Vertical stretch and a reflection in the x-axis

10. (a) $y = x^2 + 1$



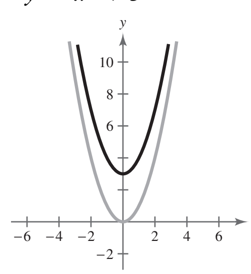
Vertical shift one unit upward

(b) $y = x^2 - 1$



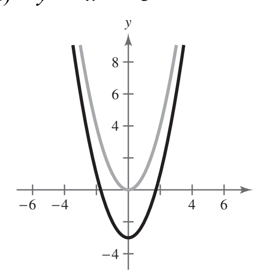
Vertical shift one unit downward

(c) $y = x^2 + 3$



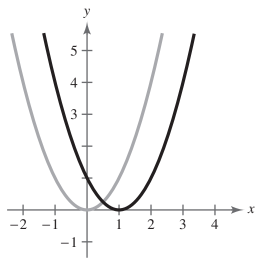
Vertical shift three units upward

(d) $y = x^2 - 3$



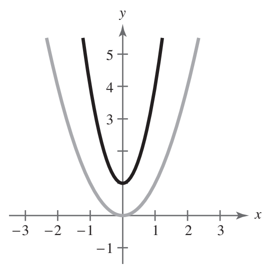
Vertical shift three units downward

11. (a) $y = (x - 1)^2$



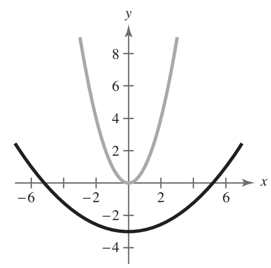
Horizontal shift one unit to the right

(b) $y = (3x)^2 + 1$



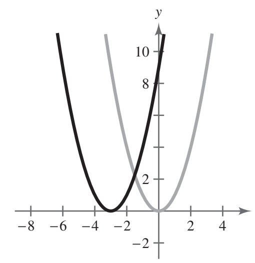
Horizontal shrink and a vertical shift one unit upward

(c) $y = \left(\frac{1}{3}x\right)^2 - 3$



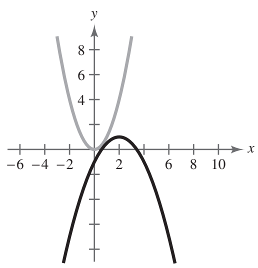
Horizontal stretch and a vertical shift three units downward

(d) $y = (x + 3)^2$



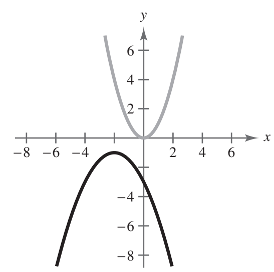
Horizontal shift three units to the left

12. (a) $y = -\frac{1}{2}(x - 2)^2 + 1$



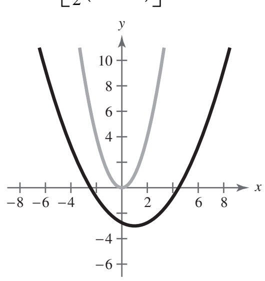
Horizontal shift two units to the right, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), reflection in the x-axis, and vertical shift one unit upward

(c) $y = -\frac{1}{2}(x + 2)^2 - 1$



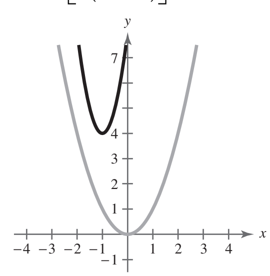
Horizontal shift two units to the left, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), reflection in x-axis, and vertical shift one unit downward

(b) $y = \left[\frac{1}{2}(x - 1)\right]^2 - 3$



Horizontal shift one unit to the right, horizontal stretch (each x-value is multiplied by 2), and vertical shift three units downward

(d) $y = \left[2(x + 1)\right]^2 + 4$



Horizontal shift one unit to the left, horizontal shrink (each x-value is multiplied by $\frac{1}{2}$), and vertical shift four units upward

13. $f(x) = x^2 - 6x$

$= (x^2 - 6x + 9) - 9$

$= (x - 3)^2 - 9$

Vertex: $(3, -9)$

Axis of symmetry: $x = 3$

Find x-intercepts:

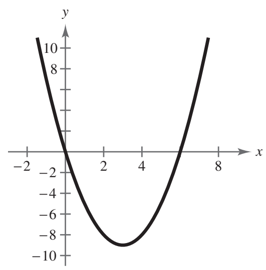
$x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0$

$x - 6 = 0 \Rightarrow x = 6$

x-intercepts: $(0, 0), (6, 0)$



14. $g(x) = x^2 - 8x$

$= (x^2 - 8x + 16) - 16$

$= (x - 4)^2 - 16$

Vertex: $(4, -16)$

Axis of symmetry: $x = 4$

Find x-intercepts:

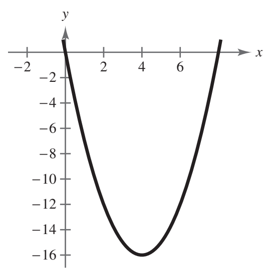
$x^2 - 8x = 0$

$x(x - 8) = 0$

$x = 0$

$x - 8 = 0 \Rightarrow x = 8$

x-intercepts: $(0, 0), (8, 0)$



15. $h(x) = x^2 - 8x + 16 = (x - 4)^2$

Vertex: $(4, 0)$

Axis of symmetry: $x = 4$

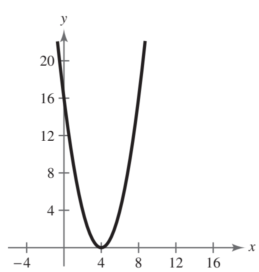
Find x-intercepts:

$(x - 4)^2 = 0$

$x - 4 = 0$

$x = 4$

x-intercept: $(4, 0)$



16. $g(x) = x^2 + 2x + 1 = (x + 1)^2$

Vertex: $(-1, 0)$

Axis of symmetry: $x = -1$

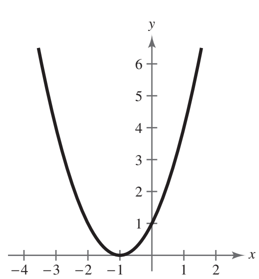
Find x-intercepts:

$(x + 1)^2 = 0$

$x + 1 = 0$

$x = -1$

x-intercept: $(-1, 0)$



17. $f(x) = x^2 - 6x + 2$

$= (x^2 - 6x + 9) - 9 + 2$

$= (x^2 - 6x + 9) - 7$

$= (x - 3)^2 - 7$

Vertex: $(3, -7)$

Axis of symmetry: $x = 3$

Find x-intercepts:

$x^2 - 6x + 2 = 0$

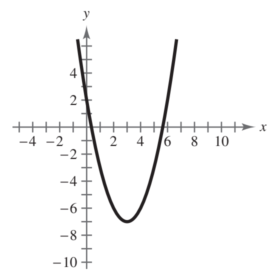
$x^2 - 6x = -2$

$x^2 - 6x + 9 = -2 + 9$

$(x - 3)^2 = 7$

$x = 3 \pm \sqrt{7}$

x-intercepts: $(3 \pm 7, 0)$



18. $h(x) = x^2 + 16x + 61$

$= (x^2 + 16x + 64) - 64 + 61$

$= (x + 8)^2 - 3$

Vertex: $(-8, -3)$

Axis of symmetry: $x = -8$

Find x-intercepts:

$x^2 + 16x + 61 = 0$

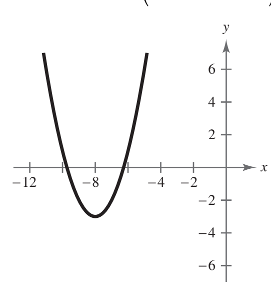
$x^2 + 16x + 64 = -61 + 64$

$(x + 8)^2 = 3$

$x + 8 = \pm\sqrt{3}$

$x = -8 \pm \sqrt{3}$

x-intercepts: $(-8 \pm \sqrt{3}, 0)$



19. $f(x) = x^2 - 8x + 21$

$= (x^2 - 8x + 16) - 16 + 21$

$= (x - 4)^2 + 5$

Vertex: $(4, 5)$

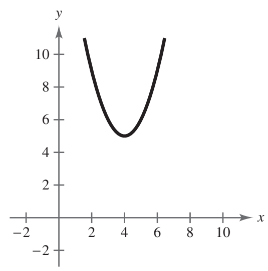
Axis of symmetry: $x = 4$

Find x-intercepts:

$$\begin{aligned} x^2 - 8x + 21 &= 0 \\ x^2 - 8x &= -21 \\ x^2 - 8x + 16 &= -21 + 16 \\ (x - 4)^2 &= -5 \\ x - 4 &= \pm\sqrt{-5} \\ x &= 4 \pm \sqrt{5}\,i \end{aligned}$$

Not a real number

No x-intercepts



20. $f(x) = x^2 + 12x + 40$

$= (x^2 + 12x + 36) - 36 + 40$

$= (x + 6)^2 + 4$

Vertex: $(-6, 4)$

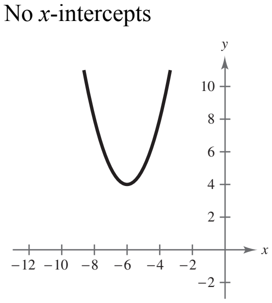
Axis of symmetry: $x = -6$

Find x-intercepts:

$$\begin{aligned} x^2 + 12x + 40 &= 0 \\ x^2 + 12x &= -40 \\ x^2 + 12x + 36 &= -40 + 36 \\ (x + 6)^2 &= -4 \\ x + 6 &= \pm 2i \\ x &= -6 \pm 2i \end{aligned}$$

Not a real number

No x-intercepts



21. $f(x) = x^2 - x + \frac{5}{4}$

$= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + \frac{5}{4}$

$= \left(x - \frac{1}{2}\right)^2 + 1$

Vertex: $\left(\frac{1}{2}, 1\right)$

Axis of symmetry: $x = \frac{1}{2}$

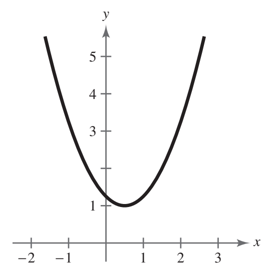
Find x-intercepts:

$$x^2 - x + \frac{5}{4} = 0$$

$$x = \frac{1 \pm \sqrt{1 - 5}}{2}$$

Not a real number

No x-intercepts



22. $f(x) = x^2 + 3x + \dfrac{1}{4}$

$$= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{1}{4}$$

$$= \left(x + \frac{3}{2}\right)^2 - 2$$

Vertex: $\left(-\dfrac{3}{2}, -2\right)$

Axis of symmetry: $x = -\dfrac{3}{2}$

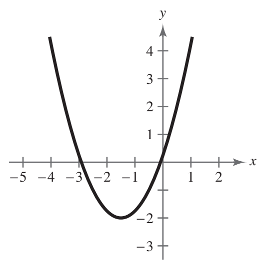
Find x-intercepts:

$$x^2 + 3x + \frac{1}{4} = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 1}}{2}$$

$$= -\frac{3}{2} \pm \sqrt{2}$$

x-intercepts: $\left(-\dfrac{3}{2} - \sqrt{2}, 0\right), \left(-\dfrac{3}{2} + \sqrt{2}, 0\right)$



23. $f(x) = -x^2 + 2x + 5$

$$= -\left(x^2 - 2x + 1\right) - (-1) + 5$$

$$= -(x - 1)^2 + 6$$

Vertex: $(1, 6)$

Axis of symmetry: $x = 1$

Find x-intercepts:

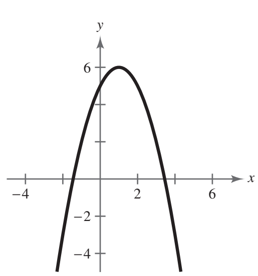
$$-x^2 + 2x + 5 = 0$$

$$x^2 - 2x - 5 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$= 1 \pm \sqrt{6}$$

x-intercepts: $\left(1 - \sqrt{6}, 0\right), \left(1 + \sqrt{6}, 0\right)$



24. $f(x) = -x^2 - 4x + 1 = -\left(x^2 + 4x\right) + 1$

$$= -\left(x^2 + 4x + 4\right) - (-4) + 1$$

$$= -(x + 2)^2 + 5$$

Vertex: $(-2, 5)$

Axis of symmetry: $x = -2$

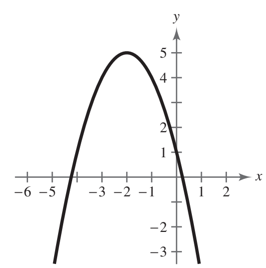
Find x-intercepts: $-x^2 - 4x + 1 = 0$

$$x^2 + 4x - 1 = 0$$

$$x = \frac{-4 \pm \sqrt{16 + 4}}{2}$$

$$= -2 \pm \sqrt{5}$$

x-intercepts: $\left(-2 - \sqrt{5}, 0\right), \left(-2 + \sqrt{5}, 0\right)$



25. $h(x) = 4x^2 - 4x + 21$

$$= 4\left(x^2 - x + \frac{1}{4}\right) - 4\left(\frac{1}{4}\right) + 21$$

$$= 4\left(x - \frac{1}{2}\right)^2 + 20$$

Vertex: $\left(\dfrac{1}{2}, 20\right)$

Axis of symmetry: $x = \dfrac{1}{2}$

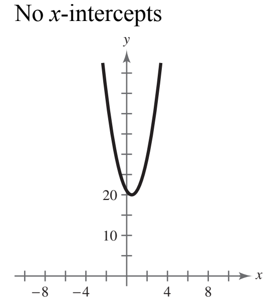
Find x-intercepts:

$$4x^2 - 4x + 21 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 336}}{2(4)}$$

Not a real number

No x-intercepts



26. $f(x) = 2x^2 - x + 1$

$$= 2\left(x^2 - \frac{1}{2}x\right) + 1$$

$$= 2\left(x - \frac{1}{4}\right)^2 - 2\left(\frac{1}{16}\right) + 1$$

$$= 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8}$$

Vertex: $\left(\frac{1}{4}, \frac{7}{8}\right)$

Axis of symmetry: $x = \frac{1}{4}$

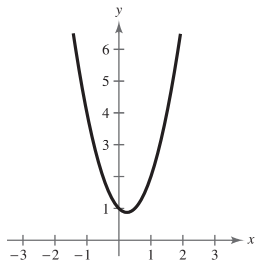
Find x-intercepts:

$2x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1 - 8}}{2(2)}$$

Not a real number

No x-intercepts

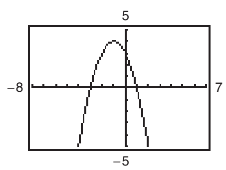


27. $f(x) = -\left(x^2 + 2x - 3\right) = -(x + 1)^2 + 4$

Vertex: $(-1, 4)$

Axis of symmetry: $x = -1$

x-intercepts: $(-3, 0), (1, 0)$



28. $f(x) = -\left(x^2 + x - 30\right)$

$$= -\left(x^2 + x\right) + 30$$

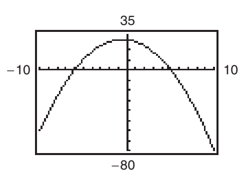
$$= -\left(x^2 + x + \tfrac{1}{4}\right) + \tfrac{1}{4} + 30$$

$$= -\left(x + \tfrac{1}{2}\right)^2 + \tfrac{121}{4}$$

Vertex: $\left(-\frac{1}{2}, \frac{121}{4}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

x-intercepts: $(-6, 0), (5, 0)$

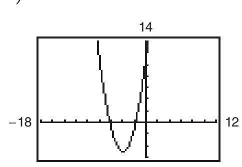


29. $g(x) = x^2 + 8x + 11 = (x + 4)^2 - 5$

Vertex: $(-4, -5)$

Axis of symmetry: $x = -4$

x-intercepts: $\left(-4 \pm \sqrt{5}, 0\right)$



30. $f(x) = x^2 + 10x + 14$

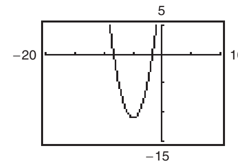
$$= \left(x^2 + 10x + 25\right) - 25 + 14$$

$$= (x + 5)^2 - 11$$

Vertex: $(-5, -11)$

Axis of symmetry: $x = -5$

x-intercepts: $\left(-5 \pm \sqrt{11}, 0\right)$



31. $f(x) = -2x^2 + 12x - 18$

$$= -2\left(x^2 - 6x + 9 - 9\right) - 18$$

$$= -2\left(x^2 - 6x + 9\right) + 18 - 18$$

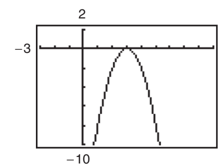
$$= -2\left(x^2 - 6x + 9\right)$$

$$= -2(x - 3)^2$$

Vertex: $(3, 0)$

Axis of symmetry: $x = 3$

x-intercept: $(3, 0)$



32. $f(x) = -4x^2 + 24x - 41$

$$= -4\left(x^2 - 6x\right) - 41$$

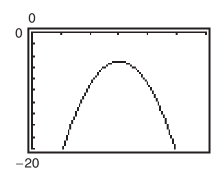
$$= -4\left(x^2 - 6x + 9\right) + 36 - 41$$

$$= -4(x - 3)^2 - 5$$

Vertex: $(3, -5)$

Axis of symmetry: $x = 3$

No x-intercepts

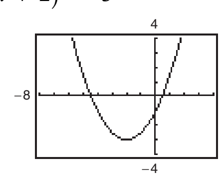


33. $g(x) = \frac{1}{2}\left(x^2 + 4x - 2\right) = \frac{1}{2}(x + 2)^2 - 3$

Vertex: $(-2, -3)$

Axis of symmetry: $x = -2$

x-intercepts: $\left(-2 \pm \sqrt{6}, 0\right)$



34. $f(x) = \frac{3}{5}\left(x^2 + 6x - 5\right)$

$$= \tfrac{3}{5}\left(x^2 + 6x + 9\right) - \tfrac{27}{5} - 3$$

$$= \tfrac{3}{5}(x + 3)^2 - \tfrac{42}{5}$$

Vertex: $\left(-3, -\frac{42}{5}\right)$

Axis of symmetry: $x = -3$

x-intercepts: $\left(-3 \pm \sqrt{14}, 0\right)$

35. $(-2, -1)$ is the vertex.

$f(x) = a(x + 2)^2 - 1$

Because the graph passes through $(0, 3)$,

$3 = a(0 + 2)^2 - 1$
$3 = 4a - 1$
$4 = 4a$
$1 = a.$

So, $y = (x + 2)^2 - 1$.

36. $(-2, 2)$ is the vertex.

$y = a(x + 2)^2 + 2$

Because the graph passes through $(-1, 0)$,

$0 = a(-1 + 2)^2 + 2$
$-2 = a.$

So, $y = -2(x + 2)^2 + 2$.

37. $(-2, 5)$ is the vertex.

$f(x) = a(x + 2)^2 + 5$

Because the graph passes through $(0, 9)$,

$9 = a(0 + 2)^2 + 5$
$4 = 4a$
$1 = a.$

So, $f(x) = 1(x + 2)^2 + 5 = (x + 2)^2 + 5$.

38. $(-3, -10)$ is the vertex.

$f(x) = a(x + 3)^2 - 10$

Because the graph passes through $(0, 8)$,

$8 = a(0 + 3)^2 - 10$
$8 = 9a - 10$
$18 = 9a$
$2 = a.$

So, $f(x) = 2(x + 3)^2 - 10$.

39. $(1, -2)$ is the vertex.

$f(x) = a(x - 1)^2 - 2$

Because the graph passes through $(-1, 14)$,

$14 = a(-1 - 1)^2 - 2$
$14 = 4a - 2$
$16 = 4a$
$4 = a.$

So, $f(x) = 4(x - 1)^2 - 2$.

40. $(2, 3)$ is the vertex.

$f(x) = a(x - 2)^2 + 3$

Because the graph passes through $(0, 2)$,

$2 = a(0 - 2)^2 + 3$
$2 = 4a + 3$
$-1 = 4a$
$-\frac{1}{4} = a.$

So, $f(x) = -\frac{1}{4}(x - 2)^2 + 3$.

41. $(5, 12)$ is the vertex.

$f(x) = a(x - 5)^2 + 12$

Because the graph passes through $(7, 15)$,

$15 = a(7 - 5)^2 + 12$
$3 = 4a \Rightarrow a = \frac{3}{4}.$

So, $f(x) = \frac{3}{4}(x - 5)^2 + 12$.

42. $(-2, -2)$ is the vertex.

$f(x) = a(x + 2)^2 - 2$

Because the graph passes through $(-1, 0)$,

$0 = a(-1 + 2)^2 - 2$
$0 = a - 2$
$2 = a.$

So, $f(x) = 2(x + 2)^2 - 2$.

43. $\left(-\frac{1}{4}, \frac{3}{2}\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

Because the graph passes through $(-2, 0)$,

$0 = a\left(-2 + \frac{1}{4}\right)^2 + \frac{3}{2}$

$-\frac{3}{2} = \frac{49}{16}a \Rightarrow a = -\frac{24}{49}.$

So, $f(x) = -\frac{24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}.$

44. $\left(\frac{5}{2}, -\frac{3}{4}\right)$ is the vertex.

$f(x) = a\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}$

Because the graph passes through $(-2, 4)$,

$4 = a\left(-2 - \frac{5}{2}\right)^2 - \frac{3}{4}$

$4 = \frac{81}{4}a - \frac{3}{4}$

$\frac{19}{4} = \frac{81}{4}a$

$\frac{19}{81} = a.$

So, $f(x) = \frac{19}{81}\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}.$

45. $\left(-\frac{5}{2}, 0\right)$ is the vertex.

$f(x) = a\left(x + \frac{5}{2}\right)^2$

Because the graph passes through $\left(-\frac{7}{2}, -\frac{16}{3}\right)$,

$-\frac{16}{3} = a\left(-\frac{7}{2} + \frac{5}{2}\right)^2$

$-\frac{16}{3} = a.$

So, $f(x) = -\frac{16}{3}\left(x + \frac{5}{2}\right)^2.$

46. $(6, 6)$ is the vertex.

$f(x) = a(x - 6)^2 + 6$

Because the graph passes through $\left(\frac{61}{10}, \frac{3}{2}\right)$,

$\frac{3}{2} = a\left(\frac{61}{10} - 6\right)^2 + 6$

$\frac{3}{2} = \frac{1}{100}a + 6$

$-\frac{9}{2} = \frac{1}{100}a$

$-450 = a.$

So, $f(x) = -450(x - 6)^2 + 6.$

47. $y = x^2 - 2x - 3$

$0 = x^2 - 2x - 3$

$0 = (x - 3)(x + 1)$

$x = 3$ or $x = -1$

x-intercepts: $(3, 0), (-1, 0)$

48. $y = x^2 - 4x - 5$

$0 = x^2 - 4x - 5$

$0 = (x - 5)(x + 1)$

$x = 5$ or $x = -1$

x-intercepts: $(5, 0), (-1, 0)$

49. $y = 2x^2 + 5x - 3$

$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$x + 3 = 0 \Rightarrow x = -3$

x-intercepts: $\left(\frac{1}{2}, 0\right), (-3, 0)$

50. $y = -2x^2 + 5x + 3$

$0 = -2x^2 + 5x + 3$

$0 = 2x^2 - 5x - 3$

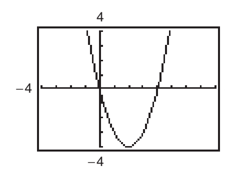
$0 = (2x + 1)(x - 3)$

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

$x - 3 = 0 \Rightarrow x = 3$

x-intercepts: $\left(-\frac{1}{2}, 0\right), (3, 0)$

51. $f(x) = x^2 - 4x$



x-intercepts: $(0, 0), (4, 0)$

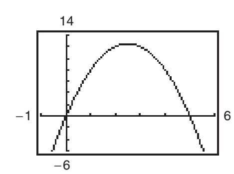
$0 = x^2 - 4x$

$0 = x(x - 4)$

$x = 0$ or $x = 4$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

52. $f(x) = -2x^2 + 10x$



x-intercepts: $(0, 0), (5, 0)$

$0 = -2x^2 + 10x$

$0 = -2x(x - 5)$

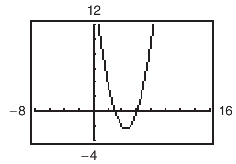
$-2x = 0 \Rightarrow x = 0$

$x - 5 = 0 \Rightarrow x = 5$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

53. $f(x) = x^2 - 9x + 18$

x-intercepts: $(3, 0), (6, 0)$



$0 = x^2 - 9x + 18$

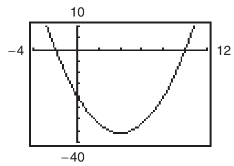
$0 = (x - 3)(x - 6)$

$x = 3$ or $x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

54. $f(x) = x^2 - 8x - 20$

x-intercepts: $(-2, 0), (10, 0)$



$0 = x^2 - 8x - 20$

$0 = (x + 2)(x - 10)$

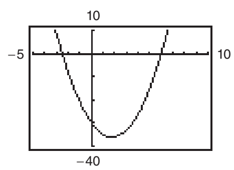
$x + 2 = 0 \Rightarrow x = -2$

$x - 10 = 0 \Rightarrow x = 10$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

55. $f(x) = 2x^2 - 7x - 30$

x-intercepts: $\left(-\frac{5}{2}, 0\right), (6, 0)$



$0 = 2x^2 - 7x - 30$

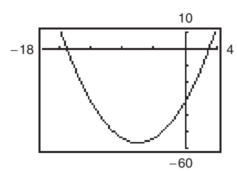
$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2}$ or $x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

56. $f(x) = \frac{7}{10}\left(x^2 + 12x - 45\right)$

x-intercepts: $(-15, 0), (3, 0)$



$0 = \frac{7}{10}\left(x^2 + 12x - 45\right)$

$0 = (x + 15)(x - 3)$

$x + 15 = 0 \Rightarrow x = -15$

$x - 3 = 0 \Rightarrow x = 3$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

57. $f(x) = [x - (-3)](x - 3)$ opens upward

$= (x + 3)(x - 3)$

$= x^2 - 9$

$g(x) = -[x - (-3)](x - 3)$

$= -(x + 3)(x - 3)$

$= -\left(x^2 - 9\right)$

$= -x^2 + 9$ opens downward

Note: $f(x) = a(x + 3)(x - 3)$ has x-intercepts $(-3, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

58. $f(x) = [x - (-5)](x - 5)$

$= (x + 5)(x - 5)$

$= x^2 - 25$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 25$

Note: $f(x) = a\left(x^2 - 25\right)$ has x-intercepts $(-5, 0)$ and $(5, 0)$ for all real numbers $a \neq 0$.

59. $f(x) = [x - (-1)](x - 4)$ opens upward

$= (x + 1)(x - 4)$

$= x^2 - 3x - 4$

$g(x) = -[x - (-1)](x - 4)$ opens downward

$= -(x + 1)(x - 4)$

$= -\left(x^2 - 3x - 4\right)$

$= -x^2 + 3x + 4$

Note: $f(x) = a(x + 1)(x - 4)$ has x-intercepts $(-1, 0)$ and $(4, 0)$ for all real numbers $a \neq 0$.

60. $f(x) = [x - (-2)](x - 3)$ opens upward

$= (x + 2)(x - 3)$

$= x^2 - x - 6$

$g(x) = -[x - (-2)](x - 3)$ opens downward

$= -(x + 2)(x - 3)$

$= -\left(x^2 - x - 6\right)$

$= -x^2 + x + 6$

Note: $f(x) = a(x + 2)(x - 3)$ has x-intercepts $(-2, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

61. $f(x) = [x - (-3)][x - (-\frac{1}{2})](2)$ opens upward

$= (x + 3)(x + \frac{1}{2})(2)$

$= (x + 3)(2x + 1)$

$= 2x^2 + 7x + 3$

$g(x) = -(2x^2 + 7x + 3)$ opens downward

$= -2x^2 - 7x - 3$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$ and $(-\frac{1}{2}, 0)$ for all real numbers $a \neq 0$.

62. $f(x) = [x - (-5)][x - (-\frac{3}{2})](2)$ opens upward

$= (x + 5)(x + \frac{3}{2})(2)$

$= (x + 5)(2x + 3)$

$= 2x^2 + 13x - 15$

$g(x) = -(2x^2 + 13x + 15)$ opens downward

$= -2x^2 - 13x - 15$

Note: $f(x) = a(x + 5)(2x + 3)$ has x-intercepts $(-5, 0)$ and $(-\frac{3}{2}, 0)$ for all real numbers $a \neq 0$.

63. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + y = 110 \Rightarrow y = 110 - x.$

The product is $P(x) = xy = x(110 - x) = 110x - x^2$.

$$\begin{aligned} P(x) &= -x^2 + 110x \\ &= -(x^2 - 110x + 3025 - 3025) \\ &= -[(x - 55)^2 - 3025] \\ &= -(x - 55)^2 + 3025 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

64. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + y = S \Rightarrow y = S - x.$

The product is $P(x) = xy = x(S - x) = Sx - x^2$.

$$\begin{aligned} P(x) &= Sx - x^2 \\ &= -x^2 + Sx \\ &= -\left(x^2 - Sx + \frac{S^2}{4} - \frac{S^2}{4}\right) \\ &= -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4} \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is $S^2/4$. This happens when $x = y = S/2$.

65. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + 2y = 24 \Rightarrow y = \frac{24 - x}{2}.$

The product is $P(x) = xy = x\left(\frac{24 - x}{2}\right)$.

$$\begin{aligned} P(x) &= \frac{1}{2}(-x^2 + 24x) \\ &= -\frac{1}{2}(x^2 - 24x + 144 - 144) \\ &= -\frac{1}{2}[(x - 12)^2 - 144] = -\frac{1}{2}(x - 12)^2 + 72 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 72. This happens when $x = 12$ and $y = (24 - 12)/2 = 6$. So, the numbers are 12 and 6.

66. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x + 3y = 42 \Rightarrow y = \frac{42 - x}{3}.$

The product is $P(x) = xy = x\left(\frac{42 - x}{3}\right)$.

$$\begin{aligned} P(x) &= \frac{1}{3}(-x^2 + 42x) \\ &= -\frac{1}{3}(x^2 - 42x + 441 - 441) \\ &= -\frac{1}{3}[(x - 21)^2 - 441] = -\frac{1}{3}(x - 21)^2 + 147 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 147. This happens when $x = 21$ and $y = \frac{42 - 21}{3} = 7$. So, the numbers are 21 and 7.

67. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

The vertex occurs at $-\frac{b}{2a} = \frac{-24/9}{2(-4/9)} = 3$. The maximum height is $y(3) = -\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16$ feet.

68. $y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$

(a) The ball height when it is punted is the y-intercept.

$y = -\frac{16}{2025}(0)^2 + \frac{9}{5}(0) + 1.5 = 1.5$ feet

(b) The vertex occurs at $x = -\frac{b}{2a} = -\frac{9/5}{2(-16/2025)} = \frac{3645}{32}$.

The maximum height is $f\left(\frac{3645}{32}\right) = -\frac{16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + 1.5$

$= -\frac{6561}{64} + \frac{6561}{32} + 1.5 = -\frac{6561}{64} + \frac{13{,}122}{64} + \frac{96}{64} = \frac{6657}{64}$ feet ≈ 104.02 feet.

(c) The length of the punt is the positive x-intercept.

$0 = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$

$x = \frac{-(9/5) \pm \sqrt{(9/5)^2 - (4)(1.5)(-16/2025)}}{-32/2025} \approx \frac{1.8 \pm 1.81312}{-0.01580247}$

$x \approx -0.83031$ or $x \approx 228.64$

The punt is about 228.64 feet.

69. $C = 800 - 10x + 0.25x^2 = 0.25x^2 - 10x + 800$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{-10}{2(0.25)} = 20$.

The cost is minimum when $x = 20$ fixtures.

70. $P = 230 + 20x - 0.5x^2$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{20}{2(-0.5)} = 20$.

Because x is in hundreds of dollars, $20 \times 100 = 2000$ dollars is the amount spent on advertising that gives maximum profit.

71. $R(p) = -25p^2 + 1200p$

(a) $R(20) = \$14{,}000$ thousand $= \$14{,}000{,}000$

$R(25) = \$14{,}375$ thousand $= \$14{,}375{,}000$

$R(30) = \$13{,}500$ thousand $= \$13{,}500{,}000$

(b) The revenue is a maximum at the vertex.

$-\frac{b}{2a} = \frac{-1200}{2(-25)} = 24$

$R(24) = 14{,}400$

The unit price that will yield a maximum revenue of \$14,400,000 is \$24.

72. $R(p) = -12p^2 + 150p$

(a) $R(4) = -12(4)^2 + 150(4) = \$408$

$R(6) = -12(6)^2 + 150(6) = \$468$

$R(8) = -12(8)^2 + 150(8) = \$432$

(b) The vertex occurs at

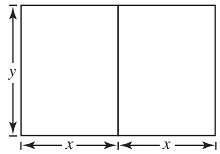
$p = -\frac{b}{2a} = -\frac{150}{2(-12)} = \$6.25$.

Revenue is maximum when price $= \$6.25$ per pet.

The maximum revenue is

$R(6.25) = -12(6.25)^2 + 150(6.25) = \$468.75$.

73. (a)



$$4x + 3y = 200 \Rightarrow y = \frac{1}{3}(200 - 4x) = \frac{4}{3}(50 - x)$$

$$A = 2xy = 2x\left[\frac{4}{3}(50 - x)\right] = \frac{8}{3}x(50 - x) = \frac{8x(50 - x)}{3}$$

(b) To find the dimensions that produce a maximum enclosed area, you can find the vertex.

To do this, either write the quadratic function in standard form or use $x = -\dfrac{b}{2a}$, so the coordinates of the vertex are $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$.

$$\begin{aligned} A &= \frac{8}{3}x(50 - x) \\ &= -\frac{8}{3}(x^2 - 50x) \\ &= -\frac{8}{3}(x^2 - 50x + 625 - 625) \\ &= -\frac{8}{3}\left[(x - 25)^2 - 625\right] \\ &= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3} \end{aligned}$$

So the vertex is $\left(25, \dfrac{5000}{3}\right)$ from the standard form, or is $x = -\dfrac{b}{2a} = -\dfrac{\frac{400}{3}}{2\left(-\frac{8}{3}\right)} = \dfrac{400}{16} = 25$ and $A(25) = \dfrac{5000}{3}$.

When $x = 25$ feet and $y = \dfrac{(200 - 4(25))}{3} = \dfrac{100}{3}$ feet.

The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet, and the maximum enclosed area is $\dfrac{5000}{3} \approx 1666.67$ square feet.

74. (a) Area of window $=$ Area of rectangle $+$ Area of semicircle $= xy + \frac{1}{2}\pi(\text{radius})^2 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = xy + \frac{\pi x^2}{8}$

To eliminate the y in the equation for area, introduce a secondary equation.

Perimeter $=$ perimeter of rectangle $+$ perimeter of semicircle

$$16 = 2y + x + \frac{1}{2}(\text{circumference})$$

$$16 = 2y + x + \frac{1}{2}(2\pi \cdot \text{radius})$$

$$16 = 2y + x + \pi\left(\frac{x}{2}\right)$$

$$y = 8 - \frac{1}{2}x - \frac{\pi x}{4}$$

Substitute the secondary equation into the area equation.

$$\text{Area} = xy + \frac{\pi x^2}{8} = x\left(8 - \frac{1}{2}x - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8} = \frac{1}{8}\left(64x - 4x^2 - \pi x^2\right)$$

(b) The area is maximum at the vertex.

$$\text{Area} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8} = \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 8x$$

$$x = -\frac{b}{2a} = \frac{-8}{2\left(-\frac{1}{2} - \frac{\pi}{8}\right)} \approx 4.48$$

$$y = 8 - \frac{1}{2}(4.48) - \frac{\pi(4.48)}{4} \approx 2.24$$

The area will be at a maximum when the width is about 4.48 feet and the length is about 2.24 feet.

75. True. The equation $-12x^2 - 1 = 0$ has no real solution, so the graph has no x-intercepts.

76. True. The vertex of $f(x)$ is $\left(-\frac{5}{4}, \frac{53}{4}\right)$ and the vertex of $g(x)$ is $\left(-\frac{5}{4}, -\frac{71}{4}\right)$.

77. $f(x) = -x^2 + bx - 75$, maximum value: 25

The maximum value, 25, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(-1)} = \frac{b}{2}$$

$$f(x) = -x^2 + bx - 75$$

$$f\left(\frac{b}{2}\right) = -\left(\frac{b}{2}\right)^2 + b\left(\frac{b}{2}\right) - 75$$

$$25 = -\frac{b^2}{4} + \frac{b^2}{2} - 75$$

$$100 = \frac{b^2}{4}$$

$$400 = b^2$$

$$\pm 20 = b$$

78. $f(x) = x^2 + bx - 25$, minimum value: –50

The minimum value, –50, is the y-coordinate of the vertex.

Find the x-coordinate:

$$x = -\frac{b}{2a} = -\frac{b}{2(1)} = -\frac{b}{2}$$

$$f(x) = x^2 + bx - 25$$

$$f\left(-\frac{b}{2}\right) = \left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) - 25$$

$$-50 = \frac{b^2}{4} - \frac{b^2}{2} - 25$$

$$-25 = \frac{-b^2}{4}$$

$$100 = b^2$$

$$\pm 10 = b$$

79. $f(x) = ax^2 + bx + c$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

$$f\left(-\frac{b}{2a}\right) = a\left(\frac{b^2}{4a^2}\right) + b\left(-\frac{b}{2a}\right) + c$$

$$= \frac{b^2}{4a} - \frac{b^2}{2a} + c$$

$$= \frac{b^2 - 2b^2 + 4ac}{4a} = \frac{4ac - b^2}{4a}$$

So, the vertex occurs at

$$\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

80. (a) Since the graph of P opens upward, the value of a is positive.

(b) Since the graph of P opens upward, the vertex of the parabola is a relative minimum at $t = \frac{-b}{2a}$.

(c) Because of the symmetrical property of the graph of a parabola, and since the company made the same yearly profit in 2008 and 2016, the midpoint of the interval $8 \le t \le 16$ or $t = 12$ corresponds to the year 2012, when the company made the least profit.

81. If $f(x) = ax^2 + bx + c$ has two real zeros, then by the Quadratic Formula they are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The average of the zeros of f is

$$\frac{\frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}}{2} = \frac{\frac{-2b}{2a}}{2} = -\frac{b}{2a}.$$

This is the x-coordinate of the vertex of the graph.

Section 2.2 Polynomial Functions of Higher Degree

1. continuous

2. Leading Coefficient Test

3. n; $n - 1$

4. (a) solution; (b) $(x - a)$; (c) x-intercept

5. touches; crosses

6. repeated zero; multiplicity

7. standard

8. Intermediate Value

9. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (c).

10. $f(x) = 2x^3 - 3x + 1$ has intercepts $(0, 1), (1, 0), \left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (f).

11. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $\left(\pm 2\sqrt{3}, 0\right)$. Matches graph (a).

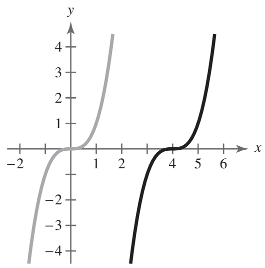
12. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (e).

13. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (d).

14. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0), (1, 0), (-1, 0), (3, 0), (-3, 0)$. Matches graph (b).

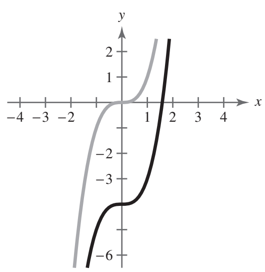
15. $y = x^3$

(a) $f(x) = (x - 4)^3$



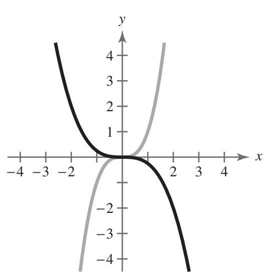
Horizontal shift four units to the right

(b) $f(x) = x^3 - 4$



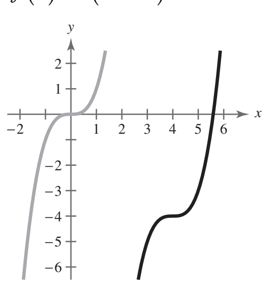
Vertical shift four units downward

(c) $f(x) = -\frac{1}{4}x^3$



Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

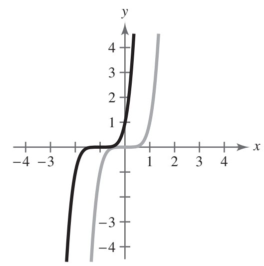
(d) $f(x) = (x - 4)^3 - 4$



Horizontal shift four units to the right and vertical shift four units downward

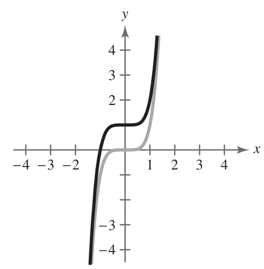
16. $y = x^5$

(a) $f(x) = (x + 1)^5$



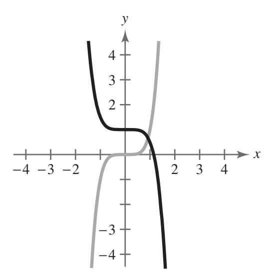
Horizontal shift one unit to the left

(b) $f(x) = x^5 + 1$



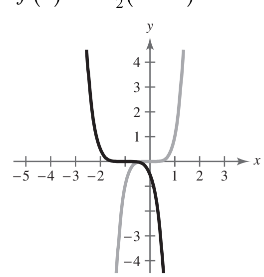
Vertical shift one unit upward

(c) $f(x) = 1 - \frac{1}{2}x^5$



Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, and vertical shift one unit upward

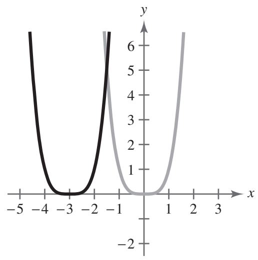
(d) $f(x) = -\frac{1}{2}(x + 1)^5$



Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, and horizontal shift one unit to the left

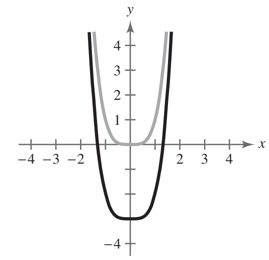
17. $y = x^4$

(a) $f(x) = (x + 3)^4$



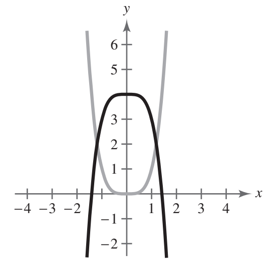
Horizontal shift three units to the left

(b) $f(x) = x^4 - 3$



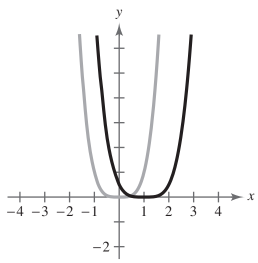
Vertical shift three units downward

(c) $f(x) = 4 - x^4$



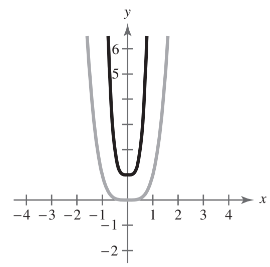
Reflection in the x-axis and then a vertical shift four units upward

(d) $f(x) = \frac{1}{2}(x - 1)^4$



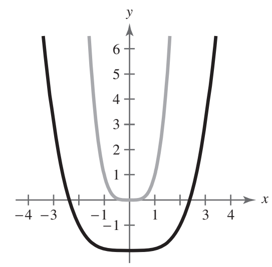
Horizontal shift one unit to the right and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

(e) $f(x) = (2x)^4 + 1$



Vertical shift one unit upward and a horizontal shrink (each y-value is multiplied by 16)

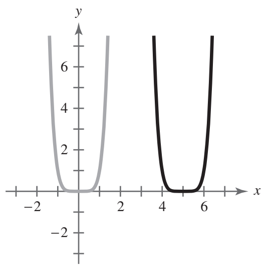
(f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$



Vertical shift two units downward and a horizontal stretch $\left(\text{each } y\text{-value is multipied by } \frac{1}{16}\right)$

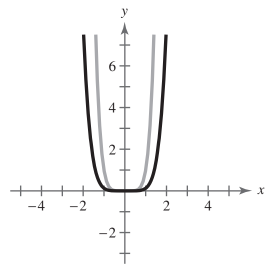
18. $y = x^6$

(a) $f(x) = (x - 5)^6$



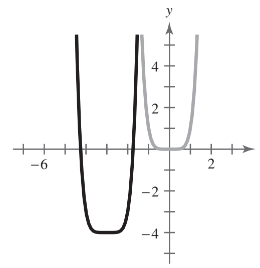
Horizontal shift to the right five units.

(b) $f(x) = \frac{1}{8}x^6$



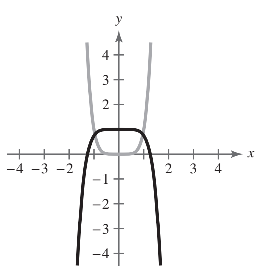
Vertical shrink (each y-value is multiplied by $\frac{1}{8}$).

(c) $f(x) = (x + 3)^6 - 4$



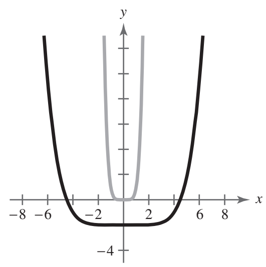
Horizontal shift three units to the left and a vertical shift four units downward.

(d) $f(x) = -\frac{1}{4}x^6 + 1$



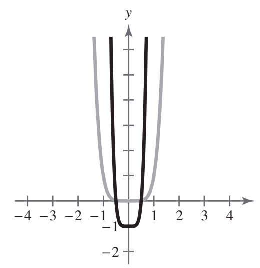
Reflection in the x-axis, vertical shrink (each y-value is multiplied by $\frac{1}{4}$), and vertical shift one unit upward

(e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$



Horizontal stretch (each x-value is multiplied by 4), and vertical shift two units downward

(f) $f(x) = (2x)^6 - 1$



Horizontal shrink (each x-value is multiplied by $\frac{1}{2}$), and vertical shift one unit downward

19. $f(x) = 12x^3 + 4x$

Degree: 3

Leading coefficient: 12

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

20. $f(x) = 2x^2 - 3x + 1$

Degree: 2

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

21. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: -3

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

22. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

23. $g(x) = 6x - x^3 + x^2$

Degree: 3

Leading coefficient: -1

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

24. $g(x) = 8 + \frac{1}{4}x^5 - x^4$

Degree: 5

Leading coefficient: $\frac{1}{4}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

25. $f(x) = 9.8x^6 - 1.2x^3$

Degree: 6

Leading coefficient: 9.8

The degree is even and the leading coefficient is positive.

The graph rises to the left and rises to the right.

26. $f(x) = 1 - 0.5x^5 - 2.7x^3$

Degree: 5

Leading coefficient: -0.5

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

27. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

Degree: 3

Leading coefficient: $-\frac{7}{8}$

The degree is odd and the leading coefficient is negative.

The graph rises to the left and falls to the right.

28. $h(t) = -\frac{4}{3}(t - 6t^3 + 2t^4 + 9)$

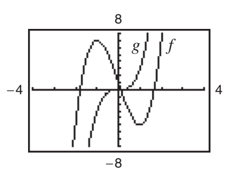
Degree: 4

Leading coefficient: $-\frac{8}{3}$

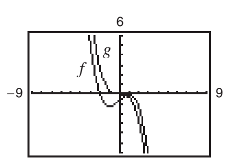
The degree is even and the leading coefficient is negative.

The graph falls to the left and falls to the right.

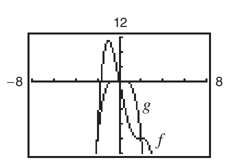
29. $f(x) = 3x^3 - 9x + 1$; $g(x) = 3x^3$



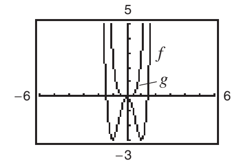
30. $f(x) = -\frac{1}{3}(x^3 - 3x + 2)$, $g(x) = -\frac{1}{3}x^3$



31. $f(x) = -(x^4 - 4x^3 + 16x)$; $g(x) = -x^4$



32. $f(x) = 3x^4 - 6x^2$, $g(x) = 3x^4$



33. $f(x) = x^2 - 36$

(a) $0 = x^2 - 36$

$0 = (x + 6)(x - 6)$

$x + 6 = 0 \qquad x - 6 = 0$

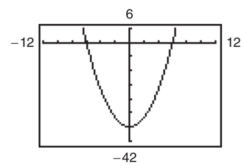
$x = -6 \qquad x = 6$

Zeros: ± 6

(b) Each zero has a multiplicity of one (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



34. $f(x) = 81 - x^2$

(a) $0 = 81 - x^2$

$0 = (9 - x)(9 + x)$

$9 - x = 0 \qquad 9 + x = 0$

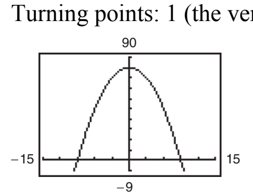
$9 = x \qquad x = -9$

Zeros: ± 9

(b) Each zero has a multiplicity of one (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



35. $h(t) = t^2 - 6t + 9$

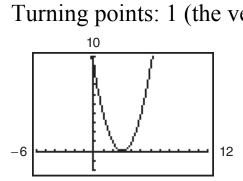
(a) $0 = t^2 - 6t + 9 = (t - 3)^2$

Zero: $t = 3$

(b) $t = 3$ has a multiplicity of 2 (even multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



36. $f(x) = x^2 + 10x + 25$

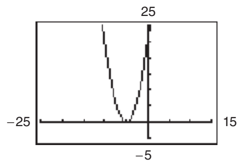
(a) $0 = x^2 + 10x + 25 = (x + 5)^2$

Zero: $x = -5$

(b) $x = -5$ has a multiplicity of 2 (even multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



37. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

(a) $0 = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

$= \frac{1}{3}(x^2 + x - 2)$

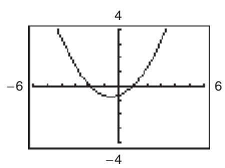
$= \frac{1}{3}(x + 2)(x - 1)$

Zeros: $x = -2, x = 1$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



38. $f(x) = \dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2}$

(a) For $\dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2} = 0, a = \dfrac{1}{2}, b = \dfrac{5}{2}, c = -\dfrac{3}{2}.$

$$x = \frac{-\dfrac{5}{2} \pm \sqrt{\left(\dfrac{5}{2}\right)^2 - 4\left(\dfrac{1}{2}\right)\left(-\dfrac{3}{2}\right)}}{1}$$

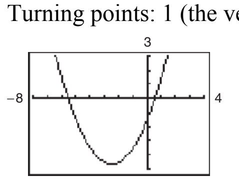
$$= -\frac{5}{2} \pm \sqrt{\frac{37}{4}}$$

Zeros: $x = \dfrac{-5 \pm \sqrt{37}}{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



39. $g(x) = 5x(x^2 - 2x - 1)$

(a) $0 = 5x(x^2 - 2x - 1)$

$0 = x(x^2 - 2x - 1)$

For $x^2 - 2x - 1 = 0, a = 1, b = -2, c = -1.$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

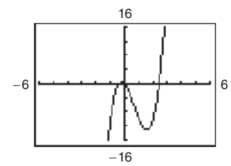
$$= 1 \pm \sqrt{2}$$

Zeros: $x = 0, x = 1 \pm \sqrt{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



40. $f(t) = t^2(3t^2 - 10t + 7)$

(a) $0 = t^2(3t^2 - 10t + 7)$

$0 = t^2(3t - 7)(t - 1)$

$t^2 = 0 \Rightarrow t = 0$

$3t - 7 = 0 \Rightarrow t = \frac{7}{3}$

$t - 1 = 0 \Rightarrow t = 1$

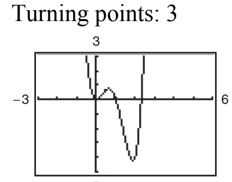
Zeros: $t = 0, t = \frac{7}{3}, t = 1$

(b) $t = 0$ has a multiplicity of 2 (even multiplicity).

$t = \frac{7}{3}$ and $t = 1$ each have a multiplicity of 1 (odd multiplicity).

(c) Turning points: 3

(d)



41. $f(x) = 3x^3 - 12x^2 + 3x$

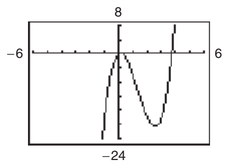
(a) $0 = 3x^3 - 12x^2 + 3x = 3x(x^2 - 4x + 1)$

Zeros: $x = 0, x = 2 \pm \sqrt{3}$ (by the Quadratic Formula)

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



42. $f(x) = x^4 - x^3 - 30x^2$

(a) $0 = x^4 - x^3 - 30x^2$

$0 = x^2(x^2 - x - 30)$

$0 = x^2(x - 6)(x + 5)$

$x^2 = 0 \quad x - 6 = 0 \quad x + 5 = 0$

$x = 0 \quad x = 6 \quad x = -5$

Zeros: $x = 0, x = 6, x = -5$

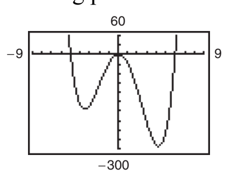
(b) The multiplicity of $x = 0$ is 2 (even multiplicity).

The multiplicity of $x = 6$ is 1 (odd multiplicity).

The multiplicity of $x = -5$ is 1 (odd multiplicity).

(c) Turning points: 3

(d)



43. $g(t) = t^5 - 6t^3 + 9t$

(a) $0 = t^5 - 6t^3 + 9t = t(t^4 - 6t^2 + 9) = t(t^2 - 3)^2$

$= t(t + \sqrt{3})^2(t - \sqrt{3})^2$

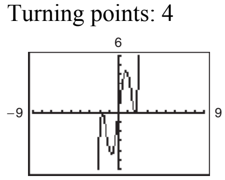
Zeros: $t = 0, t = \pm\sqrt{3}$

(b) $t = 0$ has a multiplicity of 1 (odd multiplicity).

$t = \pm\sqrt{3}$ each have a multiplicity of 2 (even multiplicity).

(c) Turning points: 4

(d)



44. (a) $f(x) = x^5 + x^3 - 6x$

$0 = x(x^4 + x^2 - 6)$

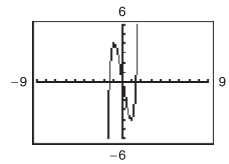
$0 = x(x^2 + 3)(x^2 - 2)$

Zeros: $x = 0, \pm\sqrt{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



45. $f(x) = 3x^4 + 9x^2 + 6$

(a) $0 = 3x^4 + 9x^2 + 6$

$0 = 3(x^4 + 3x^2 + 2)$

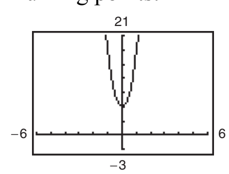
$0 = 3(x^2 + 1)(x^2 + 2)$

No real zeros

(b) No multiplicity

(c) Turning points: 1

(d)



46. $f(t) = 2t^4 - 2t^2 - 40$

(a) $0 = 2t^4 - 2t^2 - 40$

$0 = 2(t^4 - t^2 - 20)$

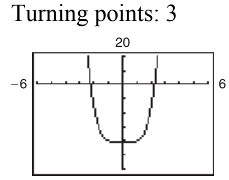
$0 = 2(t^2 + 4)(t^2 - 5)$

Zeros: $t = \pm\sqrt{5}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 3

(d)



47. $g(x) = x^3 + 3x^2 - 4x - 12$

(a) $0 = x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3)$

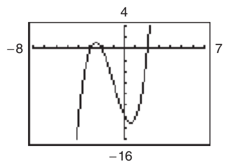
$= (x^2 - 4)(x + 3) = (x - 2)(x + 2)(x + 3)$

Zeros: $x = \pm 2, x = -3$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



48. $f(x) = x^3 - 4x^2 - 25x + 100$

(a) $0 = x^2(x - 4) - 25(x - 4)$

$0 = (x^2 - 25)(x - 4)$

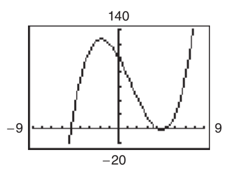
$0 = (x + 5)(x - 5)(x - 4)$

Zeros: $x = \pm 5, 4$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

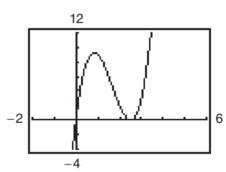
(c) Turning points: 2

(d)



49. $y = 4x^3 - 20x^2 + 25x$

(a)



(b) x-intercepts: $(0, 0), \left(\frac{5}{2}, 0\right)$

(c) $0 = 4x^3 - 20x^2 + 25x$

$0 = x(4x^2 - 20x + 25)$

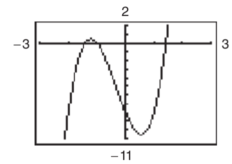
$0 = x(2x - 5)^2$

$x = 0, \frac{5}{2}$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

50. $y = 4x^3 + 4x^2 - 8x - 8$

(a)



(b) $(-1, 0), (-1.41, 0), (1.41, 0)$

(c) $0 = 4x^3 + 4x^2 - 8x - 8$

$0 = 4x^2(x + 1) - 8(x + 1)$

$0 = (4x^2 - 8)(x + 1)$

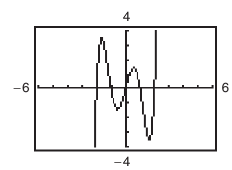
$0 = 4(x^2 - 2)(x + 1)$

$x = \pm\sqrt{2}, -1$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

51. $y = x^5 - 5x^3 + 4x$

(a)



(b) x-intercepts: $(0, 0), (\pm 1, 0), (\pm 2, 0)$

(c) $0 = x^5 - 5x^3 + 4x$

$0 = x(x^2 - 1)(x^2 - 4)$

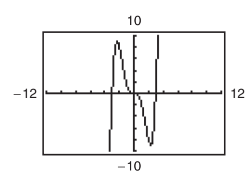
$0 = x(x + 1)(x - 1)(x + 2)(x - 2)$

$x = 0, \pm 1, \pm 2$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

52. $y = \frac{1}{5}x^5 - \frac{9}{5}x^3$

(a)



(b) x-intercepts: $(0, 0), (3, 0), (-3, 0)$

(c) $0 = \frac{1}{5}x^5 - \frac{9}{5}x^3$

$0 = \frac{1}{5}x^3(x^2 - 9)$

$\frac{1}{5}x^3 = 0, \Rightarrow x = 0$

$x^2 - 9 = 0, \Rightarrow x = \pm 3$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

53. $f(x) = (x - 0)(x - 7)$
$= x^2 - 7x$

Note: $f(x) = ax(x - 7)$ has zeros 0 and 7 for all real numbers $a \neq 0$.

54. $f(x) = (x - (-2))(x - 5)$
$= (x + 2)(x - 5)$
$= x^2 - 3x - 10$

Note: $f(x) = a(x + 2)(x - 5)$ has zeros -2 and 5 for all real numbers $a \neq 0$.

55. $f(x) = (x - 0)(x + 2)(x + 4)$
$= x(x^2 + 6x + 8)$
$= x^3 + 6x^2 + 8x$

Note: $f(x) = ax(x + 2)(x + 4)$ has zeros $0, -2,$ and -4 for all real numbers $a \neq 0$.

56. $f(x) = (x - 0)(x - 1)(x - 6)$
$= x(x^2 - 7x + 6)$
$= x^3 - 7x^2 + 6x$

Note: $f(x) = ax(x - 1)(x - 6)$ has zeros 0, 1, and 6 for all real numbers $a \neq 0$.

57. $f(x) = (x - 4)(x + 3)(x - 3)(x - 0)$
$= (x - 4)(x^2 - 9)x$
$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, -3, 3,$ and 0 for all real numbers $a \neq 0$.

58. $f(x) = (x - (-2))(x - (-1))(x - 0)(x - 1)(x - 2)$
$= x(x + 2)(x + 1)(x - 1)(x - 2)$
$= x(x^2 - 4)(x^2 - 1)$
$= x(x^4 - 5x^2 + 4)$
$= x^5 - 5x^3 + 4x$

Note: $f(x) = ax(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, -1, 0, 1,$ and 2 for all real numbers $a \neq 0$.

59. $f(x) = \left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right]$
$= \left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right]$
$= (x - 1)^2 - \left(\sqrt{2}\right)^2$
$= x^2 - 2x + 1 - 2$
$= x^2 - 2x - 1$

Note: $f(x) = a(x^2 - 2x - 1)$ has zeros $1 + \sqrt{2}$ and $1 - \sqrt{2}$ for all real numbers $a \neq 0$.

60. $f(x) = \left[x - \left(4 + \sqrt{3}\right)\right]\left[x - \left(4 - \sqrt{3}\right)\right]$
$= \left[(x - 4) - \sqrt{3}\right]\left[(x - 4) + \sqrt{3}\right]$
$= (x - 4)^2 - \left(\sqrt{3}\right)^2$
$= x^2 - 8x + 16 - 3$
$= x^2 - 8x + 13$

Note: $f(x) = a(x^2 - 8x + 13)$ has zeros $4 + \sqrt{3}$ and $4 - \sqrt{3}$ for all real numbers $a \neq 0$.

61. $f(x) = (x - 2)\left[x - \left(2 + \sqrt{5}\right)\right]\left[x - \left(2 - \sqrt{5}\right)\right]$
$= (x - 2)\left[(x - 2) - \sqrt{5}\right]\left[(x - 2) + \sqrt{5}\right]$
$= (x - 2)\left[(x - 2)^2 - 5\right]$
$= (x - 2)\left[x^2 - 4x + 4 - 5\right]$
$= (x - 2)(x^2 - 4x - 1)$
$= x^3 - 6x^2 + 7x + 2$

Note: $f(x) = a(x^3 - 6x^2 + 7x + 2)$ has zeros 2, $2 + \sqrt{5}$, and $2 - \sqrt{5}$ for all real numbers $a \neq 0$.

62. $f(x) = (x - 3)\left[x - \left(2 + \sqrt{7}\right)\right]\left[x - \left(2 - \sqrt{7}\right)\right]$
$= (x - 3)\left[(x - 2) - \sqrt{7}\right]\left[(x - 2) + \sqrt{7}\right]$
$= (x - 3)\left[(x - 2)^2 - 7\right]$
$= (x - 3)\left[x^2 - 4x + 4 - 7\right]$
$= (x - 3)(x^2 - 4x - 3)$
$= x^3 - 7x^2 + 9x + 9$

Note: $f(x) = a(x^3 - 7x^2 + 9x + 9)$ has zeros 3, $2 + \sqrt{7}$ and $2 - \sqrt{7}$ for all real numbers $a \neq 0$.

63. $f(x) = (x + 3)(x + 3) = x^2 + 6x + 9$

Note: $f(x) = a(x^2 + 6x + 9)$, $a \neq 0$, has degree 2 and zero $x = -3$.

64. $f(x) = (x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$

Note: $f(x) = a(x^2 - 2)$, $a \neq 0$, has a degree 2 and zeros $x = \pm\sqrt{2}$.

65. $f(x) = (x - 0)(x + 5)(x - 1)$

$$= x(x^2 + 4x - 5)$$

$$= x^3 + 4x^2 - 5x$$

Note: $f(x) = ax(x^2 + 4x - 5)$, $a \neq 0$, has degree 3 and zeros $x = 0, -5,$ and 1.

66. $f(x) = (x + 2)(x + 2)(x - 6)$

$$= (x + 2)^2(x - 6)$$

$$= x^3 - 2x^2 - 20x - 24$$

Note: $f(x) = a(x^3 - 2x^2 - 20x - 24)$, $a \neq 0$, has degree 3 and zeros $x = -2$ and 6.

67. $f(x) = (x - (-5))^2(x - 1)(x - 2) = x^4 + 7x^3 - 3x^2 - 55x + 50$

or $f(x) = (x - (-5))(x - 1)^2(x - 2) = x^4 + x^3 - 15x^2 + 23x - 10$

or $f(x) = (x - (-5))(x - 1)(x - 2)^2 = x^4 - 17x^2 + 36x - 20$

Note: Any nonzero scalar multiple of these functions would also have degree 4 and zeros $x = -5, 1,$ and 2.

68. $f(x) = (x + 1)(x + 1)(x + 4)(x + 4)$

$$= (x + 1)^2(x + 4)^2$$

$$= x^4 + 10x^3 + 33x^2 + 40x + 16$$

Note: $f(x) = a(x^4 + 10x^3 + 33x^2 + 40x + 16)$, $a \neq 0$, has degree 4 and zeros $x = -1$ and -4.

69. $f(x) = (x - 0)(x - 0)(x - 0)(x - \sqrt{3})(x - (-\sqrt{3}))$

$$= x^3(x - \sqrt{3})(x + \sqrt{3})$$

$$= x^3(x^2 - 3)$$

$$= x^5 - 3x^3$$

Note: $f(x) = a(x^5 - 3x^3)$, $a \neq 0$, has degree 5 and zeros $x = 0, \sqrt{3}$, and $-\sqrt{3}$.

70. $f(x) = (x + 1)^2(x - 4)(x - 7)(x - 8) = x^5 - 17x^4 + 79x^3 - 11x^2 - 332x - 224$

or $f(x) = (x + 1)(x - 4)^2(x - 7)(x - 8) = x^5 - 22x^4 + 169x^3 - 496x^2 + 208x + 896$

or $f(x) = (x + 1)(x - 4)(x - 7)^2(x - 8) = x^5 - 25x^4 + 223x^3 - 787x^2 + 532x + 1568$

or $f(x) = (x + 1)(x - 4)(x - 7)(x - 8)^2 = x^5 - 26x^4 + 241x^3 - 884x^2 + 640x + 1792$

Note: Any nonzero scalar multiple of these functions would also have degree 5 and zeros $x = -1, 4, 7,$ and 8.

71. $f(t) = \frac{1}{4}(t^2 - 2t + 15) = \frac{1}{4}(t - 1)^2 + \frac{7}{2}$

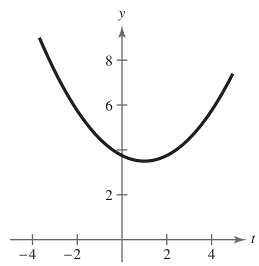
(a) Rises to the left; rises to the right

(b) No real zeros (no x-intercepts)

(c)

t	-1	0	1	2	3
$f(t)$	4.5	3.75	3.5	3.75	4.5

(d) The graph is a parabola with vertex $\left(1, \frac{7}{2}\right)$.



72. $g(x) = -x^2 + 10x - 16 = -(x - 2)(x - 8)$

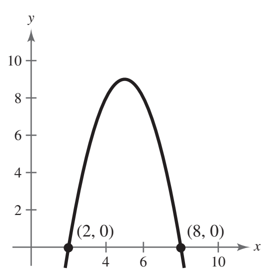
(a) Falls to the left; falls to the right

(b) Zeros: 2, 8

(c)

x	1	3	5	7	9
$g(x)$	-7	5	9	5	-7

(d)



73. $f(x) = x^3 - 25x = x(x + 5)(x - 5)$

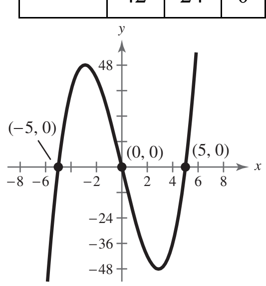
(a) Falls to the left; rises to the right

(b) Zeros: 0, −5, 5

(c)

x	-2	-1	0	1	2
$f(x)$	42	24	0	-24	-42

(d)



74. $g(x) = x^4 - 9x^2 = x^2(x + 3)(x - 3)$

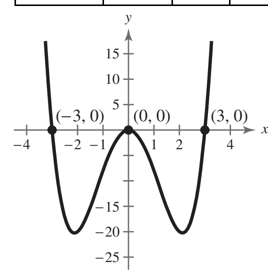
(a) Rises to the left; rises to the right

(b) Zeros: −3, 0, 3

(c)

x	-2	-1	0	1	2
$f(x)$	-24	-8	0	-8	-24

(d)



75. $f(x) = -8 + \frac{1}{2}x^4 = \frac{1}{2}(x^4 - 16)$

$= \frac{1}{2}(x^2 + 4)(x - 2)(x + 2)$

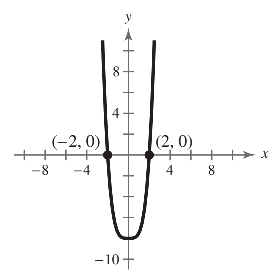
(a) Rises to the left; rises to the right

(b) Zeros $x = \pm 2$:

(c)

x	-2	-1	0	1	2
$f(x)$	0	$-\frac{15}{2}$	-8	$-\frac{15}{2}$	0

(d)



76. $f(x) = 8 - x^3 = (2 - x)(4 + 2x + x^2)$

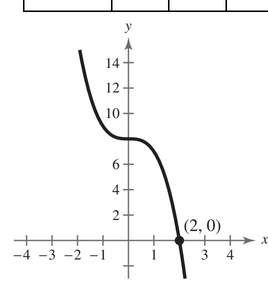
(a) Rises to the left; falls to the right

(b) Zero: 2

(c)

x	-2	-1	0	1	2
$f(x)$	16	9	8	7	0

(d)



77. $f(x) = 3x^3 - 15x^2 + 18x = 3x(x - 2)(x - 3)$

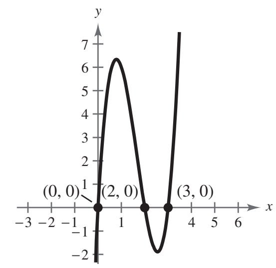
(a) Falls to the left; rises to the right

(b) Zeros: 0, 2, 3

(c)

x	0	1	2	2.5	3	3.5
$f(x)$	0	6	0	−1.875	0	7.875

(d)



78. $f(x) = -4x^3 + 4x^2 + 15x$

$= -x(4x^2 - 4x - 15)$

$= -x(2x - 5)(2x + 3)$

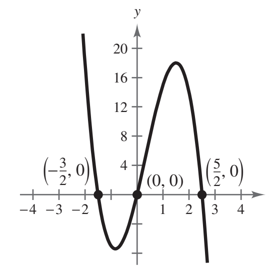
(a) Rises to the left; falls to the right

(b) Zeros: $-\frac{3}{2}, 0, \frac{5}{2}$

(c)

x	−3	−2	−1	0	1	2	3
$f(x)$	99	18	−7	0	15	14	−27

(d)



79. $f(x) = -5x^2 - x^3 = -x^2(5 + x)$

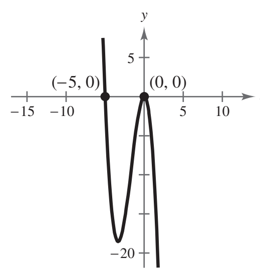
(a) Rises to the left; falls to the right

(b) Zeros: 0, −5

(c)

x	−5	−4	−3	−2	−1	0	1
$f(x)$	0	−16	−18	−12	−4	0	−6

(d)



80. $f(x) = -48x^2 + 3x^4$

$= 3x^2(x^2 - 16)$

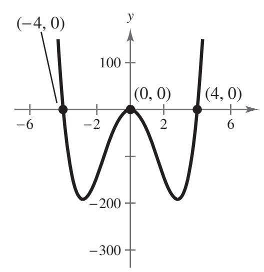
(a) Rises to the left; rises to the right

(b) Zeros; 0, ±4

(c)

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$f(x)$	675	0	−189	−144	−45	0	−45	−144	−189	0	675

(d)



81. $f(x) = 9x^2(x + 2)^2$

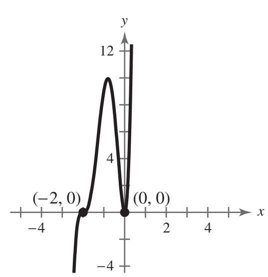
(a) Falls to the left, rises to the right

(b) Zeros: $x = 0, -2$

(c)

x	−3	−2	−1	0	1
$f(x)$	81	0	9	0	81

(d)



82. $h(x) = \frac{1}{3}x^3(x - 4)^2$

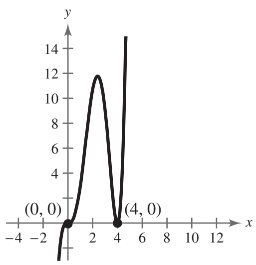
(a) Falls to the left; rises to the right

(b) Zeros: 0, 4

(c)

x	-1	0	1	2	3	4	5
$h(x)$	$-\frac{25}{3}$	0	3	$\frac{32}{3}$	9	0	$\frac{125}{3}$

(d)



83. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$

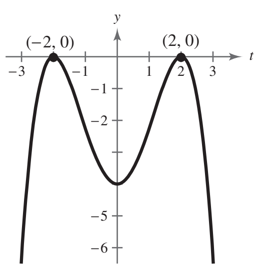
(a) Falls to the left; falls to the right

(b) Zeros: 2, -2

(c)

t	-3	-2	-1	0	1	2	3
$g(t)$	$-\frac{25}{4}$	0	$-\frac{9}{4}$	-4	$-\frac{9}{4}$	0	$-\frac{25}{4}$

(d)



84. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

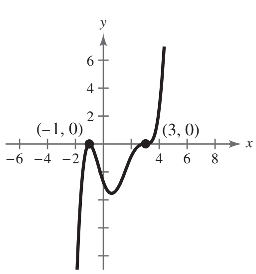
(a) Falls to the left; rises to the right

(b) Zeros: -1, 3

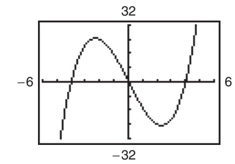
(c)

x	-2	-1	0	1	2	4
$g(x)$	-12.5	0	-2.7	-3.2	-0.9	2.5

(d)

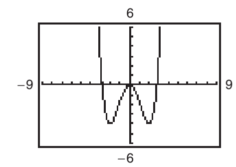


85. $f(x) = x^3 - 16x = x(x - 4)(x + 4)$



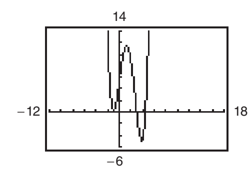
Zeros: 0 of multiplicity 1; 4 of multiplicity 1; and -4 of multiplicity 1

86. $f(x) = \frac{1}{4}x^4 - 2x^2$



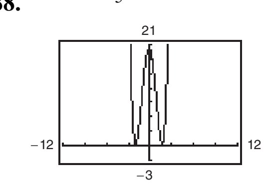
Zeros: $-2\sqrt{2}$ and $2\sqrt{2}$ of multiplicity 1; 0 of multiplicity 2

87. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$



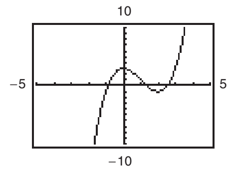
Zeros: -1 of multiplicity 2; 3 of multiplicity 1; $\frac{9}{2}$ of multiplicity 1

88. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$



Zeros: $-2, \frac{5}{3}$, both with multiplicity 2

89. $f(x) = x^3 - 3x^2 + 3$

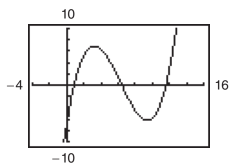


x	y
−3	−51
−2	−17
−1	−1
0	3
1	1
2	−1
2	−21
3	3
4	19

The function has three zeros. They are in the intervals $[-1, 0]$, $[1, 2]$, and $[2, 3]$. They are $x \approx -0.879, 1.347, 2.532$.

90. $f(x) = 0.11x^3 - 2.07x^2 + 9.81x - 6.88$

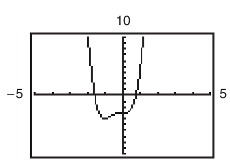
The function has three zeros. They are in the intervals $[0, 1]$, $[6, 7]$, and $[11, 12]$. They are approximately 0.845, 6.385, and 11.588.



x	y
0	−6.88
1	0.97
2	5.34
3	6.89
4	6.28
5	4.17
6	1.12

x	y
7	−1.91
8	−4.56
9	−6.07
10	−5.78
11	3.03
12	2.84

91. $g(x) = 3x^4 + 4x^3 - 3$

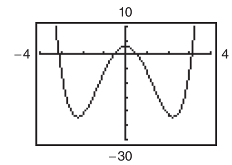


x	y
−4	509
−3	132
−2	13
−1	−4
0	−3
1	4
2	77
3	348

The function has two zeros. They are in the intervals $[-2, -1]$ and $[0, 1]$. They are $x \approx -1.585, 0.779$.

92. $h(x) = x^4 - 10x^2 + 3$

The function has four zeros. They are in the intervals $[-4, -3]$, $[-1, 0]$, $[0, 1]$, and $[3, 4]$. They are approximately ± 3.113 and ± 0.556.



x	y
−4	99
−3	−6
−2	−21
−1	−6
0	3
1	−6
2	−21
3	−6
4	99

93. (a) Volume $= l \cdot w \cdot h$

height $= x$

length $=$ width $= 36 - 2x$

So, $V(x) = (36 - 2x)(36 - 2x)(x) = x(36 - 2x)^2$.

(b) Domain: $0 < x < 18$

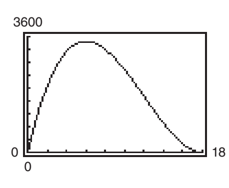
The length and width must be positive.

(c)

Box Height	Box Width	Box Volume, V
1	$36 - 2(1)$	$1[36 - 2(1)]^2 = 1156$
2	$36 - 2(2)$	$2[36 - 2(2)]^2 = 2048$
3	$36 - 2(3)$	$3[36 - 2(3)]^2 = 2700$
4	$36 - 2(4)$	$4[36 - 2(4)]^2 = 3136$
5	$36 - 2(5)$	$5[36 - 2(5)]^2 = 3380$
6	$36 - 2(6)$	$6[36 - 2(6)]^2 = 3456$
7	$36 - 2(7)$	$7[36 - 2(7)]^2 = 3388$

The volume is a maximum of 3456 cubic inches when the height is 6 inches and the length and width are each 24 inches. So the dimensions are $6 \times 24 \times 24$ inches.

(d)



The maximum point on the graph occurs at $x = 6$. This agrees with the maximum found in part (c).

94. (a) $\text{Volume} = l \cdot w \cdot h = (24 - 2x)(24 - 4x)x$

$= 2(12 - x) \cdot 4(6 - x)x$

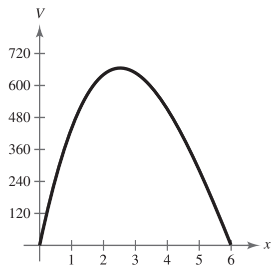
$= 8x(12 - x)(6 - x)$

(b) $x > 0, \quad 12 - x > 0, \quad 6 - x > 0$

$x < 12 \qquad x < 6$

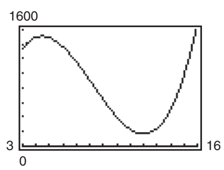
Domain: $0 < x < 6$

(c)



$x \approx 2.5$ corresponds to a maximum of 665 cubic inches.

95. (a)

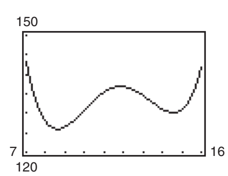


Using trace and zoom features, the relative maximum is approximately $(4.44, 1512.60)$ and the relative minimum is approximately $(11.97, 189.37)$.

(b) The revenue is increasing on $(3, 4.44)$ and $(11.97, 16)$ and decreasing on $(4.44, 11.97)$.

(c) Answers will vary. *Sample answer:* The revenue for the software company was increasing from 2003 to midway through 2004 when it reached a maximum of approximately \$1.5 trillion. Then from 2004 to 2012 the revenue was decreasing. It decreased to \$189 million. From 2012 to 2016 the revenue has been increasing.

96. (a)



Using trace and zoom features, the relative maximum is approximately $(11.85, 136.84)$ and the relative minimum is approximately $(8.60, 125.93)$ and $(14.55, 130.16)$.

(b) The revenue is increasing on $(8.60, 11.85)$ and $(14.55, 16)$ and decreasing on $(7, 8.60)$ and $(11.85, 14.55)$.

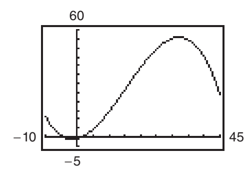
(c) Answers will vary. *Sample answer:* The revenue for the construction company was decreasing from 2007 to midway through 2008 when it reached a minimum of approximately \$125 million. Then from 2008 to late 2011 the revenue was increasing to a maximum of \$137 million. The revenue once again was decreasing from 2012 to 2015. The lower revenue was about \$130 million. Since 2015 to 2016, revenue has been increasing.

97. $R = \dfrac{1}{100{,}000}\left(-x^3 + 600x^2\right)$

The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$. The point is $(200, 160)$ which corresponds to spending \$2,000,000 on advertising to obtain a revenue of \$160 million.

98. $G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839, 2 \le t \le 34$

(a)



(b) The tree is growing most rapidly at $t \approx 15$.

(c) $y = -0.009t^2 + 0.274t + 0.458$

$-\dfrac{b}{2a} = \dfrac{-0.274}{2(-0.009)} \approx 15.222$

$y(15.222) \approx 2.543$

$\text{Vertex} \approx (15.22, 2.54)$

(d) The x-value of the vertex in part (c) is approximately equal to the value found in part (b).

99. True. A polynomial function only falls to the right when the leading coefficient is negative.

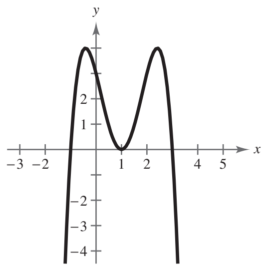
100. False. A fifth-degree polynomial can have at most four turning points.

101. False. The range of an even function cannot be $(-\infty, \infty)$. An even function's graph will fall to the left and right or rise to the left and right.

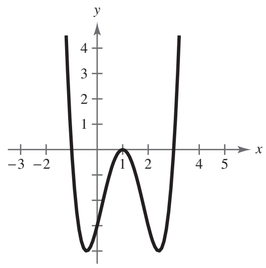
102. False. f has at least one real zero between $x = 2$ and $x = 6$.

103. Answers will vary. *Sample answers:*

$a_4 < 0$

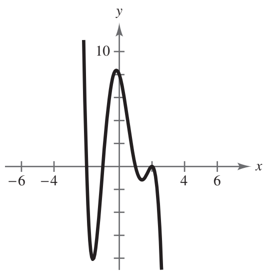


$a_4 > 0$

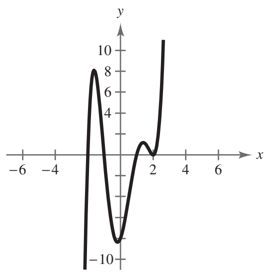


104. Answers will vary. *Sample answers:*

$a_5 < 0$



$a_5 > 0$



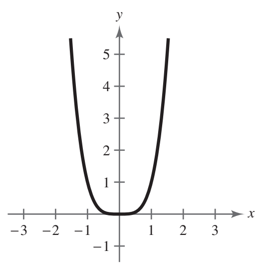
105. $f(x) = x^4$; $f(x)$ is even.

(a) $g(x) = f(x) + 2$

Vertical shift two units upward

$$g(-x) = f(-x) + 2$$
$$= f(x) + 2$$
$$= g(x)$$

Even



(b) $g(x) = f(x + 2)$

Horizontal shift two units to the left

Neither odd nor even

(c) $g(x) = f(-x) = (-x)^4 = x^4$

Reflection in the y-axis. The graph looks the same.

Even

(d) $g(x) = -f(x) = -x^4$

Reflection in the x-axis

Even

(e) $g(x) = f\left(\frac{1}{2}x\right) = \frac{1}{16}x^4$

Horizontal stretch

Even

(f) $g(x) = \frac{1}{2}f(x) = \frac{1}{2}x^4$

Vertical shrink

Even

(g) $g(x) = f\left(x^{3/4}\right) = \left(x^{3/4}\right)^4 = x^3, x \geq 0$

Neither odd nor even

(h) $g(x) = (f \circ f)(x) = f(f(x)) = f\left(x^4\right) = \left(x^4\right)^4 = x^{16}$

Even

106. (a) Degree: 3

Leading coefficient: Positive

(b) Degree: 2

Leading coefficient: Positive

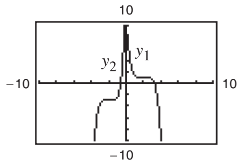
(c) Degree: 4

Leading coefficient: Positive

(d) Degree: 5

Leading coefficient: Positive

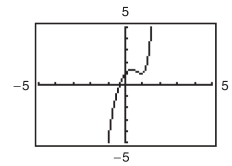
107.



(a) $y_1 = -\frac{1}{3}(x-2)^5 + 1$ is decreasing and $y_2 = \frac{3}{5}(x+2)^5 - 3$ is increasing.

(b) It is possible for $g(x) = a(x-h)^5 + k$ to be strictly increasing if $a > 0$ and strictly decreasing if $a < 0$.

(c) f cannot be written in the form $f(x) = a(x-h)^5 + k$ because f is not strictly increasing or strictly decreasing.



Section 2.3 Polynomial and Synthetic Division

1. $f(x)$ is the dividend; $d(x)$ is the divisor; $q(x)$ is the quotient; $r(x)$ is the remainder

2. proper

3. improper

4. synthetic division

5. Factor

6. Remainder

7. $y_1 = \dfrac{x^2}{x+2}$ and $y_2 = x - 2 + \dfrac{4}{x+2}$

$$
\begin{array}{r}
x - 2 \\
x + 2 \overline{\smash{\big)}\, x^2 + 0x + 0} \\
\underline{x^2 + 2x} \phantom{{}+0} \\
-2x + 0 \\
\underline{-2x - 4} \\
4
\end{array}
$$

So, $\dfrac{x^2}{x+2} = x - 2 + \dfrac{4}{x+2}$ and $y_1 = y_2$.

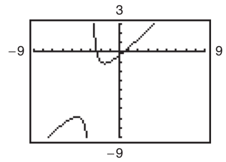
8. $y_1 = \dfrac{x^3 - 3x^2 + 4x - 1}{x+3}$ and $y_2 = x^2 - 6x + 22 - \dfrac{67}{x+3}$

$$
\begin{array}{r}
x^2 - 6x + 22 \\
x + 3 \overline{\smash{\big)}\, x^3 - 3x^2 + 4x - 1} \\
\underline{-(x^3 + 3x^2)} \phantom{{}+4x-1} \\
-6x^2 + 4x - 1 \\
\underline{-6x^2 - 18x} \phantom{{}-1} \\
22x - 1 \\
\underline{22x + 66} \\
-67
\end{array}
$$

So, $\dfrac{x^3 - 3x^2 + 4x - 1}{x+3} = x^2 - 6x + 22 - \dfrac{67}{x+3}$ and $y_1 = y_2$.

9. $y_1 = \dfrac{x^2 + 2x - 1}{x + 3}, y_2 = x - 1 + \dfrac{2}{x + 3}$

(a) and (b)

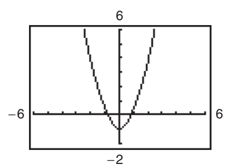


(c)
$$\begin{array}{r} x - 1 \\ x + 3\overline{\big)x^2 + 2x - 1} \\ \underline{x^2 + 3x} \\ -x - 1 \\ \underline{-x - 3} \\ 2 \end{array}$$

So, $\dfrac{x^2 + 2x - 1}{x + 3} = x - 1 + \dfrac{2}{x + 3}$ and $y_1 = y_2$.

10. $y_1 = \dfrac{x^4 + x^2 - 1}{x^2 + 1}, y_2 = x^2 - \dfrac{1}{x^2 + 1}$

(a) and (b)



(c)
$$\begin{array}{r} x^2 \\ x^2 + 0x + 1\overline{\big)x^4 + 0x^3 + x^2 + 0x - 1} \\ \underline{x^4 + 0x^3 + x^2 \qquad\quad} \\ -1 \end{array}$$

So, $\dfrac{x^4 + x^2 - 1}{x^2 + 1} = x^2 - \dfrac{1}{x^2 + 1}$ and $y_1 = y_2$.

11.
$$\begin{array}{r} 2x + 4 \\ x + 3\overline{\big)2x^2 + 10x + 12} \\ \underline{2x^2 + 6x \quad} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

$\dfrac{2x^2 + 10x + 12}{x + 3} = 2x + 4, x \neq 3$

12.
$$\begin{array}{r} 5x + 3 \\ x - 4\overline{\big)5x^2 - 17x - 12} \\ \underline{5x^2 - 20x \quad} \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

$\dfrac{5x^2 - 17x - 12}{x - 4} = 5x + 3, x \neq 4$

13.
$$\begin{array}{r} x^2 - 3x + 1 \\ 4x + 5\overline{\big)4x^3 - 7x^2 - 11x + 5} \\ \underline{4x^3 + 5x^2 \qquad\qquad} \\ -12x^2 - 11x \quad \\ \underline{-12x^2 - 15x} \quad \\ 4x + 5 \\ \underline{4x + 5} \\ 0 \end{array}$$

$\dfrac{4x^3 - 7x^2 - 11x + 5}{4x + 5} = x^2 - 3x + 1, x \neq -\dfrac{5}{4}$

14.
$$\begin{array}{r} 2x^2 - 4x + 3 \\ 3x - 2\overline{\big)6x^3 - 16x^2 + 17x - 6} \\ \underline{6x^3 - 4x^2 \qquad\qquad} \\ -12x^2 + 17x \quad \\ \underline{-12x^2 + 8x} \quad \\ 9x - 6 \\ \underline{9x - 6} \\ 0 \end{array}$$

$\dfrac{6x^3 - 16x^2 + 17x - 6}{3x - 2} = 2x^2 - 4x + 3, x \neq \dfrac{2}{3}$

15.
$$\begin{array}{r} x^3 + 3x^2 \qquad -1 \\ x + 2\overline{\big)x^4 + 5x^3 + 6x^2 - x - 2} \\ \underline{x^4 + 2x^3 \qquad\qquad\qquad} \\ 3x^3 + 6x^2 \qquad\quad \\ \underline{3x^3 + 6x^2} \qquad\quad \\ -x - 2 \\ \underline{-x - 2} \\ 0 \end{array}$$

$\dfrac{x^4 + 5x^3 + 6x^2 - x - 2}{x + 2} = x^3 + 3x^2 - 1, x \neq -2$

16.
$$\begin{array}{r} x^2 + 7x + 18 \\ x - 3\overline{\big)x^3 + 4x^2 - 3x - 12} \\ \underline{x^3 - 3x^2 \qquad\qquad} \\ 7x^2 - 3x \quad \\ \underline{7x^2 - 21x} \quad \\ 18x - 12 \\ \underline{18x - 54} \\ 42 \end{array}$$

$\dfrac{x^3 + 4x^2 - 3x - 12}{x - 3} = x^2 + 7x + 18 + \dfrac{42}{x - 3}$

17.
$$\begin{array}{r} 6 \\ x + 1\overline{\big)6x + 5} \\ \underline{6x + 6} \\ -1 \end{array}$$

$\dfrac{6x + 5}{x + 1} = 6 - \dfrac{1}{x + 1}$

18.
$$\begin{array}{rl} & 3 \\ 3x+2 & \overline{)9x-4} \\ & \underline{9x+6} \\ & -10 \end{array}$$

$$\frac{9x-4}{3x+2} = 3 - \frac{10}{3x+2}$$

19.
$$\begin{array}{rl} & x \\ x^2+0x+1 & \overline{)x^3+0x^2+0x-9} \\ & \underline{x^3+0x^2+x} \\ & -x-9 \end{array}$$

$$\frac{x^3-9}{x^2+1} = x - \frac{x+9}{x^2+1}$$

20.
$$\begin{array}{rl} & x \\ x^4+0x^3+0x^2+0x-1 & \overline{)x^5+0x^4+0x^3+0x^2+0x+7} \\ & \underline{x^5+0x^4+0x^3+0x^2-x} \\ & x+7 \end{array}$$

$$\frac{x^5+7}{x^4-1} = x + \frac{x+7}{x^4-1}$$

21.
$$\begin{array}{rl} & 2x-8 \\ x^2+0x+1 & \overline{)2x^3-8x^2+3x-9} \\ & \underline{2x^3+0x^2+2x} \\ & -8x^2+x-9 \\ & \underline{-8x^2-0x-8} \\ & x-1 \end{array}$$

$$\frac{2x^3-8x^2+3x-9}{x^2+1} = 2x-8+\frac{x-1}{x^2+1}$$

22.
$$\begin{array}{rl} & x^2+6x+9 \\ x^2-x-3 & \overline{)x^4+5x^3+0x^2-20x-16} \\ & \underline{x^4-x^3-3x^2} \\ & 6x^3+3x^2-20x \\ & \underline{6x^3-6x^2-18x} \\ & 9x^2-2x-16 \\ & \underline{9x^2-9x-27} \\ & 7x+11 \end{array}$$

$$\frac{x^4+5x^3-20x-16}{x^2-x-3} = x^2+6x+9+\frac{7x+11}{x^2-x-3}$$

23.
$$\begin{array}{rl} & x+3 \\ x^3-3x^2+3x-1 & \overline{)x^4+0x^3+0x^2+0x+0} \\ & \underline{x^4-3x^3+3x^2-x} \\ & 3x^3-3x^2+x+0 \\ & \underline{3x^3-9x^2+9x-3} \\ & 6x^2-8x+3 \end{array}$$

$$\frac{x^4}{(x-1)^3} = x+3+\frac{6x^2-8x+3}{(x-1)^3}$$

24.
$$\begin{array}{rl} & 2x \\ x^2-2x+1 & \overline{)2x^3-4x^2-15x+5} \\ & \underline{2x^3-4x^2+2x} \\ & -17x+5 \end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2} = 2x-\frac{17x-5}{x^2-2x+1}$$

25.
$$\begin{array}{r|rrrr} 4 & 2 & -10 & 14 & -24 \\ & & 8 & -8 & 24 \\ \hline & 2 & -2 & 6 & 0 \end{array}$$

$$\frac{2x^3-10x^2+14x-24}{x-4} = 2x^2-2x+6,\ x \neq 4$$

26.
$$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$

$$\frac{5x^3+18x^2+7x-6}{x+3} = 5x^2+3x-2,\ x \neq -3$$

27.
$$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3+7x^2-x+26}{x-3} = 6x^2+25x+74+\frac{248}{x-3}$$

28.
$$\begin{array}{r|rrrr} -4 & 2 & 12 & 14 & -3 \\ & & -8 & -16 & 8 \\ \hline & 2 & 4 & -2 & 5 \end{array}$$

$$\frac{2x^3+12x^2+14x-3}{x+4} = 2x^2+4x-2+\frac{5}{x+4}$$

29.
$$\begin{array}{r|rrrr} -2 & 4 & 8 & -9 & -18 \\ & & -8 & 0 & 18 \\ \hline & 4 & 0 & -9 & 0 \end{array}$$

$$\frac{4x^3+8x^2-9x-18}{x+2} = 4x^2-9,\ x \neq -2$$

30.
$$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3 - 18x^2 - 16x + 32}{x - 2} = 9x^2 - 16, x \neq 2$$

31.
$$\begin{array}{r|rrrr} -10 & -1 & 0 & 75 & -250 \\ & & 10 & -100 & 250 \\ \hline & -1 & 10 & -25 & 0 \end{array}$$

$$\frac{-x^3 + 75x - 250}{x + 10} = -x^2 + 10x - 25, x \neq -10$$

32.
$$\begin{array}{r|rrrr} 6 & 3 & -16 & 0 & -72 \\ & & 18 & 12 & 72 \\ \hline & 3 & 2 & 12 & 0 \end{array}$$

$$\frac{3x^3 - 16x^2 - 72}{x - 6} = 3x^2 + 2x + 12, x \neq 6$$

33.
$$\begin{array}{r|rrrr} 4 & 1 & -3 & 0 & 5 \\ & & 4 & 4 & 16 \\ \hline & 1 & 1 & 4 & 21 \end{array}$$

$$\frac{x^3 - 3x^2 + 5}{x - 4} = x^2 + x + 4 + \frac{21}{x - 4}$$

34.
$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3 + 6x + 8}{x + 2} = 5x^2 - 10x + 26 - \frac{44}{x + 2}$$

35.
$$\begin{array}{r|rrrrr} 6 & 10 & -50 & 0 & 0 & -800 \\ & & 60 & 60 & 360 & 2160 \\ \hline & 10 & 10 & 60 & 360 & 1360 \end{array}$$

$$\frac{10x^4 - 50x^3 - 800}{x - 6} = 10x^3 + 10x^2 + 60x + 360 + \frac{1360}{x - 6}$$

36.
$$\begin{array}{r|rrrrrr} -3 & 1 & -13 & 0 & 0 & -120 & 80 \\ & & -3 & 48 & -144 & 432 & -936 \\ \hline & 1 & -16 & 48 & -144 & 312 & -856 \end{array}$$

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3} = x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x + 3}$$

37.
$$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3 + 512}{x + 8} = x^2 - 8x + 64, x \neq -8$$

38.
$$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$

$$\frac{x^3 - 729}{x - 9} = x^2 + 9x + 81, x \neq 9$$

39.
$$\begin{array}{r|rrrrr} 2 & -3 & 0 & 0 & 0 & 0 \\ & & -6 & -12 & -24 & -48 \\ \hline & -3 & -6 & -12 & -24 & -48 \end{array}$$

$$\frac{-3x^4}{x - 2} = -3x^3 - 6x^2 - 12x - 24 - \frac{48}{x - 2}$$

40.
$$\begin{array}{r|rrrrrr} 2 & -2 & 0 & 0 & 0 & 0 & 0 \\ & & 4 & -8 & 16 & -32 & 64 \\ \hline & -2 & 4 & -8 & 16 & -32 & 64 \end{array}$$

$$\frac{-2x^5}{x + 2} = -2x^4 + 4x^3 - 8x^2 + 16x - 32 + \frac{64}{x + 2}$$

41.
$$\begin{array}{r|rrrrr} 6 & -1 & 0 & 0 & 180 & 0 \\ & & -6 & -36 & -216 & -216 \\ \hline & -1 & -6 & -36 & -36 & -216 \end{array}$$

$$\frac{180x - x^4}{x - 6} = -x^3 - 6x^2 - 36x - 36 - \frac{216}{x - 6}$$

42.
$$\begin{array}{r|rrrr} -1 & -1 & 2 & -3 & 5 \\ & & 1 & -3 & 6 \\ \hline & -1 & 3 & -6 & 11 \end{array}$$

$$\frac{5 - 3x + 2x^2 - x^3}{x + 1} = -x^2 + 3x - 6 + \frac{11}{x + 1}$$

43.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}} = 4x^2 + 14x - 30,\ x \neq -\frac{1}{2}$$

44.
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$

$$\frac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}} = 3x^2 + \frac{1}{2}x + \frac{3}{4} + \frac{49}{8x - 12}$$

45. $f(x) = x^3 - x^2 - 10x + 7,\ k = 3$
$$\begin{array}{r|rrrr} 3 & 1 & -1 & -10 & 7 \\ & & 3 & 6 & -12 \\ \hline & 1 & 2 & -4 & -5 \end{array}$$

$$f(x) = (x - 3)(x^2 + 2x - 4) - 5$$
$$f(3) = 3^3 - 3^2 - 10(3) + 7 = -5$$

46. $f(x) = x^3 - 4x^2 - 10x + 8,\ k = -2$
$$\begin{array}{r|rrrr} -2 & 1 & -4 & -10 & 8 \\ & & -2 & 12 & -4 \\ \hline & 1 & -6 & 2 & 4 \end{array}$$

$$f(x) = (x + 2)(x^2 - 6x + 2) + 4$$
$$f(-2) = (-2)^3 - 4(-2)^2 - 10(-2) + 8 = 4$$

47. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14,\ k = -\frac{2}{3}$
$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$$f(x) = \left(x + \tfrac{2}{3}\right)(15x^3 - 6x + 4) + \tfrac{34}{3}$$
$$f\left(-\tfrac{2}{3}\right) = 15\left(-\tfrac{2}{3}\right)^4 + 10\left(-\tfrac{2}{3}\right)^3 - 6\left(-\tfrac{2}{3}\right)^2 + 14 = \tfrac{34}{3}$$

48. $f(x) = 10x^3 - 22x^2 - 3x + 4,\ k = \frac{1}{5}$
$$\begin{array}{r|rrrr} \frac{1}{5} & 10 & -22 & -3 & 4 \\ & & 2 & -4 & -\frac{7}{5} \\ \hline & 10 & -20 & -7 & \frac{13}{5} \end{array}$$

$$f(x) = \left(x - \tfrac{1}{5}\right)(10x^2 - 20x - 7) + \tfrac{13}{5}$$
$$f\left(\tfrac{1}{5}\right) = 10\left(\tfrac{1}{5}\right)^3 - 22\left(\tfrac{1}{5}\right)^2 - 3\left(\tfrac{1}{5}\right) + 4 = \tfrac{13}{5}$$

49. $f(x) = -4x^3 + 6x^2 + 12x + 4,\ k = 1 - \sqrt{3}$
$$\begin{array}{r|rrrr} 1 - \sqrt{3} & -4 & 6 & 12 & 4 \\ & & -4 + 4\sqrt{3} & -10 + 2\sqrt{3} & -4 \\ \hline & -4 & 2 + 4\sqrt{3} & 2 + 2\sqrt{3} & 0 \end{array}$$

$$f(x) = \left(x - 1 + \sqrt{3}\right)\left[-4x^2 + \left(2 + 4\sqrt{3}\right)x + \left(2 + 2\sqrt{3}\right)\right]$$
$$f\left(1 - \sqrt{3}\right) = -4\left(1 - \sqrt{3}\right)^3 + 6\left(1 - \sqrt{3}\right)^2 + 12\left(1 - \sqrt{3}\right) + 4 = 0$$

50. $f(x) = -3x^3 + 8x^2 + 10x - 8, k = 2 + \sqrt{2}$

$$\begin{array}{r|rrrr}
2+\sqrt{2} & -3 & 8 & 10 & -8 \\
 & & -6-3\sqrt{2} & -2-4\sqrt{2} & 8 \\
\hline
 & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & 0
\end{array}$$

$$f(x) = \left(x - 2 - \sqrt{2}\right)\left[-3x^2 + \left(2 - 3\sqrt{2}\right)x + 8 - 4\sqrt{2}\right]$$

$$f\left(2 + \sqrt{2}\right) = -3\left(2 + \sqrt{2}\right)^3 + 8\left(2 + \sqrt{2}\right)^2 + 10\left(2 + \sqrt{2}\right) - 8 = 0$$

51. $f(x) = 2x^3 - 7x + 3$

(a) Using the Remainder Theorem:

$f(1) = 2(1)^3 - 7(1) + 3 = -2$

Using synthetic division:

$$\begin{array}{r|rrrr}
1 & 2 & 0 & -7 & 3 \\
 & & 2 & 2 & -5 \\
\hline
 & 2 & 2 & -5 & -2
\end{array}$$

Verify using long division:

$$\begin{array}{r}
2x^2 + 2x - 5 \\
x - 1\overline{\big)2x^3 + 0x^2 - 7x + 3} \\
\underline{2x^3 - 2x^2} \qquad\qquad \\
2x^2 - 7x \qquad \\
\underline{2x^2 - 2x} \qquad \\
-5x + 3 \\
\underline{-5x + 5} \\
-2
\end{array}$$

(b) Using the Remainder Theorem:

$f(-2) = 2(-2)^3 - 7(-2) + 3 = 1$

Using synthetic division:

$$\begin{array}{r|rrrr}
-2 & 2 & 0 & -7 & 3 \\
 & & -4 & 8 & -2 \\
\hline
 & 2 & -4 & 1 & 1
\end{array}$$

Verify using long division:

$$\begin{array}{r}
2x^2 - 4x + 1 \\
x + 2\overline{\big)2x^3 + 0x^2 - 7x + 3} \\
\underline{2x^3 + 4x^2} \qquad\qquad \\
-4x^2 - 7x \qquad \\
\underline{-4x^2 - 8x} \qquad \\
x + 3 \\
\underline{x + 2} \\
1
\end{array}$$

(c) Using the Remainder Theorem:

$f(3) = 2(3)^3 - 7(3) + 3 = 36$

Using synthetic division:

$$\begin{array}{r|rrrr}
3 & 2 & 0 & -7 & 3 \\
 & & 6 & 18 & 33 \\
\hline
 & 2 & 6 & 11 & 36
\end{array}$$

Verify using long division:

$$\begin{array}{r}
2x^2 + 6x + 11 \\
x - 3\overline{\big)2x^3 + 0x^2 - 7x + 3} \\
\underline{2x^3 - 6x^2} \qquad\qquad \\
6x^2 - 7x + 3 \\
\underline{6x^2 - 18x} \qquad \\
11x + 3 \\
\underline{11x - 33} \\
36
\end{array}$$

(d) Using the Remainder Theorem:

$f(2) = 2(2)^3 - 7(2) + 3 = 5$

Using synthetic division:

$$\begin{array}{r|rrrr}
2 & 2 & 0 & -7 & 3 \\
 & & 4 & 8 & 2 \\
\hline
 & 2 & 4 & 1 & 5
\end{array}$$

Verify using long division:

$$\begin{array}{r}
2x^2 + 4x + 1 \\
x - 2\overline{\big)2x^3 + 0x^2 - 7x + 3} \\
\underline{2x^3 - 4x^2} \qquad\qquad \\
4x^2 - 7x \qquad \\
\underline{4x^2 - 8x} \qquad \\
x + 3 \\
\underline{x - 2} \\
5
\end{array}$$

52. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a) Using the Remainder Theorem:

$$g(2) = 2(2)^6 + 3(2)^4 - (2)^2 + 3 = 175$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^5 + 4x^4 + 11x^3 + 22x^2 + 43x + 86 \\ x - 2\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \underline{2x^6 - 4x^5} \qquad\qquad\qquad\qquad\qquad\qquad\quad \\ 4x^5 + 3x^4 \qquad\qquad\qquad\qquad\qquad\quad \\ \underline{4x^5 - 8x^4} \qquad\qquad\qquad\qquad\qquad\quad \\ 11x^4 + 0x^3 \qquad\qquad\qquad\qquad \\ \underline{11x^4 - 22x^3} \qquad\qquad\qquad\qquad \\ 22x^3 - x^2 \qquad\qquad\quad \\ \underline{22x^3 - 44x^2} \qquad\qquad\quad \\ 43x^2 + 0x \qquad\quad \\ \underline{43x^2 - 86x} \qquad\quad \\ 86x + 3 \\ \underline{86x - 172} \\ 175 \end{array}$$

(b) Using the Remainder Theorem:

$$g(1) = 2(1)^6 + 3(1)^4 - (1)^2 + 3 = 7$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 2 & 2 & 5 & 5 & 4 & 4 \\ \hline & 2 & 2 & 5 & 5 & 4 & 4 & 7 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^5 + 2x^4 + 5x^3 + 5x^2 + 4x + 4 \\ x - 1\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \underline{2x^6 - 2x^5} \qquad\qquad\qquad\qquad\qquad\qquad\quad \\ 2x^5 + 3x^4 \qquad\qquad\qquad\qquad\qquad\quad \\ \underline{2x^5 - 2x^4} \qquad\qquad\qquad\qquad\qquad\quad \\ 5x^4 + 0x^3 \qquad\qquad\qquad\qquad \\ \underline{5x^4 - 5x^3} \qquad\qquad\qquad\qquad \\ 5x^3 - x^2 \qquad\qquad\quad \\ \underline{5x^3 - 5x^2} \qquad\qquad\quad \\ 4x^2 + 0x \qquad\quad \\ \underline{4x^2 - 4x} \qquad\quad \\ 4x + 3 \\ \underline{4x - 4} \\ 7 \end{array}$$

(c) Using the Remainder Theorem:

$$g(3) = 2(3)^6 + 3(3)^4 - (3)^2 + 3 = 1695$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & 564 & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & 564 & 1695 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^5 + 6x^4 + 21x^3 + 63x^2 + 188x + 564 \\ x - 3\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \underline{2x^6 - 6x^5} \qquad\qquad\qquad\qquad\qquad\qquad\quad \\ 6x^5 + 3x^4 \qquad\qquad\qquad\qquad\qquad\quad \\ \underline{6x^4 - 18x^4} \qquad\qquad\qquad\qquad\qquad\quad \\ 21x^4 + 0x^3 \qquad\qquad\qquad\qquad \\ \underline{21x^4 - 63x^3} \qquad\qquad\qquad\qquad \\ 63x^3 - x^2 \qquad\qquad\quad \\ \underline{63x^3 - 189x^2} \qquad\qquad\quad \\ 188x^2 + 0x \qquad\quad \\ \underline{188x^2 - 564x} \qquad\quad \\ 564x + 3 \\ \underline{564x - 1692} \\ 1695 \end{array}$$

(d) Using the Remainder Theorem:

$$g(-1) = 2(-1)^6 + 3(-1)^4 - (-1)^2 + 3 = 7$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} -1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & -2 & 2 & -5 & 5 & -4 & 4 \\ \hline & 2 & -2 & 5 & -5 & 4 & -4 & 7 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^5 - 2x^4 + 5x^3 - 5x^2 + 4x - 4 \\ x + 1\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \underline{2x^6 + 2x^5} \qquad\qquad\qquad\qquad\qquad\qquad\quad \\ -2x^5 + 3x^4 \qquad\qquad\qquad\qquad\qquad\quad \\ \underline{-2x^5 - 2x^4} \qquad\qquad\qquad\qquad\qquad\quad \\ 5x^4 + 0x^3 \qquad\qquad\qquad\qquad \\ \underline{5x^4 + 5x^3} \qquad\qquad\qquad\qquad \\ -5x^3 - x^2 \qquad\qquad\quad \\ \underline{-5x^3 - 5x^2} \qquad\qquad\quad \\ 4x^2 + 0x \qquad\quad \\ \underline{4x^2 + 4x} \qquad\quad \\ -4x + 3 \\ \underline{-4x - 4} \\ 7 \end{array}$$

53. $h(x) = x^3 - 5x^2 - 7x + 4$

(a) Using the Remainder Theorem:

$$h(3) = (3)^3 - 5(3)^2 - 7(3) + 4 = -35$$

Using synthetic division:

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 2x - 13 \\ x - 3 \overline{\big) x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 - 3x^2} \qquad\qquad \\ -2x^2 - 7x \qquad \\ \underline{-2x^2 + 6x} \qquad \\ -13x + 4 \\ \underline{-13x + 39} \\ -35 \end{array}$$

(b) Using the Remainder Theorem:

$$h\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^3 - 5\left(\tfrac{1}{2}\right)^2 - 7\left(\tfrac{1}{2}\right) + 4 = -\frac{5}{8}$$

Using synthetic division:

$$\begin{array}{r|rrrr} \frac{1}{2} & 1 & -5 & -7 & 4 \\ & & \frac{1}{2} & -\frac{9}{4} & -\frac{37}{8} \\ \hline & 1 & -\frac{9}{2} & -\frac{37}{4} & -\frac{5}{8} \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - \frac{9}{2}x - \frac{37}{4} \\ x - \frac{1}{2} \overline{\big) x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 - \frac{1}{2}x^2} \qquad\qquad \\ -\frac{9}{2}x^2 - 7x + 4 \\ \underline{-\frac{9}{2}x^2 + \frac{9}{4}x} \qquad \\ -\frac{37}{4}x + 4 \\ \underline{-\frac{37}{4}x + \frac{37}{8}} \\ -\frac{5}{8} \end{array}$$

(c) Using the Remainder Theorem:

$$h(-2) = (-2)^3 - 5(-2)^2 - 7(-2) + 4 = -10$$

Using synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 7x + 7 \\ x + 2 \overline{\big) x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 + 2x^2} \qquad\qquad \\ -7x^2 - 7x \qquad \\ \underline{-7x^2 - 14x} \qquad \\ 7x + 4 \\ \underline{7x + 14} \\ -10 \end{array}$$

(d) Using the Remainder Theorem:

$$h(-5) = (-5)^3 - 5(-5)^2 - 7(-5) + 4 = -211$$

Using synthetic division:

$$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 10x + 43 \\ x + 5 \overline{\big) x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 + 5x^2} \qquad\qquad \\ -10x^2 - 7x \qquad \\ \underline{-10x^2 - 50x} \qquad \\ 43x + 4 \\ \underline{43x + 215} \\ -211 \end{array}$$

54. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) Using the Remainder Theorem:

$f(1) = 4(1)^4 - 16(1)^3 + 7(1) + 20 = 15$

Using synthetic division:

$$\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array}$$

Verify using long division:

$$\begin{array}{rl} & 4x^3 - 12x^2 - 5x - 5 \\ x - 1 & \overline{)\,4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ & \underline{4x^4 - 4x^3} \\ & \quad -12x^3 + 7x^2 \\ & \quad \underline{-12x^3 + 12x^2} \\ & \qquad -5x^2 + 0x \\ & \qquad \underline{-5x^2 + 5x} \\ & \qquad\quad -5x + 20 \\ & \qquad\quad \underline{-5x + 5} \\ & \qquad\qquad 15 \end{array}$$

(b) Using the Remainder Theorem:

$f(-2) = 4(-2)^4 - 16(-2)^3 + 7(-2)^2 + 20 = 240$

Using synthetic division:

$$\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array}$$

Verify using long division:

$$\begin{array}{rl} & 4x^3 - 24x^2 + 55x - 110 \\ x + 2 & \overline{)\,4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ & \underline{4x^3 + 8x^3} \\ & \quad -24x^3 + 7x^2 \\ & \quad \underline{-24x^3 - 48x^2} \\ & \qquad 55x^2 + 0x \\ & \qquad \underline{55x^2 + 110x} \\ & \qquad\quad -110x + 20 \\ & \qquad\quad \underline{-110x - 220} \\ & \qquad\qquad 240 \end{array}$$

(c) Using the Remainder Theorem:

$f(5) = 4(5)^4 - 16(5)^3 + 7(5)^2 + 20 = 695$

Using synthetic division:

$$\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array}$$

Verify using long division:

$$\begin{array}{rl} & 4x^3 + 4x^2 + 27x + 135 \\ x - 5 & \overline{)\,4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ & \underline{4x^4 - 20x^3} \\ & \quad 4x^3 + 7x^2 \\ & \quad \underline{4x^3 - 20x^2} \\ & \qquad 27x^2 + 0x \\ & \qquad \underline{27x^2 - 135x} \\ & \qquad\quad 135x + 20 \\ & \qquad\quad \underline{135x - 675} \\ & \qquad\qquad 695 \end{array}$$

(d) Using the Remainder Theorem:

$f(-10) = 4(-10)^4 - 16(-10)^3 + 7(-10)^2 + 20 = 56{,}720$

Using synthetic division:

$$\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56{,}700 \\ \hline & 4 & -56 & 567 & -5670 & 56{,}720 \end{array}$$

Verify using long division:

$$\begin{array}{rl} & 4x^3 - 56x^2 + 567x - 5670 \\ x + 10 & \overline{)\,4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ & \underline{4x^4 + 40x^3} \\ & \quad -56x^3 + 7x^2 \\ & \quad \underline{-56x^3 - 560x^2} \\ & \qquad 567x^2 + 0x \\ & \qquad \underline{567x^2 + 5670x} \\ & \qquad\quad -5670x + 20 \\ & \qquad\quad \underline{-5670x - 56{,}700} \\ & \qquad\qquad 56{,}720 \end{array}$$

55.

$$\begin{array}{r|rrrr} -3 & 1 & 6 & 11 & 6 \\ & & -3 & -9 & -6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$\begin{aligned} x^3 + 6x^2 + 11x + 6 &= (x + 3)(x^2 + 3x + 2) \\ &= (x + 3)(x + 2)(x + 1) \end{aligned}$$

Zeros: $-3, -2, -1$

56.

$$\begin{array}{r|rrrr} -6 & 1 & 0 & -52 & -96 \\ & & -6 & 36 & 96 \\ \hline & 1 & -6 & -16 & 0 \end{array}$$

$$\begin{aligned} x^3 - 52x - 96 &= (x + 6)(x^2 - 6x - 16) \\ &= (x + 6)(x - 8)(x + 2) \end{aligned}$$

Zeros: $-6, 8, -2$

57.
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$$\begin{aligned} 2x^3 - 15x^2 + 27x - 10 &= \left(x - \tfrac{1}{2}\right)\left(2x^2 - 14x + 20\right) \\ &= (2x - 1)(x - 2)(x - 5) \end{aligned}$$

Zeros: $\frac{1}{2}, 2, 5$

58.
$$\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

$$\begin{aligned} 48x^3 - 80x^2 + 41x - 6 &= \left(x - \tfrac{2}{3}\right)\left(48x^2 - 48x + 9\right) \\ &= \left(x - \tfrac{2}{3}\right)(4x - 3)(12x - 3) \\ &= (3x - 2)(4x - 3)(4x - 1) \end{aligned}$$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

59.
$$\begin{array}{r|rrrr} \sqrt{3} & 1 & 2 & -3 & -6 \\ & & \sqrt{3} & 3 + 2\sqrt{3} & 6 \\ \hline & 1 & 2 + \sqrt{3} & 2\sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{3} & 1 & 2 + \sqrt{3} & 2\sqrt{3} \\ & & -\sqrt{3} & -2\sqrt{3} \\ \hline & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 - 3x - 6 = \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x + 2)$$

Zeros: $-\sqrt{3}, \sqrt{3}, -2$

60.
$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 2 & -2 & -4 \\ & & \sqrt{2} & 2\sqrt{2} + 2 & 4 \\ \hline & 1 & 2 + \sqrt{2} & 2\sqrt{2} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{2} & 1 & 2 + \sqrt{2} & 2\sqrt{2} \\ & & -\sqrt{2} & -2\sqrt{2} \\ \hline & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 - 2x - 4 = \left(x - \sqrt{2}\right)(x + 2)\left(x + \sqrt{2}\right)$$

Zeros: $-2, -\sqrt{2}, \sqrt{2}$

61.
$$\begin{array}{r|rrrr} 1 + \sqrt{3} & 1 & -3 & 0 & 2 \\ & & 1 + \sqrt{3} & 1 - \sqrt{3} & -2 \\ \hline & 1 & -2 + \sqrt{3} & 1 - \sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 - \sqrt{3} & 1 & -2 + \sqrt{3} & 1 - \sqrt{3} \\ & & 1 - \sqrt{3} & -1 + \sqrt{3} \\ \hline & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} x^3 - 3x^2 + 2 &= \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right](x - 1) \\ &= (x - 1)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{3}\right) \end{aligned}$$

Zeros: $1, 1 - \sqrt{3}, 1 + \sqrt{3}$

62.
$$\begin{array}{r|rrrr} 2 - \sqrt{5} & 1 & -1 & -13 & -3 \\ & & 2 - \sqrt{5} & 7 - 3\sqrt{5} & 3 \\ \hline & 1 & 1 - \sqrt{5} & -6 - 3\sqrt{5} & 0 \end{array}$$

$$\begin{array}{r|rrr} 2 + \sqrt{5} & 1 & 1 - \sqrt{5} & -6 - 3\sqrt{5} \\ & & 2 + \sqrt{5} & 6 + 3\sqrt{5} \\ \hline & 1 & 3 & 0 \end{array}$$

$$x^3 - x^2 - 13x - 3 = \left(x - 2 + \sqrt{5}\right)\left(x - 2 - \sqrt{5}\right)(x + 3)$$

Zeros: $2 - \sqrt{5}, 2 + \sqrt{5}, -3$

63. $f(x) = 2x^3 + x^2 - 5x + 2$; Factors: $(x + 2), (x - 1)$

(a)
$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 2 & -3 & 1 \\ & & 2 & -1 \\ \hline & 2 & -1 & 0 \end{array}$$

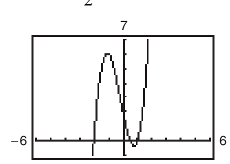
Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor of $f(x)$ is $(2x - 1)$.

(c) $f(x) = (2x - 1)(x + 2)(x - 1)$

(d) Zeros: $\frac{1}{2}, -2, 1$

(e)



64. $f(x) = 3x^3 - x^2 - 8x - 4;$

Factors: $(x + 1), (x - 2)$

(a)
$$\begin{array}{r|rrrr} -1 & 3 & -1 & -8 & -4 \\ & & -3 & 4 & 4 \\ \hline & 3 & -4 & -4 & 0 \end{array}$$

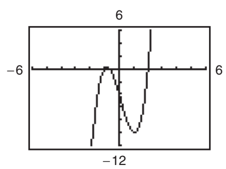
$$\begin{array}{r|rrr} 2 & 3 & -4 & -4 \\ & & 6 & 4 \\ \hline & 3 & 2 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $(3x + 2)$.

(c) $f(x) = 3x^3 - x^2 - 8x - 4$
$= (3x + 2)(x + 1)(x - 2)$

(d) Zeros: $-\frac{2}{3}, -1, 2$

(e)


65. $f(x) = x^4 - 8x^3 + 9x^2 + 38x - 40;$

Factors: $(x - 5), (x + 2)$

(a)
$$\begin{array}{r|rrrrr} 5 & 1 & -8 & 9 & 38 & -40 \\ & & 5 & -15 & -30 & 40 \\ \hline & 1 & -3 & -6 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -6 & 8 \\ & & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

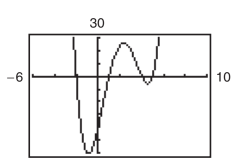
Both are factors of $f(x)$ because the remainders are zero.

(b) $x^2 - 5x + 4 = (x - 1)(x - 4)$

The remaining factors are $(x - 1)$ and $(x - 2)$.

(c) $f(x) = (x - 5)(x + 2)(x - 1)(x - 4)$

(d) Zeros: $-2, 1, 4, 5$

(e)


66. $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24;$

Factors: $(x + 2), (x - 4)$

(a)
$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

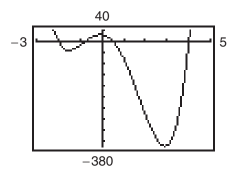
Both are factors of $f(x)$ because the remainders are zero.

(b) $8x^2 + 2x - 3 = (4x + 3)(2x - 1)$

The remaining factors are $(4x + 3)$ and $(2x - 1)$.

(c) $f(x) = (4x + 3)(2x - 1)(x + 2)(x - 4)$

(d) Zeros: $-\frac{3}{4}, \frac{1}{2}, -2, 4$

(e)


67. $f(x) = 6x^3 + 41x^2 - 9x - 14;$

Factors: $(2x + 1), (3x - 2)$

(a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{2}{3} & 6 & 38 & -28 \\ & & 4 & 28 \\ \hline & 6 & 42 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $6x + 42 = 6(x + 7)$

This shows that $\dfrac{f(x)}{\left(x + \frac{1}{2}\right)\left(x - \frac{2}{3}\right)} = 6(x + 7),$

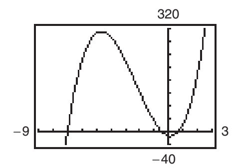
so $\dfrac{f(x)}{(2x + 1)(3x - 2)} = x + 7.$

The remaining factor is $(x + 7)$.

(c) $f(x) = (x + 7)(2x + 1)(3x - 2)$

(d) Zeros: $-7, -\frac{1}{2}, \frac{2}{3}$

(e)



68. $f(x) = 10x^3 - 11x^2 - 72x + 45;$

Factors: $(2x + 5), (5x - 3)$

(a)
$$\begin{array}{r|rrrr} -\frac{5}{2} & 10 & -11 & -72 & 45 \\ & & -25 & 90 & -45 \\ \hline & 10 & -36 & 18 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{5} & 10 & -36 & 18 \\ & & 6 & -18 \\ \hline & 10 & -30 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $10x - 30 = 10(x - 3)$

This shows that $\dfrac{f(x)}{\left(x + \frac{5}{2}\right)\left(x - \frac{3}{5}\right)} = 10(x - 3),$

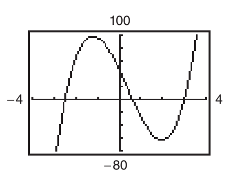
so $\dfrac{f(x)}{(2x + 5)(5x - 3)} = x - 3.$

The remaining factor is $(x - 3)$.

(c) $f(x) = (x - 3)(2x + 5)(5x - 3)$

(d) Zeros: $3, -\frac{5}{2}, \frac{3}{5}$

(e)



69. $f(x) = 2x^3 - x^2 - 10x + 5;$

Factors: $(2x - 1), \left(x + \sqrt{5}\right)$

(a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{5} & 2 & 0 & -10 \\ & & -2\sqrt{5} & 10 \\ \hline & 2 & -2\sqrt{5} & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $2x - 2\sqrt{5} = 2\left(x - \sqrt{5}\right)$

This shows that $\dfrac{f(x)}{\left(x - \frac{1}{2}\right)\left(x + \sqrt{5}\right)} = 2\left(x - \sqrt{5}\right),$

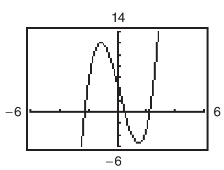
so $\dfrac{f(x)}{(2x - 1)\left(x + \sqrt{5}\right)} = x - \sqrt{5}.$

The remaining factor is $\left(x - \sqrt{5}\right)$.

(c) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(2x - 1)$

(d) Zeros: $-\sqrt{5}, \sqrt{5}, \dfrac{1}{2}$

(e)



70. $f(x) = x^3 + 3x^2 - 48x - 144;$

Factors: $\left(x + 4\sqrt{3}\right), (x + 3)$

(a)
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -48 & -144 \\ & & -3 & 0 & 144 \\ \hline & 1 & 0 & -48 & 0 \end{array}$$

$$\begin{array}{r|rrr} -4\sqrt{3} & 1 & 0 & -48 \\ & & -4\sqrt{3} & 48 \\ \hline & 1 & -4\sqrt{3} & 0 \end{array}$$

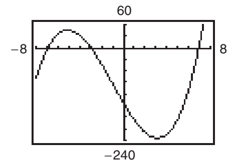
Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $\left(x - 4\sqrt{3}\right)$.

(c) $f(x) = \left(x - 4\sqrt{3}\right)\left(x + 4\sqrt{3}\right)(x + 3)$

(d) Zeros: $\pm 4\sqrt{3}, -3$

(e)



71. $f(x) = x^3 - 2x^2 - 5x + 10$

(a) The zeros of f are $x = 2$ and $x \approx \pm 2.236$.

(b) An exact zero is $x = 2$.

(c)
$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 2)\left(x^2 - 5\right) \\ &= (x - 2)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) \end{aligned}$$

72. $g(x) = x^3 + 3x^2 - 2x - 6$

(a) The zeros of g are $x = -3, x \approx -1.414, x \approx 1.414$.

(b) $x = -3$ is an exact zero.

(c)
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -2 & -6 \\ & & -3 & 0 & 6 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} g(x) &= (x + 3)\left(x^2 - 2\right) \\ &= (x + 3)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) \end{aligned}$$

73. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) The zeros of h are $t = -2, t \approx 3.732, t \approx 0.268$.

(b) An exact zero is $t = -2$.

(c)
$$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$h(t) = (t + 2)\left(t^2 - 4t + 1\right)$

By the Quadratic Formula, the zeros of $t^2 - 4t + 1$ are $2 \pm \sqrt{3}$. Thus,

$h(t) = (t + 2)\left[t - \left(2 + \sqrt{3}\right)\right]\left[t - \left(2 - \sqrt{3}\right)\right].$

74. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) The zeros of f are $s = 6, s \approx 0.764, s \approx 5.236$

(b) $s = 6$ is an exact zero.

(c)
$$\begin{array}{r|rrrr} 6 & 1 & -12 & 40 & -24 \\ & & 6 & -36 & 24 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$

$$\begin{aligned} f(s) &= (s-6)(s^2 - 6s + 4) \\ &= (s-6)\left[s - \left(3 + \sqrt{5}\right)\right]\left[s - \left(3 - \sqrt{5}\right)\right] \end{aligned}$$

75. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) The zeros of h are $x = 0, x = 3, x = 4,$ $x \approx 1.414, x \approx -1.414.$

(b) An exact zero is $x = 4.$

(c)
$$\begin{array}{r|rrrrr} 4 & 1 & -7 & 10 & 14 & -24 \\ & & 4 & -12 & -8 & 24 \\ \hline & 1 & -3 & -2 & 6 & 0 \end{array}$$

$$\begin{aligned} h(x) &= (x-4)(x^4 - 3x^3 - 2x^2 + 6x) \\ &= x(x-4)(x-3)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) \end{aligned}$$

76. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) The zeros of a are $x = 3, x = -3, x = 1.5,$ $x \approx 0.333.$

(b) An exact zero is $x = -3.$

(c)
$$\begin{array}{r|rrrrr} -3 & 6 & -11 & -51 & 99 & -27 \\ & & -18 & 87 & -108 & 27 \\ \hline & 6 & -29 & 36 & -9 & 0 \end{array}$$

$$\begin{aligned} a(x) &= (x+3)(6x^3 - 29x^2 + 36x - 9) \\ &= (x+3)(x-3)(2x-3)(3x-1) \end{aligned}$$

77. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

$$\begin{array}{r|rrrr} -8 & 1 & 1 & -64 & -64 \\ & & -8 & 56 & 64 \\ \hline & 1 & -7 & -8 & 0 \end{array}$$

$$\frac{x^3 + x^2 - 64x - 64}{x + 8} = x^2 - 7x - 8, x \neq -8$$

78. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 4 & -8 & 1 & 3 \\ & & 6 & -3 & -3 \\ \hline & 4 & -2 & -2 & 0 \end{array}$$

$$\frac{4x^3 - 8x^2 + x + 3}{x - \frac{3}{2}} = 4x^2 - 2x - 2 = 2(2x^2 - x - 1)$$

So, $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3} = 2x^2 - x - 1, x \neq \dfrac{3}{2}.$

79. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2} = \dfrac{x^4 + 6x^3 + 11x^2 + 6x}{(x+1)(x+2)}$

$$\begin{array}{r|rrrrr} -1 & 1 & 6 & 11 & 6 & 0 \\ & & -1 & -5 & -6 & 0 \\ \hline & 1 & 5 & 6 & 0 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 6 & 0 \\ & & -2 & -6 & 0 \\ \hline & 1 & 3 & 0 & 0 \end{array}$$

$$\frac{x^4 + 6x^3 + 11x^2 + 6x}{(x+1)(x+2)} = x^2 + 3x, x \neq -2, -1$$

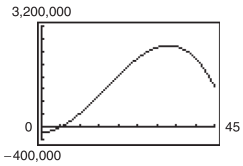
80. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = \dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{(x+2)(x-2)}$

$$\begin{array}{r|rrrrr} 2 & 1 & 9 & -5 & -36 & 4 \\ & & 2 & 22 & 34 & -4 \\ \hline & 1 & 11 & 17 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 11 & 17 & -2 \\ & & -2 & -18 & 2 \\ \hline & 1 & 9 & -1 & 0 \end{array}$$

$$\frac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = x^2 + 9x - 1, x \neq \pm 2$$

81. (a)



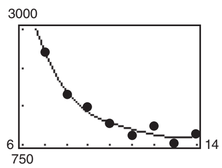
(b) Using the trace and zoom features, when $x = 25$, an advertising expense of about \$250,000 would produce the same profit of \$2,174,375.

(c) $x = 25$

$$\begin{array}{r|rrrr} 25 & -152 & 7545 & 0 & -169{,}625 \\ & & -3800 & 93{,}625 & 2{,}340{,}625 \\ \hline & -152 & 3745 & 93{,}625 & 2{,}171{,}000 \end{array}$$

So, an advertising expense of \$250,000 yields a profit of \$2,171,000, which is close to \$2,174,375.

82. (a) and (b)



$N = 2.6439t^4 - 119.788t^3 + 2044.33t^2 - 15{,}679.5t + 46{,}891$

The data fits the model well.

(c)

t	7	8	9	10	11	12	13	14
N_{model}	2567	1790	1387	1180	1052	951	887	926
N_{actual}	2576	1746	1466	1163	938	1113	801	957

The estimated values are close to the original data values.

(d)

$$\begin{array}{r|rrrrr} 14 & 2.6439 & -119.788 & 2044.33 & -15{,}679.5 & 46{,}891 \\ & & 37.0146 & -1158.8276 & 12{,}397.0336 & -45{,}954.5296 \\ \hline & 2.6439 & -82.7734 & 885.5024 & -3282.4664 & 936.4704 \end{array}$$

Because the remainder is $r \approx 936$, you can conclude that $N(14) \approx 936$. This confirms the estimated value.

83. False. If $(7x + 4)$ is a factor of f, then $-\frac{4}{7}$ is a zero of f.

84. True.

$$\begin{array}{r|rrrrrrr} \frac{1}{2} & 6 & 1 & -92 & 45 & 184 & 4 & -48 \\ & & 3 & 2 & -45 & 0 & 92 & 48 \\ \hline & 6 & 4 & -90 & 0 & 184 & 96 & 0 \end{array}$$

$f(x) = (2x - 1)(x + 1)(x - 2)(x - 3)(3x + 2)(x + 4)$

85. True. The degree of the numerator is greater than the degree of the denominator.

86. False. The equation $\dfrac{x^3 - 3x^2 + 4}{x + 1} = x^2 - 4x + 4$ is not true for $x = -1$ since this value would result in division by zero in the original equation. So, the equation should be written as $\dfrac{x^3 - 3x^2 + 4}{x + 1} = x^2 - 4x + 4$, $x \neq -1$.

87.
$$\begin{array}{r} x^{2n} + 6x^n + 9 \\ x^n + 3 \overline{\smash{\big)} x^{3n} + 9x^{2n} + 27x^n + 27} \\ \underline{x^{3n} + 3x^{2n} } \\ 6x^{2n} + 27x^n \\ \underline{6x^{2n} + 18x^n} \\ 9x^n + 27 \\ \underline{9x^n + 27} \\ 0 \end{array}$$

$$\frac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3} = x^{2n} + 6x^n + 9,\ x^n \neq -3$$

88.
$$\begin{array}{r} x^{2n} - x^n + 3 \\ x^n - 2 \overline{\smash{\big)} x^{3n} - 3x^{2n} + 5x^n - 6} \\ \underline{x^{3n} - 2x^{2n} } \\ -x^{2n} + 5x^n \\ \underline{-x^{2n} + 2x^n} \\ 3x^n - 6 \\ \underline{3x^n - 6} \\ 0 \end{array}$$

$$\frac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2} = x^{2n} - x^n + 3,\ x^n \neq 2$$

89. To divide $x^2 + 3x - 5$ by $x + 1$ using synthetic division, the value of k is $k = -1$ not $k = 1$ as shown.

$$\begin{array}{r|rrr} -1 & 1 & 3 & -5 \\ & & -1 & -2 \\ \hline & 1 & 2 & -7 \end{array} \quad \leftarrow \text{Remainder: } -7$$

90. (a) An advertising expense of \$200,000 when $x = 20$, also appears to yield a profit of about \$936,660.

(b) Evaluate l at $x = 20$.

$$\begin{array}{r|rrrr} 20 & -140.75 & 5348.3 & 0 & -76{,}560 \\ & & -28.15 & 50{,}666 & 1{,}013{,}320 \\ \hline & -140.75 & 2533.3 & 50{,}666 & 936{,}760 \end{array}$$

$P(20) = 936{,}760$

So, an advertising expense of \$20,000 will yield a profit of \$936,760.

91.
$$\begin{array}{r|rrrr} 5 & 1 & 4 & -3 & c \\ & & 5 & 45 & 210 \\ \hline & 1 & 9 & 42 & c + 210 \end{array}$$

To divide evenly, $c + 210$ must equal zero. So, c must equal -210.

92.
$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & -2 & 1 & c \\ & & -2 & 4 & -8 & 20 & -42 \\ \hline & 1 & -2 & 4 & -10 & 21 & c - 42 \end{array}$$

To divide evenly, $c - 42$ must equal zero. So, c must equal 42.

93. If $x - 4$ is a factor of $f(x) = x^3 - kx^2 + 2kx - 8$, then $f(4) = 0$.

$$\begin{aligned} f(4) &= (4)^3 - k(4)^2 + 2k(4) - 8 \\ 0 &= 64 - 16k + 8k - 8 \\ -56 &= -8k \\ 7 &= k \end{aligned}$$

Section 2.4 Complex Numbers

1. real

2. imaginary

3. pure imaginary

4. $\sqrt{-1};\ -1$

5. principal square

6. complex conjugates

7. $a + bi = 9 + 8i$
$a = 9$
$b = 8$

8. $a + bi = 10 - 5i$
$a = 10$
$b = -5$

9. $(a - 2) + (b + 1)i = 6 + 5i$
$a - 2 = 6 \Rightarrow a = 8$
$b + 1 = 5 \Rightarrow b = 4$

10. $(a + 2) + (b - 3)i = 4 + 7i$
$a + 2 = 4 \Rightarrow a = 2$
$b - 3 = 7 \Rightarrow b = 10$

11. $2 + \sqrt{-25} = 2 + 5i$

12. $4 + \sqrt{-49} = 4 + 7i$

13. $1 - \sqrt{-12} = 1 - 2\sqrt{3}\,i$

14. $2 - \sqrt{-18} = 2 - 3\sqrt{2}\,i$

15. $\sqrt{-40} = 2\sqrt{10}\,i$

16. $\sqrt{-27} = 3\sqrt{3}\,i$

17. 23

18. 50

19. $-6i + i^2 = -6i + (-1)$
$= -1 - 6\,i$

20. $-2i^2 + 4i = -2(-1) + 4\,i$
$= 2 + 4\,i$

21. $\sqrt{-0.04} = \sqrt{0.04}\,i$
$= 0.2i$

22. $\sqrt{-0.0025} = \sqrt{0.0025}\,i$
$= 0.05i$

23. $(5 + i) + (2 + 3i) = 5 + i + 2 + 3i$
$= 7 + 4i$

24. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

25. $(9 - i) - (8 - i) = 1$

26. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
$= -3 - 11i$

27. $\left(-2 + \sqrt{-8}\right) + \left(5 - \sqrt{-50}\right) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i = 3 - 3\sqrt{2}i$

28. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i = 4$

29. $13i - (14 - 7i) = 13i - 14 + 7i$
$= -14 + 20i$

30. $25 + (-10 + 11i) + 15i = 15 + 26i$

31. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2 = 5 + i$

32. $(7 - 2i)(3 - 5i) = 21 - 35i - 6i + 10i^2$
$= 21 - 41i - 10$
$= 11 - 41i$

33. $12i(1 - 9i) = 12i - 108i^2$
$= 12i + 108$
$= 108 + 12i$

34. $-8i(9 + 4i) = -72i - 32i^2$
$= 32 - 72i$

35. $\left(\sqrt{2} + 3i\right)\left(\sqrt{2} - 3i\right) = 2 - 9i^2$
$= 2 + 9 = 11$

36. $\left(4 + \sqrt{7}i\right)\left(4 - \sqrt{7}i\right) = 16 - 7i^2$
$= 16 + 7 = 23$

37. $(6 + 7i)^2 = 36 + 84i + 49i^2$
$= 36 + 84i - 49$
$= -13 + 84i$

38. $(5 - 4i)^2 = 25 - 40i + 16i^2$
$= 25 - 40i - 16$
$= 9 - 40i$

39. The complex conjugate of $9 + 2i$ is $9 - 2i$.

$(9 + 2i)(9 - 2i) = 81 - 4i^2$
$= 81 + 4$
$= 85$

40. The complex conjugate of $8 - 10i$ is $8 + 10i$.

$(8 - 10i)(8 + 10i) = 64 - 100i^2$
$= 64 + 100$
$= 164$

41. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.

$\left(-1 - \sqrt{5}i\right)\left(-1 + \sqrt{5}i\right) = 1 - 5i^2$
$= 1 + 5 = 6$

42. The complex conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.

$\left(-3 + \sqrt{2}i\right)\left(-3 - \sqrt{2}i\right) = 9 - 2i^2$
$= 9 + 2$
$= 11$

43. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.

$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right) = -20i^2 = 20$

44. The complex conjugate of $\sqrt{-15} = \sqrt{15}i$ is $-\sqrt{15}i$.

$$\left(\sqrt{15}i\right)\left(-\sqrt{15}i\right) = -15i^2 = 15$$

45. The complex conjugate of $\sqrt{6}$ is $\sqrt{6}$.

$$\left(\sqrt{6}\right)\left(\sqrt{6}\right) = 6$$

46. The complex conjugate of $1 + \sqrt{8}$ is $1 + \sqrt{8}$.

$$\begin{aligned}\left(1 + \sqrt{8}\right)\left(1 + \sqrt{8}\right) &= 1 + 2\sqrt{8} + 8\\ &= 9 + 4\sqrt{2}\end{aligned}$$

47. $$\begin{aligned}\frac{2}{4 - 5i} &= \frac{2}{4 - 5i} \cdot \frac{4 + 5i}{4 + 5i}\\ &= \frac{2(4 + 5i)}{16 + 25} = \frac{8 + 10i}{41} = \frac{8}{41} + \frac{10}{41}i\end{aligned}$$

48. $$\frac{13}{1 - i} \cdot \frac{(1 + i)}{(1 + i)} = \frac{13 + 13i}{1 - i^2} = \frac{13 + 13i}{2} = \frac{13}{2} + \frac{13}{2}i$$

49. $$\begin{aligned}\frac{5 + i}{5 - i} \cdot \frac{(5 + i)}{(5 + i)} &= \frac{25 + 10i + i^2}{25 - i^2}\\ &= \frac{24 + 10i}{26} = \frac{12}{13} + \frac{5}{13}i\end{aligned}$$

50. $$\begin{aligned}\frac{6 - 7i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} &= \frac{6 + 12i - 7i - 14i^2}{1 - 4i^2}\\ &= \frac{20 + 5i}{5} = 4 + i\end{aligned}$$

51. $$\frac{9 - 4i}{i} \cdot \frac{-i}{-i} = \frac{-9i + 4i^2}{-i^2} = -4 - 9i$$

52. $$\frac{8 + 16i}{2i} \cdot \frac{-2i}{-2i} = \frac{-16i - 32i^2}{-4i^2} = 8 - 4i$$

53. $$\begin{aligned}\frac{3i}{(4 - 5i)^2} &= \frac{3i}{16 - 40i + 25i^2} = \frac{3i}{-9 - 40i} \cdot \frac{-9 + 40i}{-9 + 40i}\\ &= \frac{-27i + 120i^2}{81 + 1600} = \frac{-120 - 27i}{1681}\\ &= -\frac{120}{1681} - \frac{27}{1681}i\end{aligned}$$

54. $$\begin{aligned}\frac{5i}{(2 + 3i)^2} &= \frac{5i}{4 + 12i + 9i^2}\\ &= \frac{5i}{-5 + 12i} \cdot \frac{-5 - 12i}{-5 - 12i}\\ &= \frac{-25i - 60i^2}{25 - 144i^2}\\ &= \frac{60 - 25i}{169} = \frac{60}{169} - \frac{25}{169}i\end{aligned}$$

55. $$\begin{aligned}\frac{2}{1 + i} - \frac{3}{1 - i} &= \frac{2(1 - i) - 3(1 + i)}{(1 + i)(1 - i)}\\ &= \frac{2 - 2i - 3 - 3i}{1 + 1}\\ &= \frac{-1 - 5i}{2}\\ &= -\frac{1}{2} - \frac{5}{2}i\end{aligned}$$

56. $$\begin{aligned}\frac{2i}{2 + i} + \frac{5}{2 - i} &= \frac{2i(2 - i) + 5(2 + i)}{(2 + i)(2 - i)}\\ &= \frac{4i - 2i^2 + 10 + 5i}{4 - i^2}\\ &= \frac{12 + 9i}{5}\\ &= \frac{12}{5} + \frac{9}{5}i\end{aligned}$$

57. $$\begin{aligned}\frac{i}{3 - 2i} + \frac{2i}{3 + 8i} &= \frac{i(3 + 8i) + 2i(3 - 2i)}{(3 - 2i)(3 + 8i)}\\ &= \frac{3i + 8i^2 + 6i - 4i^2}{9 + 24i - 6i - 16i^2}\\ &= \frac{4i^2 + 9i}{9 + 18i + 16}\\ &= \frac{-4 + 9i}{25 + 18i} \cdot \frac{25 - 18i}{25 - 18i}\\ &= \frac{-100 + 72i + 225i - 162i^2}{625 + 324}\\ &= \frac{62 + 297i}{949} = \frac{62}{949} + \frac{297}{949}i\end{aligned}$$

58. $$\begin{aligned}\frac{1 + i}{i} - \frac{3}{4 - i} &= \frac{(1 + i)(4 - i) - 3i}{i(4 - i)}\\ &= \frac{4 - i + 4i - i^2 - 3i}{4i - i^2}\\ &= \frac{5}{1 + 4i} \cdot \frac{1 - 4i}{1 - 4i}\\ &= \frac{5 - 20i}{1 - 16i^2}\\ &= \frac{5}{17} - \frac{20}{17}i\end{aligned}$$

59. $$\begin{aligned}\sqrt{-6} \cdot \sqrt{-2} &= \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}i^2 = \left(2\sqrt{3}\right)(-1)\\ &= -2\sqrt{3}\end{aligned}$$

60. $$\begin{aligned}\sqrt{-5} \cdot \sqrt{-10} &= \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)\\ &= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}\end{aligned}$$

61. $\left(\sqrt{-15}\right)^2 = \left(\sqrt{15}i\right)^2 = 15i^2 = -15$

62. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

63. $$\begin{aligned}\sqrt{-8} + \sqrt{-50} &= \sqrt{8}\,i + \sqrt{50}\,i\\ &= 2\sqrt{2}\,i + 5\sqrt{2}\,i\\ &= 7\sqrt{2}\,i\end{aligned}$$

64. $$\begin{aligned}\sqrt{-45} - \sqrt{-5} &= \sqrt{45}\,i - \sqrt{5}\,i\\ &= 3\sqrt{5}\,i - \sqrt{5}\,i\\ &= 2\sqrt{5}\,i\end{aligned}$$

65. $$\begin{aligned}\left(3 + \sqrt{-5}\right)\left(7 - \sqrt{-10}\right) &= \left(3 + \sqrt{5}i\right)\left(7 - \sqrt{10}i\right)\\ &= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2\\ &= \left(21 + \sqrt{50}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i\\ &= \left(21 + 5\sqrt{2}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i\end{aligned}$$

66. $$\begin{aligned}\left(2 - \sqrt{-6}\right)^2 &= \left(2 - \sqrt{6}i\right)\left(2 - \sqrt{6}i\right)\\ &= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2\\ &= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)\\ &= 4 - 6 - 4\sqrt{6}i\\ &= -2 - 4\sqrt{6}i\end{aligned}$$

67. $x^2 - 2x + 2 = 0;\ a = 1, b = -2, c = 2$

$$\begin{aligned}x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}\\ &= \frac{2 \pm \sqrt{-4}}{2}\\ &= \frac{2 \pm 2i}{2}\\ &= 1 \pm i\end{aligned}$$

68. $x^2 + 6x + 10 = 0;\ a = 1, b = 6, c = 10$

$$\begin{aligned}x &= \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}\\ &= \frac{-6 \pm \sqrt{-4}}{2}\\ &= \frac{-6 + 2i}{2}\\ &= -3 \pm i\end{aligned}$$

69. $4x^2 + 16x + 17 = 0;\ a = 4, b = 16, c = 17$

$$\begin{aligned}x &= \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)}\\ &= \frac{-16 \pm \sqrt{-16}}{8}\\ &= \frac{-16 \pm 4i}{8}\\ &= -2 \pm \frac{1}{2}i\end{aligned}$$

70. $9x^2 - 6x + 37 = 0;\ a = 9, b = -6, c = 37$

$$\begin{aligned}x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}\\ &= \frac{6 \pm \sqrt{-1296}}{18}\\ &= \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i\end{aligned}$$

71. $4x^2 + 16x + 21 = 0;\ a = 4, b = 16, c = 21$

$$\begin{aligned}x &= \frac{-16 \pm \sqrt{(16)^2 - 4(4)(21)}}{2(4)}\\ &= \frac{-16 \pm \sqrt{-80}}{8}\\ &= \frac{-16 \pm \sqrt{80}\ i}{8}\\ &= \frac{-16 \pm 4\sqrt{5}\ i}{8}\\ &= -2 \pm \frac{\sqrt{5}}{2}i\end{aligned}$$

72. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$
$$= \frac{4 \pm \sqrt{-176}}{32}$$
$$= \frac{4 \pm 4\sqrt{11}i}{32}$$
$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

73. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{-72}}{6}$$
$$= \frac{12 \pm 6\sqrt{2}i}{6}$$
$$= 2 \pm \sqrt{2}i$$

74. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14,\ b = -12,\ c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}$$
$$= \frac{12 \pm \sqrt{-136}}{28}$$
$$= \frac{12 \pm 2\sqrt{34}i}{28}$$
$$= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

75. $1.4x^2 - 2x + 10 = 0 \Rightarrow 14x^2 - 20x + 100 = 0;$

$a = 14,\ b = -20,\ c = 100$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(14)(100)}}{2(14)}$$
$$= \frac{20 \pm \sqrt{-5200}}{28}$$
$$= \frac{20 \pm 20\sqrt{13}\,i}{28}$$
$$= \frac{20}{28} \pm \frac{20\sqrt{13}\,i}{28}$$
$$= \frac{5}{7} \pm \frac{5\sqrt{13}}{7}i$$

76. $4.5x^2 - 3x + 12 = 0;\ a = 4.5,\ b = -3,\ c = 12$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)}$$
$$= \frac{3 \pm \sqrt{-207}}{9}$$
$$= \frac{3 \pm 3\sqrt{23}i}{9}$$
$$= \frac{1}{3} \pm \frac{\sqrt{23}}{3}i$$

77. $-6i^3 + i^2 = -6i^2i + i^2$
$$= -6(-1)i + (-1)$$
$$= 6i - 1$$
$$= -1 + 6i$$

78. $4i^2 - 2i^3 = 4i^2 - 2i^2i = 4(-1) - 2(-1)i = -4 + 2i$

79. $-14i^5 = -14i^2i^2i = -14(-1)(-1)(i) = -14i$

80. $(-i)^3 = (-1)(i^3) = (-1)i^2i = (-1)(-1)i = i$

81. $\left(\sqrt{-72}\right)^3 = \left(6\sqrt{2}i\right)^3$
$$= 6^3\left(\sqrt{2}\right)^3 i^3$$
$$= 216\left(2\sqrt{2}\right)i^2i$$
$$= 432\sqrt{2}(-1)i$$
$$= -432\sqrt{2}i$$

82. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^2i^2i^2 = 8(-1)(-1)(-1) = -8$

83. $\frac{1}{i^3} = \frac{1}{i^2i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = i$

84. $\dfrac{1}{(2i)^3} = \dfrac{1}{8i^3} = \dfrac{1}{8i^2 i} = \dfrac{1}{-8i} = \dfrac{1}{-8i} \cdot \dfrac{8i}{8i} = \dfrac{8i}{-64i^2} = \dfrac{1}{8}i$

85. $(3i)^4 = 81i^4 = 81i^2i^2 = 81(-1)(-1) = 81$

86. $(-i)^6 = i^6 = i^2i^2i^2 = (-1)(-1)(-1) = -1$

87. (a) $z_1 = 9 + 16i, z_2 = 20 - 10i$

(b) $\dfrac{1}{z} = \dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{1}{9 + 16i} + \dfrac{1}{20 - 10i} = \dfrac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \dfrac{29 + 6i}{340 + 230i}$

$$z = \left(\frac{340 + 230i}{29 + 6i}\right)\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11{,}240 + 4630i}{877} = \frac{11{,}240}{877} + \frac{4630}{877}i$$

88. (a)
$$\begin{aligned}(-1 + \sqrt{3}i)^3 &= (-1)^3 + 3(-1)^2(\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2 + (\sqrt{3}i)^3\\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3\\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^2 i\\ &= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i\\ &= 8\end{aligned}$$

(b)
$$\begin{aligned}(-1 - \sqrt{3}i)^3 &= (-1)^3 + 3(-1)^2(-\sqrt{3}i) + 3(-1)(-\sqrt{3}i)^2 + (-\sqrt{3}i)^3\\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3\\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^2 i\\ &= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i\\ &= 8\end{aligned}$$

89. False.

Sample answer: $(1 + i) + (3 + i) = 4 + 2i$ which is not a real number.

90. False.

If $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

91. True.

$$\begin{aligned}x^4 - x^2 + 14 &= 56\\ (-i\sqrt{6})^4 - (-i\sqrt{6})^2 + 14 &\stackrel{?}{=} 56\\ 36 + 6 + 14 &\stackrel{?}{=} 56\\ 56 &= 56\end{aligned}$$

92. False.

$$\begin{aligned}i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= (i^2)^{22} + (i^2)^{75} - (i^2)^{37} - (i^2)^{54} i + (i^2)^{30} i\\ &= (-1)^{22} + (-1)^{75} - (-1)^{37} - (-1)^{54} i + (-1)^{30} i\\ &= 1 - 1 + 1 - i + i = 1\end{aligned}$$

93. $i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i^4 i = i$

$i^6 = i^4 i^2 = -1$

$i^7 = i^4 i^3 = -i$

$i^8 = i^4 i^4 = 1$

$i^9 = i^4 i^4 i = i$

$i^{10} = i^4 i^4 i^2 = -1$

$i^{11} = i^4 i^4 i^3 = -i$

$i^{12} = i^4 i^4 i^4 = 1$

The pattern $i, -1, -i, 1$ repeats. Divide the exponent by 4.

If the remainder is 1, the result is i.

If the remainder is 2, the result is -1.

If the remainder is 3, the result is $-i$.

If the remainder is 0, the result is 1.

94. (i) D

(ii) F

(iii) B

(iv) E

(v) A

(vi) C

95. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

96. $(a_1 + b_1 i)(a_2 + b_2 i) = a_1 a_2 + a_1 b_2 i + a_2 b_1 i + b_1 b_2 i^2$

$= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1)i$

The complex conjugate of this product is

$(a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1)i.$

The product of the complex conjugates is

$(a_1 - b_1 i)(a_2 - b_2 i) = a_1 a_2 - a_1 b_2 i - a_2 b_1 i - b_1 b_2 i^2$

$= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1)i.$

So, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

97. $(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2)i$

The complex conjugate of this sum is $(a_1 + a_2) - (b_1 + b_2)i.$

The sum of the complex conjugates is $(a_1 - b_1 i) + (a_2 - b_2 i) = (a_1 + a_2) - (b_1 + b_2)i.$

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

Section 2.5 Zeros of Polynomial Functions

1. Fundamental Theorem of Algebra

2. Linear Factorization Theorem

3. Rational Zero

4. complex conjugate

5. linear; quadratic; quadratic

6. irreducible; reals

7. Descartes's Rule of Signs

8. lower; upper

9. f is a 3rd degree polynomial function, so there are three zeros.

10. f is a 4th degree polynomial function, so there are four zeros.

11. f is a 5th degree polynomial function, so there are five zeros.

12. f is a 6th degree polynomial function, so there are six zeros.

13. f is a 2nd degree polynomial function, so there are two zeros.

14. $h(t) = (t - 1)^2 - (t + 1)^2$

$= (t^2 - 2t + 1) - (t^2 + 2t + 1)$

$= -4t$

h is a 1st degree polynomial function, so there is one real zero.

15. $f(x) = x^3 + 2x^2 - x - 2$

Possible rational zeros: $\pm 1, \pm 2$

Zeros shown on graph: $-2, -1, 1, 2$

16. $f(x) = x^3 - 4x^2 - 4x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Zeros shown on graph: $-2, 2, 4$

17. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Zeros shown on graph: $-1, \frac{3}{2}, 3, 5$

18. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{4}$

Zeros shown on graph: $-1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

19. $f(x) = x^3 - 7x - 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(x^2 + 3x + 2) \\ &= (x - 3)(x + 2)(x + 1) \end{aligned}$$

So, the rational zeros are $-2, -1$, and 3.

20. $f(x) = x^3 - 13x + 12$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -13 & -12 \\ & & 3 & 9 & -12 \\ \hline & & 3 & -4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(x^2 + 3x - 4) \\ &= (x - 3)(x + 4)(x - 1) \end{aligned}$$

So, the rational zeros are $3, -4$, and 1.

21. $g(t) = t^3 - 4t^2 + 4$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

After testing all six possible rational zeros by synthetic division, you can conclude there are no rational zeros.

22. $p(x) = x^3 - 19x + 30$

Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -19 & 30 \\ & & 3 & 9 & -30 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

$$\begin{aligned} p(x) &= (x - 3)(x^2 + 3x - 10) \\ &= (x - 3)(x + 5)(x - 2) \end{aligned}$$

So, the rational zeros are $3, -5, 2$.

23. $h(t) = t^3 + 8t^2 + 13t + 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -6 & 1 & 8 & 13 & 6 \\ & & -6 & -12 & -6 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$$\begin{aligned} t^3 + 8t^2 + 13t + 6 &= (t + 6)(t^2 + 2t + 1) \\ &= (t + 6)(t + 1)(t + 1) \end{aligned}$$

So, the rational zeros are -1 and -6.

24. $g(x) = x^3 + 8x^2 + 12x + 18$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$.

After testing all six possible rational zeros by synthetic division, you can conclude there are no rational zeros.

25. $C(x) = 2x^3 + 3x^2 - 1$

Possible rational zeros: $\pm 1, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 0 & -1 \\ & & -2 & -1 & 1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} 2x^3 + 3x^2 - 1 &= (x + 1)(2x^2 + x - 1) \\ &= (x + 1)(x + 1)(2x - 1) \\ &= (x + 1)^2(2x - 1) \end{aligned}$$

So, the rational zeros are -1 and $\frac{1}{2}$.

26. $f(x) = 3x^3 - 19x^2 + 33x - 9$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm\frac{1}{3}$

$$\begin{array}{r|rrrr} 3 & 3 & -19 & 33 & -9 \\ & & 9 & -30 & 9 \\ \hline & 3 & -10 & 3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-3)(3x^2 - 10x + 3) \\ &= (x-3)(3x-1)(x-3) \end{aligned}$$

So, the rational zeros are 3 and $\frac{1}{3}$.

27. $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24,$
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{8}{9}$

$$\begin{array}{r|rrrrr} -2 & 9 & -9 & -58 & 4 & 24 \\ & & -18 & 54 & 8 & -24 \\ \hline & 9 & -27 & -4 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 9 & -27 & -4 & 12 \\ & & 27 & 0 & -12 \\ \hline & 9 & 0 & -4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x+2)(x-3)(9x^2 - 4) \\ &= (x+2)(x-3)(3x-2)(3x+2) \end{aligned}$$

So, the rational zeros are $-2, 3, \frac{2}{3},$ and $-\frac{2}{3}$.

28. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$

Possible rational zeros: $\pm1, \pm5, \pm25, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}$

$$\begin{array}{r|rrrrr} 5 & 2 & -15 & 23 & 15 & -25 \\ & & 10 & -25 & -10 & 25 \\ \hline & 2 & -5 & -2 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -5 & -2 & 5 \\ & & 2 & -3 & -5 \\ \hline & 2 & -3 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 & 2 & -3 & -5 \\ & & -2 & 5 \\ \hline & 2 & -5 & 0 \end{array}$$

$$f(x) = (x-5)(x-1)(x+1)(2x-5)$$

So, the rational zeros are $5, 1, -1$ and $\frac{5}{2}$.

29. $-5x^3 + 11x^2 - 4x - 2 = 0$

Possible rational zeros: $\dfrac{\pm1, \pm2}{\pm1, \pm5} = \pm\dfrac{1}{5}, \pm\dfrac{2}{5}, \pm1, \pm2$

$$\begin{array}{r|rrrr} 1 & -5 & 11 & -4 & -2 \\ & & -5 & 6 & 2 \\ \hline & -5 & 6 & 2 & 0 \end{array}$$

$$\begin{aligned} (x-1)(-5x^2 + 6x + 2) &= 0 \\ -5x^2 + 6x + 2 &= 0 \\ 5x^2 - 6x - 2 &= 0 \end{aligned}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(5)(-2)}}{2(5)}$$

$$x = \frac{6 \pm \sqrt{76}}{10}$$

$$x = \frac{2(3 \pm \sqrt{19})}{10}$$

$$x = \frac{3 \pm \sqrt{19}}{5}$$

So, the real zeros are $x = 1, x = \dfrac{3}{5} \pm \dfrac{\sqrt{19}}{5}$.

30. $8x^3 + 10x^2 - 15x - 6 = 0$

Possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4, \pm 8} = \pm\dfrac{1}{8}, \pm\dfrac{1}{4}, \pm\dfrac{3}{8}, \pm\dfrac{1}{2}, \pm\dfrac{3}{4}, \pm 1, \pm\dfrac{3}{2}, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -2 & 8 & 10 & -15 & -6 \\ & & -16 & 12 & 6 \\ \hline & 8 & -6 & -3 & 0 \end{array}$$

$$(x + 2)(8x^2 - 6x - 3) = 0$$

$$8x^2 - 6x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(8)(-3)}}{2(8)}$$

$$x = \frac{6 \pm \sqrt{132}}{16}$$

$$x = \frac{2(3 \pm \sqrt{33})}{16}$$

$$x = \frac{3 \pm \sqrt{33}}{8}$$

So, the real zeros are $x = -2, x = \dfrac{3}{8} \pm \dfrac{\sqrt{33}}{8}$.

31. $x^4 + 6x^3 + 3x^2 - 16x + 6 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrrr} 1 & 1 & 6 & 3 & -16 & 6 \\ & & 1 & 7 & 10 & -6 \\ \hline & 1 & 7 & 10 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 10 & -6 \\ & & -3 & -12 & 6 \\ \hline & 1 & 4 & -2 & 0 \end{array}$$

$$(x - 1)(x + 3)(x^2 + 4x - 2) = 0$$

$$x^2 + 4x - 2 = 0$$

$$x^2 + 4x + 4 = 2 + 4$$

$$(x + 2)^2 = 6$$

$$x + 2 = \pm\sqrt{6}$$

$$x = -2 \pm \sqrt{6}$$

So the real zeros are $x = 1, -3, -2 \pm 2\sqrt{6}$.

32. $x^4 + 8x^3 + 14x^2 - 17x - 42 = 0$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21, \pm42$

$$\begin{array}{r|rrrrr} -2 & 1 & 8 & 14 & -17 & -42 \\ & & -2 & -12 & -4 & 42 \\ \hline & 1 & 6 & 2 & -21 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 6 & 2 & -21 \\ & & -3 & -9 & 21 \\ \hline & 1 & 3 & -7 & 0 \end{array}$$

$$(x + 2)(x + 3)(x^2 + 3x - 7) = 0$$

$$x^2 + 3x - 7 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-7)}}{2(1)}$$

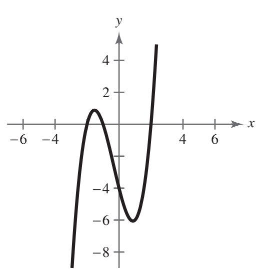
$$x = \frac{-3 \pm \sqrt{37}}{2}$$

So, the real zeros are $x = -2, -3, \dfrac{-3}{2} \pm \dfrac{\sqrt{37}}{2}$.

33. $f(x) = x^3 + x^2 - 4x - 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4$

(b)

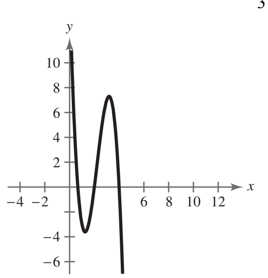


(c) Real zeros: $-2, -1, 2$

34. $f(x) = -3x^3 + 20x^2 - 36x + 16$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

(b)

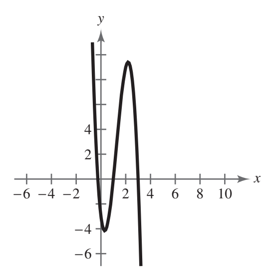


(c) Real zeros: $\frac{2}{3}, 2, 4$

35. $f(x) = -4x^3 + 15x^2 - 8x - 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

(b)

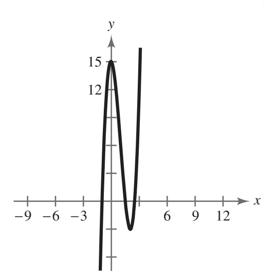


(c) Real zeros: $-\frac{1}{4}, 1, 3$

36. $f(x) = 4x^3 - 12x^2 - x + 15$

(a) Possible rational zeros: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

(b)

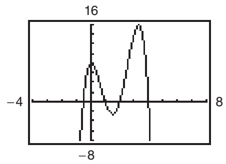


(c) Real zeros: $-1, \frac{3}{2}, \frac{5}{2}$

37. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

(b)

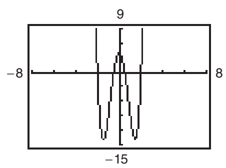


(c) Real zeros: $-\frac{1}{2}, 1, 2, 4$

38. $f(x) = 4x^4 - 17x^2 + 4$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{4}$

(b)

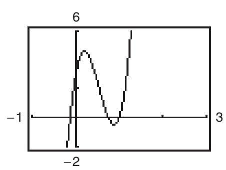


(c) Real zeros: $-2, -\frac{1}{2}, \frac{1}{2}, 2$

39. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) Possible rational zeros: $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4},$
$\pm \frac{1}{8}, \pm \frac{3}{8}, \pm \frac{1}{16}, \pm \frac{3}{16}, \pm \frac{1}{32}, \pm \frac{3}{32}$

(b)

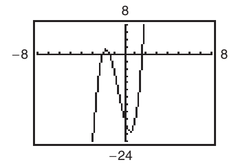


(c) Real zeros: $-\frac{1}{8}, \frac{3}{4}, 1$

40. $f(x) = 4x^3 + 7x^2 - 11x - 18$

(a) Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18,$
$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{9}{4}$

(b)



(c) Real zeros: $-2, \dfrac{1}{8} \pm \dfrac{\sqrt{145}}{8}$

41. $f(x) = (x - 1)(x - 5i)(x + 5i)$
$= (x - 1)(x^2 + 25)$
$= x^3 - x^2 + 25x - 25$

Note: $f(x) = a(x^3 - x^2 + 25x - 25)$, where a is any nonzero real number, has the zeros 1 and $\pm 5i$.

42. $f(x) = (x - 4)(x - 3i)(x + 3i)$
$= (x - 4)(x^2 + 9)$
$= x^3 - 4x^2 + 9x - 36$

Note: $f(x) = a(x^3 - 4x^2 + 9x - 36)$, where a is any real number, has the zeros $4, 3i,$ and $-3i$.

43. If $1 + i$ is a zero, so is its conjugate, $1 - i$.

$f(x) = (x - 2)(x - 2)(x - (1 + i))(x - (1 - i))$
$= (x^2 - 4x + 4)(x^2 - 2x + 2)$
$= x^4 - 6x^3 + 14x^2 - 16x + 8$

Note: $f(x) = a(x^4 - 6x^3 + 14x^2 - 16x + 8)$, where a is any nonzero real number, has the zeros 2, 2 and $1 \pm i$.

44. If $3 - 2i$ is a zero, so is its conjugate, $3 + 2i$.

$f(x) = (x + 1)(x - 5)(x - (3 - 2i))(x - (3 + 2i))$
$= (x^2 - 4x - 5)(x^2 - 6x + 13)$
$= x^4 - 10x^3 + 32x^2 - 22x - 65$

Note: $f(x) = a(x^4 - 10x^3 + 32x^2 - 22x - 65)$, where a is any nonzero real number, has the zeros $-1, 5$ and $3 \pm 2i$.

45. If $3 + \sqrt{2}i$ is a zero, so is its conjugate, $3 - \sqrt{2}i$.

$$f(x) = (3x - 2)(x + 1)\left[x - \left(3 + \sqrt{2}i\right)\right]\left[x - \left(3 - \sqrt{2}i\right)\right]$$
$$= (3x - 2)(x + 1)\left[(x - 3) - \sqrt{2}i\right]\left[(x - 3) + \sqrt{2}i\right]$$
$$= (3x^2 + x - 2)\left[(x - 3)^2 - \left(\sqrt{2}i\right)^2\right]$$
$$= (3x^2 + x - 2)(x^2 - 6x + 9 + 2)$$
$$= (3x^2 + x - 2)(x^2 - 6x + 11)$$
$$= 3x^4 - 17x^3 + 25x^2 + 23x - 22$$

Note: $f(x) = a(3x^4 - 17x^3 + 25x^2 + 23x - 22)$, where a is any nonzero real number, has the zeros $\frac{2}{3}, -1,$ and $3 \pm \sqrt{2}i$.

46. If $1 + \sqrt{3}i$ is a zero, so is its conjugate, $1 - \sqrt{3}i$.

$$\begin{aligned} f(x) &= (2x + 5)(x + 5)\left(x - 1 - \sqrt{3}i\right)\left(x - 1 + \sqrt{3}i\right) \\ &= \left(2x^2 + 15x + 25\right)\left(x^2 - 2x + 4\right) \\ &= 2x^4 + 11x^3 + 3x^2 + 10x + 100 \end{aligned}$$

Note: $f(x) = a\left(2x^4 + 11x^3 + 3x^2 + 10x + 100\right)$, where a is any real number, has the zeros $-\frac{5}{2}, -5$, and $1 \pm \sqrt{3}i$.

47.

$$\begin{aligned} f(x) &= a(x + 2)(x - 1)(x - i)(x + i) \\ &= a\left(x^2 + x - 2\right)\left(x^2 + 1\right) \\ &= a\left(x^4 + x^3 - x^2 + x - 2\right) \end{aligned}$$

Since $f(0) = -4$

$$\begin{aligned} -4 &= a\left((0)^4 + (0)^3 - (0)^2 + (0) - 2\right) \\ -4 &= -2a \\ a &= 2 \end{aligned}$$

So, $f(x) = 2\left(x^4 + x^3 - x^2 + x - 2\right)$
$= 2x^4 + 2x^3 - 2x^2 + 2x - 4.$

48.

$$\begin{aligned} f(x) &= a(x + 1)(x - 2)\left(x - \sqrt{2}i\right)\left(x + \sqrt{2}i\right) \\ &= a\left(x^2 - x - 2\right)\left(x^2 + 2\right) \\ &= a\left(x^4 - x^3 - 2x - 4\right) \end{aligned}$$

Since $f(1) = 12$

$$\begin{aligned} 12 &= a\left((1)^4 - (1)^3 - 2(1) - 4\right) \\ 12 &= -6a \\ a &= -2 \end{aligned}$$

So, $f(x) = -2\left(x^4 - x^3 - 2x - 4\right)$
$= -2x^4 + 2x^3 + 4x + 8.$

49.

$$\begin{aligned} f(x) &= a(x + 3)\left(x - \left(1 + \sqrt{3}i\right)\right)\left(x - \left(1 - \sqrt{3}i\right)\right) \\ &= a(x + 3)\left(x^2 - 2x + 4\right) \\ &= a\left(x^3 + x^2 - 2x + 12\right) \end{aligned}$$

Since $f(-2) = 12$

$$\begin{aligned} 12 &= a\left((-2)^3 + (-2)^2 - 2(-2) + 12\right) \\ 12 &= 12a \\ a &= 1 \end{aligned}$$

So, $f(x) = (1)\left(x^4 - x^3 - 2x - 4\right)$
$= x^3 + x^2 - 2x + 12.$

50.

$$\begin{aligned} f(x) &= a(x + 2)\left(x - \left(1 + \sqrt{2}i\right)\right)\left(x - \left(1 - \sqrt{2}i\right)\right) \\ &= a(x + 2)\left(x^2 - 2x + 3\right) \\ &= a\left(x^3 - x + 6\right) \end{aligned}$$

Since $f(-1) = -12$

$$\begin{aligned} -12 &= a\left((-1)^3 - (-1) + 6\right) \\ -12 &= 6a \\ a &= -2 \end{aligned}$$

So, $f(x) = (2)\left(x^3 - x + 6\right)$
$= -2x^3 + 2x - 12.$

51. $f(x) = x^4 + 2x^2 - 8$

(a) $f(x) = \left(x^2 + 4\right)\left(x^2 - 2\right)$

(b) $f(x) = \left(x^2 + 4\right)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

52. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = \left(x^2 + 9\right)\left(x^2 - 3\right)$

(b) $f(x) = \left(x^2 + 9\right)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

(c) $f(x) = (x + 3i)(x - 3i)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

53. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

$$\begin{array}{r|l} & x^2 - 2x + 3 \\ \hline x^2 - 6 & x^4 - 2x^3 - 3x^2 + 12x - 18 \\ & \underline{x^4 \qquad\quad - 6x^2} \\ & -2x^3 + 3x^2 + 12x \\ & \underline{-2x^3 \qquad\quad + 12x} \\ & 3x^2 \qquad -18 \\ & \underline{3x^2 \qquad -18} \\ & 0 \end{array}$$

(a) $f(x) = \left(x^2 - 6\right)\left(x^2 - 2x + 3\right)$

(b) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x^2 - 2x + 3\right)$

(c) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

Note: Use the Quadratic Formula for (c).

54. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

$$\begin{array}{r}
x^2 - 3x - 5 \\
x^2 + 4 \overline{\smash{\big)} x^4 - 3x^3 - x^2 - 12x - 20} \\
\underline{x^4 \qquad + 4x^2 \qquad\qquad} \\
-3x^3 - 5x^2 - 12x \qquad \\
\underline{-3x^3 \qquad - 12x} \qquad \\
-5x^2 \qquad - 20 \\
\underline{-5x^2 \qquad - 20} \\
0
\end{array}$$

(a) $f(x) = (x^2 + 4)(x^2 - 3x - 5)$

(b) $f(x) = (x^2 + 4)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

Note: Use the Quadratic Formula for (b).

55. $f(x) = x^3 - x^2 + 4x - 4$

Because $2i$ is a zero, so is $-2i$.

$$\begin{array}{r|rrrr}
2i & 1 & -1 & 4 & -4 \\
 & & 2i & -4 - 2i & 4 \\
\hline
 & 1 & 2i - 1 & -2i & 0
\end{array}$$

$$\begin{array}{r|rrr}
-2i & 1 & 2i - 1 & -2i \\
 & & -2i & 2i \\
\hline
 & 1 & -1 & 0
\end{array}$$

$f(x) = (x - 2i)(x + 2i)(x - 1)$

The zeros of $f(x)$ are $x = 1, \pm 2i$.

Alternate Solution:

Because $x = \pm 2i$ are zeros of $f(x)$,

$(x + 2i)(x - 2i) = x^2 + 4$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r}
x - 1 \\
x^2 + 0x + 4 \overline{\smash{\big)} x^3 - x^2 + 4x - 4} \\
\underline{x^3 + 0x^2 + 4x \qquad} \\
-x^2 + 0x - 4 \\
\underline{-x^2 + 0x - 4} \\
0
\end{array}$$

$f(x) = (x^2 + 4)(x - 1)$

The zeros of $f(x)$ are $x = 1, \pm 2i$.

56. $f(x) = 2x^3 + 3x^2 + 18x + 27$

Because $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|rrrr}
3i & 2 & 3 & 18 & 27 \\
 & & 6i & 9i - 18 & -27 \\
\hline
 & 2 & 3 + 6i & 9i & 0
\end{array}$$

$$\begin{array}{r|rrr}
-3i & 2 & 3 + 6i & 9i \\
 & & -6i & -9i \\
\hline
 & 2 & 3 & 0
\end{array}$$

$f(x) = (x - 3i)(x + 3i)(2x + 3)$

The zeros of $f(x)$ are $x = \pm 3i, -\frac{3}{2}$.

Alternate Solution:

Because $x = \pm 3i$ are zeros of $f(x)$,

$(x - 3i)(x + 3i) = x^2 + 9$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r}
2x + 3 \\
x^2 + 0x + 9 \overline{\smash{\big)} 2x^3 + 3x^2 + 18x + 27} \\
\underline{2x^3 + 0x^2 + 18x \qquad} \\
3x^2 + 0x + 27 \\
\underline{3x^2 + 0x + 27} \\
0
\end{array}$$

$f(x) = (x^2 + 9)(2x + 3)$

The zeros of $f(x)$ are $x = \pm 3i, -\frac{3}{2}$.

57. $f(x) = x^3 - 8x^2 + 25x - 26$

Because $3 + 2i$ is a zero, so is $3 - 2i$.

$$\begin{array}{r|rrrr} 3+2i & 1 & -8 & 25 & -26 \\ & & 3+2i & -19-4i & 26 \\ \hline & 1 & -5+2i & 6-4i & 0 \end{array}$$

$$\begin{array}{r|rrr} 3-2i & 1 & -5+2i & 6-4i \\ & & 3-2i & -6+4i \\ \hline & 1 & -2 & 0 \end{array}$$

$f(x) = (x - (3 + 2i))(x - (3 - 2i))(x - 2)$

The zeros of $f(x)$ are $x = 3 \pm 2i, 2$.

Alternate Solution:

Because $x = 3 \pm 2i$ are zeros of $f(x), (x - (3 + 2i))(x - (3 - 2i)) = x^2 - 6x + 13$ is a factor of $f(x.)$

By long division, you have:

$$\begin{array}{r} x - 2 \\ x^2 - 6x + 13 \overline{\smash{\big)} x^3 - 8x^2 + 25x - 26} \\ \underline{x^3 - 6x^2 + 13x} \\ -2x^2 + 12x - 26 \\ \underline{-2x^2 + 12x^2 - 26} \\ 0 \end{array}$$

$f(x) = (x^2 - 6x + 13)(x - 2)$

The zeros of $f(x)$ are $x = 3 \pm 2i, 2$.

58. $f(x) = x^3 + 9x^2 + 25x + 17$

Because $-4 + i$ is a zero, so is $-4 - i$

$$\begin{array}{r|rrrr} -4+i & 1 & 9 & 25 & 17 \\ & & -4+i & -21+i & -17 \\ \hline & 1 & 5+i & 4+i & 0 \end{array}$$

$$\begin{array}{r|rrr} -4-i & 1 & 5+i & 4+i \\ & & -4-i & -4-i \\ \hline & 1 & 1 & 0 \end{array}$$

$f(x) = (x - (-4 + i))(x - (-4 - i))(x + 1)$

The zeros of $f(x)$ are $x = -4 \pm i, -1$.

Alternate Solution:

Because $x = -4 \pm i$ are zeros of $f(x), (x - (-4 + i))(x - (-4 - i)) = x^2 + 8x + 17$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r} x + 1 \\ x^2 + 8x + 17 \overline{\smash{\big)} x^3 + 9x^2 + 25x + 17} \\ \underline{x^3 + 8x^2 + 17x} \\ x^2 + 8x + 17 \\ \underline{x^2 + 8x + 17} \\ 0 \end{array}$$

$f(x) = (x^2 + 8x + 17)(x + 1)$

The zeros of $f(x)$ are $x = -4 \pm i, -1$.

59. $f(x) = x^4 - 6x^3 + 14x^2 - 18x + 9$

Because $1 - \sqrt{2}\,i$ is a zero, so is $1 + \sqrt{2}\,i$, and

$$\begin{aligned} \left[x - \left(1 - \sqrt{2}i\right)\right]\left[x - \left(1 + \sqrt{2}i\right)\right] &= \left[(x - 1) - \sqrt{2}i\right]\left[(x - 1) - \sqrt{2}i\right]\left[(x - 1) + \sqrt{2}i\right] \\ &= (x - 1)^2 - \left(\sqrt{2}i\right)^2 \\ &= x^2 - 2x + 1 - 2i^2 \\ &= x^2 - 2x + 3 \end{aligned}$$

is a factor of $f(x)$. By long division, you have:

$$\begin{array}{r} x^2 - 4x + 3 \\ x^2 - 2x + 3 \overline{\smash{\big)} x^4 - 6x^3 + 14x^2 - 18x + 9} \\ \underline{x^4 - 2x^3 + 3x^2} \\ -4x^3 + 11x^2 - 18x + 9 \\ \underline{-4x^3 + 8x^2 - 12x} \\ 3x^2 - 6x + 9 \\ \underline{3x^2 - 6x + 9} \\ 0 \end{array}$$

$$\begin{aligned} f(x) &= (x^2 - 2x + 3)(x^2 - 4x + 3) \\ &= (x^2 - 2x + 3)(x - 1)(x - 3) \end{aligned}$$

The zeros of $f(x)$ are $x = 1 \pm \sqrt{2}i, 1, 3$.

60. $f(x) = x^4 + x^3 - 3x^2 - 13x + 14$

Because $-2 + \sqrt{3}i$ is a zero, so is $-2 - \sqrt{3}i$, and

$$\begin{aligned}\left[x - \left(-2 + \sqrt{3}i\right)\right]\left[x - \left(2 - \sqrt{3}i\right)\right] &= \left[(x + 2) - \sqrt{3}i\right]\left[(x - 1) + \sqrt{3}i\right] \\ &= (x + 2)^2 - \left(\sqrt{3}i\right)^2 \\ &= x^2 + 4x + 4 - 3i^2 \\ &= x^2 + 4x + 7\end{aligned}$$

is a factor of $f(x)$. By long division, you have:

$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 4x - 7 \overline{\big) x^4 + x^3 - 3x^2 - 13x + 14} \\ \underline{x^4 + 4x^3 + 7x^2} \qquad\qquad \\ -3x^3 - 10x^2 - 13x + 14 \\ \underline{-3x^3 - 12x^2 - 21x} \qquad \\ 2x^2 + 8x + 14 \\ \underline{2x^2 + 8x + 14} \\ 0 \end{array}$$

$$\begin{aligned}f(x) &= \left(x^2 + 4x + 7\right)\left(x^2 - 3x + 2\right) \\ &= \left(x^2 + 4x + 7\right)(x - 1)(x - 2)\end{aligned}$$

The zeros of $f(x)$ are $x = -2 \pm \sqrt{3}i, 1, 2$.

61. $f(x) = x^2 + 36$
$= (x + 6i)(x - 6i)$

The zeros of $f(x)$ are $x = \pm 6i$.

62. $f(x) = x^2 + 49$
$= (x - 7i)(x + 7i)$

The zeros of $f(x)$ are $x = \pm 7i$.

63. $h(x) = x^2 - 2x + 17$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{2 \pm \sqrt{4 - 68}}{2} = \frac{2 \pm \sqrt{-64}}{2} = 1 \pm 4i.$$

$$\begin{aligned}f(x) &= \left(x - (1 + 4i)\right)\left(x - (1 - 4i)\right) \\ &= (x - 1 - 4i)(x - 1 + 4i)\end{aligned}$$

64. $g(x) = x^2 + 10x + 17$

By the Quadratic Formula, the zeros of $f(x)$ are:

$$x = \frac{-10 \pm \sqrt{100 - 68}}{2} = \frac{-10 \pm \sqrt{32}}{2} = -5 \pm 2\sqrt{2}.$$

$$\begin{aligned}f(x) &= \left(x - \left(-5 + 2\sqrt{2}\right)\right)\left(x - \left(-5 - 2\sqrt{2}\right)\right) \\ &= \left(x + 5 - 2\sqrt{2}\right)\left(x + 5 + 2\sqrt{2}\right)\end{aligned}$$

65. $f(x) = x^4 - 16$
$= \left(x^2 - 4\right)\left(x^2 + 4\right)$
$= (x - 2)(x + 2)(x - 2i)(x + 2i)$

Zeros: $\pm 2, \pm 2i$

66. $f(y) = y^4 - 256$
$= \left(y^2 - 16\right)\left(y^2 + 16\right)$
$= (y - 4)(y + 4)(y - 4i)(y + 4i)$

Zeros: $\pm 4, \pm 4i$

67. $f(z) = z^2 - 2z + 2$

By the Quadratic Formula, the zeros of $f(z)$ are

$$z = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$$

$$\begin{aligned} f(z) &= [z - (1 + i)][z - (1 - i)] \\ &= (z - 1 - i)(z - 1 + i) \end{aligned}$$

68. $h(x) = x^3 - 3x^2 + 4x - 2$

Possible rational zeros: $\pm1, \pm2$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 2$ are $x = \dfrac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$

Zeros: $1, 1 \pm i$

$h(x) = (x - 1)(x - 1 - i)(x - 1 + i)$

69. $g(x) = x^3 - 3x^2 + x + 5$

Possible rational zeros: $\pm1, \pm5$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 1 & 5 \\ & & -1 & 4 & -5 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 4x + 5$ are: $x = \dfrac{4 \pm \sqrt{16 - 20}}{2} = 2 \pm i$

Zeros: $-1, 2 \pm i$

$g(x) = (x + 1)(x - 2 - i)(x - 2 + i)$

70. $f(x) = x^3 - x^2 + x + 39$

Possible rational zeros: $\pm1, \pm3, \pm13, \pm39$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & 1 & 39 \\ & & -3 & 12 & -39 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 4x + 13$ are: $x = \dfrac{4 \pm \sqrt{16 - 52}}{2} = 2 \pm 3i$

Zeros: $-3, 2 \pm 3i$

$f(x) = (x + 3)(x - 2 - 3i)(x - 2 + 3i)$

71. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 4 & -8 \\ & & 2 & 0 & 8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$\begin{aligned} g(x) &= (x - 2)(x - 2)(x^2 + 4) \\ &= (x - 2)^2(x + 2i)(x - 2i) \end{aligned}$$

Zeros: $2, \pm 2i$

72. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

Possible rational zeros: $\pm1, \pm3, \pm9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

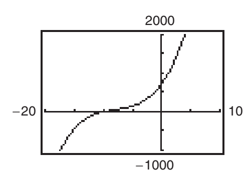
The zeros of $x^2 + 1$ are $x = \pm i$.

Zeros: $-3, \pm i$

$h(x) = (x + 3)^2(x + i)(x - i)$

73. $f(x) = x^3 + 24x^2 + 214x + 740$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm37, \pm74, \pm148, \pm185, \pm370, \pm740$



Based on the graph, try $x = -10$.

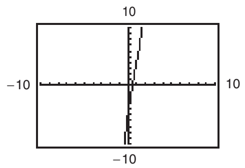
$$\begin{array}{r|rrrr} -10 & 1 & 24 & 214 & 740 \\ & & -10 & -140 & -740 \\ \hline & 1 & 14 & 74 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 14x + 74$ are $x = \dfrac{-14 \pm \sqrt{196 - 296}}{2} = -7 \pm 5i.$

The zeros of $f(x)$ are $x = -10$ and $x = -7 \pm 5i$.

74. $f(s) = 2s^3 - 5s^2 + 12s - 5$

Possible rational zeros: $\pm1, \pm5, \pm\frac{1}{2}, \pm\frac{5}{2}$



Based on the graph, try $s = \frac{1}{2}$.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & -5 & 12 & -5 \\ & & 1 & -2 & 5 \\ \hline & 2 & -4 & 10 & 0 \end{array}$$

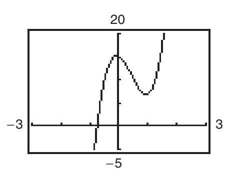
By the Quadratic Formula, the zeros of $2(s^2 - 2s + 5)$ are $s = \dfrac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i$.

The zeros of $f(s)$ are $s = \frac{1}{2}$ and $s = 1 \pm 2i$.

75. $f(x) = 16x^3 - 20x^2 - 4x + 15$

Possible rational zeros:

$\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4},$
$\pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{5}{16}, \pm\frac{15}{16}$



Based on the graph, try $x = -\frac{3}{4}$.

$$\begin{array}{c|cccc} -\frac{3}{4} & 16 & -20 & -4 & 15 \\ & & -12 & 24 & -15 \\ \hline & 16 & -32 & 20 & 0 \end{array}$$

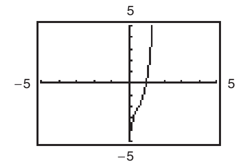
By the Quadratic Formula, the zeros of $16x^2 - 32x + 20 = 4(4x^2 - 8x + 5)$ are

$x = \dfrac{8 \pm \sqrt{64 - 80}}{8} = 1 \pm \frac{1}{2}i$.

The zeros of $f(x)$ are $x = -\frac{3}{4}$ and $x = 1 \pm \frac{1}{2}i$.

76. $f(x) = 9x^3 - 15x^2 + 11x - 5$

Possible rational zeros: $\pm1, \pm5, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{9}, \pm\frac{5}{9}$



Based on the graph, try $x = 1$.

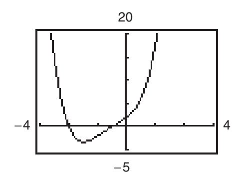
$$\begin{array}{c|cccc} 1 & 9 & -15 & 11 & -5 \\ & & 9 & -6 & 5 \\ \hline & 9 & -6 & 5 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $9x^2 - 6x + 5$ are $x = \dfrac{6 \pm \sqrt{36 - 180}}{18} = \frac{1}{3} \pm \frac{2}{3}i$.

The zeros of $f(x)$ are $x = 1$ and $x = \frac{1}{3} \pm \frac{2}{3}i$.

77. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

Possible rational zeros: $\pm1, \pm2, \pm\frac{1}{2}$



Based on the graph, try $x = -2$ and $x = -\frac{1}{2}$.

$$\begin{array}{c|ccccc} -2 & 2 & 5 & 4 & 5 & 2 \\ & & -4 & -2 & -4 & -2 \\ \hline & 2 & 1 & 2 & 1 & 0 \end{array}$$

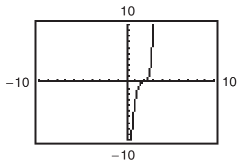
$$\begin{array}{c|cccc} -\frac{1}{2} & 2 & 1 & 2 & 1 \\ & & -1 & 0 & -1 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

The zeros of $2x^2 + 2 = 2(x^2 + 1)$ are $x = \pm i$.

The zeros of $f(x)$ are $x = -2, x = -\frac{1}{2},$ and $x = \pm i$.

78. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm32$



Based on the graph, try $x = 2$.

$$\begin{array}{r|rrrrrr} 2 & 1 & -8 & 28 & -56 & 64 & -32 \\ & & 2 & -12 & 32 & -48 & 32 \\ \hline & 1 & -6 & 16 & -24 & 16 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 16 & -24 & 16 \\ & & 2 & -8 & 16 & -16 \\ \hline & 1 & -4 & 8 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 4$ are $x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm \sqrt{3}i.$

The zeros of $g(x)$ are $x = 2$ and $x = 1 \pm \sqrt{3}i.$

79. $g(x) = 2x^3 - 3x^2 - 3$

Sign variations: 1, positive zeros: 1

$g(-x) = -2x^3 - 3x^2 - 3$

Sign variations: 0, negative zeros: 0

80. $h(x) = 4x^2 - 8x + 3$

Sign variations: 2, positive zeros: 2 or 0

$h(-x) = 4x^2 + 8x + 3$

Sign variations: 0, negative zeros: 0

81. $h(x) = 2x^3 + 3x^2 + 1$

Sign variations: 0, positive zeros: 0

$h(-x) = -2x^3 + 3x^2 + 1$

Sign variations: 1, negative zeros: 1

82. $h(x) = 2x^4 - 3x - 2$

Sign variations: 1, positive zeros: 1

$h(-x) = 2x^4 + 3x - 2$

Sign variations: 1, negative zeros: 1

83. $g(x) = 6x^4 + 2x^3 - 3x^2 + 2$

Sign variations: 2, positive zeros: 2 or 0

$g(-x) = 6x^4 - 2x^3 - 3x^2 + 2$

Sign variations: 2, negative zeros: 2 or 0

84. $f(x) = 4x^3 - 3x^2 - 2x - 1$

Sign variations: 1, positive zeros: 1

$f(-x) = -4x^3 - 3x^2 - 2x - 1$

Sign variations: 2, negative zeros: 2 or 0

85. $f(x) = 5x^3 + x^2 - x + 5$

Sign variations: 2, positive zeros: 2 or 0

$f(-x) = -5x^3 + x^2 + x + 5$

Sign variations: 1, negative zeros: 1.

86. $f(x) = 3x^3 - 2x^2 - x + 3$

Sign variations: 2, positive zeros: 2 or 0

$f(-x) = -3x^3 - 2x^2 + x + 3$

Sign variations: 1, negative zeros: 1

87. $f(x) = x^3 + 3x^2 - 2x + 1$

(a) $$\begin{array}{r|rrrr} 1 & 1 & 3 & -2 & 1 \\ & & 1 & 4 & 2 \\ \hline & 1 & 4 & 2 & 3 \end{array}$$

1 is an upper bound.

(b) $$\begin{array}{r|rrrr} -4 & 1 & 3 & -2 & 1 \\ & & -4 & 4 & -8 \\ \hline & 1 & -1 & 2 & -7 \end{array}$$

−4 is a lower bound.

88. $f(x) = x^3 - 4x^2 + 1$

(a) $$\begin{array}{r|rrrr} 4 & 1 & -4 & 0 & 1 \\ & & 4 & 0 & 0 \\ \hline & 1 & 0 & 0 & 1 \end{array}$$

4 is an upper bound.

(b) $$\begin{array}{r|rrrr} -1 & 1 & -4 & 0 & 1 \\ & & -1 & 5 & -5 \\ \hline & 1 & -5 & 5 & -4 \end{array}$$

−1 is a lower bound.

89. $f(x) = x^4 - 4x^3 + 16x - 16$

(a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 5 & 5 & 25 & 205 \\ \hline & 1 & 1 & 5 & 41 & 189 \end{array}$$

5 is an upper bound.

(b)
$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

-3 is a lower bound.

90. $f(x) = 2x^4 - 8x + 3$

(a)
$$\begin{array}{r|rrrrr} 3 & 2 & 0 & 0 & -8 & 3 \\ & & 6 & 18 & 54 & 138 \\ \hline & 2 & 6 & 18 & 46 & 141 \end{array}$$

3 is an upper bound.

(b)
$$\begin{array}{r|rrrrr} -4 & 2 & 0 & 0 & -8 & 3 \\ & & -8 & 32 & -128 & 544 \\ \hline & 2 & -8 & 32 & -136 & 547 \end{array}$$

-4 is a lower bound

91. $f(x) = 16x^3 - 12x^2 - 4x + 3$

Possible rational zeros: $\dfrac{\pm 1, \pm 3}{\pm, \pm 2, \pm 4, \pm 8, \pm 16} = \pm\frac{1}{16}, \pm\frac{1}{8}, \pm\frac{3}{16}, \pm\frac{1}{4}, \pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3$

However, the function factors by grouping.

$$\begin{aligned} 16x^2 - 12x^2 - 4x + 3 &= 0 \\ 4x^2(4x - 3) - (4x - 3) &= 0 \\ (4x - 3)(4x^2 - 1) &= 0 \\ (4x - 3)(2x - 1)(2x + 1) &= 0 \end{aligned}$$

$4x - 3 = 0 \rightarrow x = \frac{3}{4}$

$2x - 1 = 0 \rightarrow x = \frac{1}{2}$

$2x + 1 = 0 \rightarrow x = -\frac{1}{2}$

So, the zeros are $x = \frac{3}{4}, \pm\frac{1}{2}$.

92. $f(z) = 12z^3 - 4z^2 - 27z + 9$

Possible rational zeros: $\pm 1, \pm 3, \pm 9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 12 & -4 & -27 & 9 \\ & & 18 & 21 & -9 \\ \hline & 12 & 14 & -6 & 0 \end{array}$$

$$\begin{aligned} f(z) &= 2\left(z - \tfrac{3}{2}\right)\left(6z^2 + 7z - 3\right) \\ &= (2z - 3)(3z - 1)(2z + 3) \end{aligned}$$

So, the real zeros are $-\frac{3}{2}, \frac{1}{3},$ and $\frac{3}{2}$.

93. $f(y) = 4y^3 + 3y^2 + 8y + 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 4 & 3 & 8 & 6 \\ & & -3 & 0 & -6 \\ \hline & 4 & 0 & 8 & 0 \end{array}$$

$$\begin{aligned} 4y^3 + 3y^2 + 8y + 6 &= \left(y + \tfrac{3}{4}\right)\left(4y^2 + 8\right) \\ &= \left(y + \tfrac{3}{4}\right)4\left(y^2 + 2\right) \\ &= (4y + 3)\left(y^2 + 2\right) \end{aligned}$$

So, the only real zero is $-\frac{3}{4}$.

94. $g(x) = 3x^3 - 2x^2 + 15x - 10$

Possible rational zeros:
$\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 15 & -10 \\ & & 2 & 0 & 10 \\ \hline & 3 & 0 & 15 & 0 \end{array}$$

$g(x) = \left(x - \frac{2}{3}\right)\left(3x^2 + 15\right) = (3x - 2)\left(x^2 + 5\right)$

So, the only real zero is $\frac{2}{3}$.

95. $$\begin{aligned} P(x) &= x^4 - \tfrac{25}{4}x^2 + 9 \\ &= \tfrac{1}{4}\left(4x^4 - 25x^2 + 36\right) \\ &= \tfrac{1}{4}\left(4x^2 - 9\right)\left(x^2 - 4\right) \\ &= \tfrac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2) \end{aligned}$$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

96. $f(x) = \frac{1}{2}\left(2x^3 - 3x^2 - 23x + 12\right)$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= \tfrac{1}{2}(x - 4)\left(2x^2 + 5x - 3\right) \\ &= \tfrac{1}{2}(x - 4)(2x - 1)(x + 3) \end{aligned}$$

The rational zeros are $-3, \frac{1}{2},$ and 4.

97. $$\begin{aligned} f(x) &= x^3 - \tfrac{1}{4}x^2 - x + \tfrac{1}{4} \\ &= \tfrac{1}{4}\left(4x^3 - x^2 - 4x + 1\right) \\ &= \tfrac{1}{4}\left[x^2(4x - 1) - 1(4x - 1)\right] \\ &= \tfrac{1}{4}(4x - 1)\left(x^2 - 1\right) \\ &= \tfrac{1}{4}(4x - 1)(x + 1)(x - 1) \end{aligned}$$

The rational zeros are $\frac{1}{4}$ and ± 1.

98. $f(z) = \frac{1}{6}\left(6z^3 + 11z^2 - 3z - 2\right)$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

$$\begin{aligned} f(x) &= \tfrac{1}{6}(x + 2)\left(6x^2 - x - 1\right) \\ &= \tfrac{1}{6}(x + 2)(3x + 1)(2x - 1) \end{aligned}$$

The rational zeros are $-2, -\frac{1}{3},$ and $\frac{1}{2}$.

99. $f(x) = x^3 - 1 = (x - 1)\left(x^2 + x + 1\right)$

Rational zeros: $1\ (x = 1)$

Irrational zeros: 0

Matches (d).

100. $$\begin{aligned} f(x) &= x^3 - 2 \\ &= \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right) \end{aligned}$$

Rational zeros: 0

Irrational zeros: $1\left(x = \sqrt[3]{2}\right)$

Matches (a).

101. $f(x) = x^3 - x = x(x + 1)(x - 1)$

Rational zeros: $3\ (x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

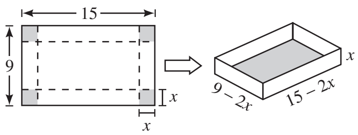
102. $$\begin{aligned} f(x) &= x^3 - 2x \\ &= x\left(x^2 - 2\right) \\ &= x\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) \end{aligned}$$

Rational zeros: $1\ (x = 0)$

Irrational zeros: $2\left(x = \pm\sqrt{2}\right)$

Matches (c).

103. (a)

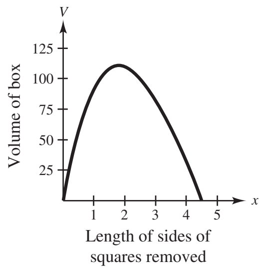


(b) $V = l \cdot w \cdot h = (15 - 2x)(9 - 2x)x$

$= x(9 - 2x)(15 - 2x)$

Because length, width, and height must be positive, you have $0 < x < \frac{9}{2}$ for the domain.

(c)



The volume is maximum when $x \approx 1.82$.

The dimensions are: $r = ak^3 + bk^2 + ck + d, f(k) = r.$

1.82 cm × 5.36 cm × 11.36 cm

(d) $56 = x(9 - 2x)(15 - 2x)$

$56 = 135x - 48x^2 + 4x^3$

$0 = 4x^3 - 48x^2 + 135x - 56$

The zeros of this polynomial are $\frac{1}{2}, \frac{7}{2},$ and 8.

x cannot equal 8 because it is not in the domain of V. [The length cannot equal -1 and the width cannot equal -7. The product of $(8)(-1)(-7) = 56$ so it showed up as an extraneous solution.]

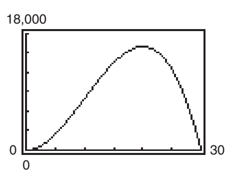
So, the volume is 56 cubic centimeters when $x = \frac{1}{2}$ centimeter or $x = \frac{7}{2}$ centimeters.

104. (a) Combined length and width:

$4x + y = 120 \Rightarrow y = 120 - 4x$

$$\text{Volume} = l \cdot w \cdot h = x^2 y = x^2(120 - 4x) = 4x^2(30 - x)$$

(b)



Dimensions with maximum volume:
20 in. × 20 in. × 40 in.

(c) $13{,}500 = 4x^2(30 - x)$

$4x^3 - 120x^2 + 13{,}500 = 0$

$x^3 - 30x^2 + 3375 = 0$

15	1	−30	0	3375
		15	−225	−3375
	1	−15	−225	0

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15, \dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

105. (a) Current bin: $V = 2 \times 3 \times 4 = 24$ cubic feet

New bin: $V = 5(24) = 120$ cubic feet

$V(x) = (2 + x)(3 + x)(4 + x) = 120$

(b) $x^3 + 9x^2 + 26x + 24 = 120$

$x^3 + 9x^2 + 26x - 96 = 0$

The only real zero of this polynomial is $x = 2$. All the dimensions should be increased by 2 feet, so the new bin will have dimensions of 4 feet by 5 feet by 6 feet.

106. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right), x \ge 1$

C is minimum when

$3x^3 - 40x^2 - 2400x - 36000 = 0.$

The only real zero is $x \approx 40$ or 4000 units.

107. False. The most complex zeros it can have is two, and the Linear Factorization Theorem guarantees that there are three linear factors, so one zero must be real.

108. False. f does not have real coefficients.

109. $g(x) = -f(x)$. This function would have the same zeros as $f(x)$, so $r_1, r_2,$ and r_3 are also zeros of $g(x)$.

110. $g(x) = 3f(x)$. This function has the same zeros as f because it is a vertical stretch of f. The zeros of g are $r_1, r_2,$ and r_3.

111. $g(x) = f(x - 5)$. The graph of $g(x)$ is a horizontal shift of the graph of $f(x)$ five units of the right, so the zeros of $g(x)$ are $5 + r_1, 5 + r_2,$ and $5 + r_3$.

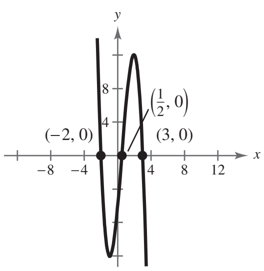
112. $g(x) = f(2x)$. Note that x is a zero of g if and only if $2x$ is a zero of f. The zeros of g are $\frac{r_1}{2}, \frac{r_2}{2}$, and $\frac{r_3}{2}$.

113. $g(x) = 3 + f(x)$. Because $g(x)$ is a vertical shift of the graph of $f(x)$, the zeros of $g(x)$ cannot be determined.

114. $g(x) = f(-x)$. Note that x is a zero of g if and only if $-x$ is a zero of f. The zeros of g are $-r_1, -r_2$, and $-r_3$.

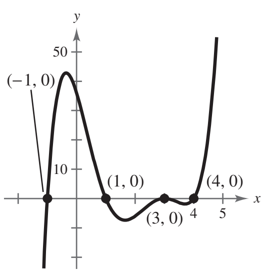
115. Zeros: $-2, \frac{1}{2}, 3$

$$f(x) = -(x + 2)(2x - 1)(x - 3)$$
$$= -2x^3 + 3x^2 + 11x - 6$$



Any nonzero scalar multiple of f would have the same three zeros. Let $g(x) = af(x), a > 0$. There are infinitely many possible functions for f.

116.



117. Because $1 + i$ is a zero of f, so is $1 - i$. From the graph, 1 is also a zero.

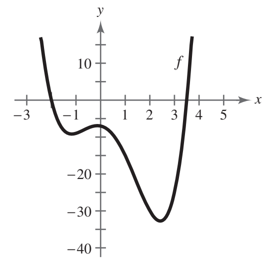
$$f(x) = (x - (1 + i))(x - (1 - i))(x - 1)$$
$$= (x^2 - 2x + 2)(x - 1)$$
$$= x^3 - 3x^2 + 4x - 2$$

118. Because $1 + i$ is a zero of f, so is $1 - i$. From the graph, -1 is also a zero.

$$f(x) = (x - (1 + i))(x - (1 - i))(x + 1)$$
$$= (x^2 - 2x + 2)(x + 1)$$
$$= x^3 - x^2 + 2$$

Because the graph rises to the left and falls to the right, $a = -1$, and $f(x) = -x^3 + x^2 - 2$.

119.



If $x = i$ is a zero, then $x = -i$ is also a zero. So the function is $f(x) = (x - 2)(x - 3.5)(x - i)(x + i)$.

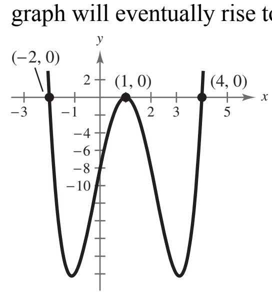
120. (a) Zeros of $f(x)$: $-2, 1, 4$

(b) The graph touches the x-axis at $x = 1$.

(c) The least possible degree of the function is 4 because there are at least four real zeros (1 is repeated) and a function can have at most the number of real zeros equal to the degree of the function. The degree cannot be odd by the definition of multiplicity.

(d) The leading coefficient of f is positive. From the information in the table, you can conclude that the graph will eventually rise to the left and to the right.

(e)



121. Because $f(i) = f(2i) = 0$, then i and $2i$ are zeros of f. Because i and $2i$ are zeros of f, so are $-i$ and $-2i$.

$$f(x) = (x - i)(x + i)(x - 2i)(x + 2i)$$
$$= (x^2 + 1)(x^2 + 4)$$
$$= x^4 + 5x^2 + 4$$

122. Because $f(2) = 0$, 2 is a zero of f. Because $f(i) = 0$, i is a zero of f. Because i is a zero of f, so is $-i$.

$$f(x) = -1(x - 2)(x + i)(x - i)$$
$$= -1(x - 2)(x^2 + 1)$$
$$= -x^3 + 2x^2 - x + 2$$

123. (a) $f(x) = (x - \sqrt{b}i)(x + \sqrt{b}i) = x^2 + b$

(b) $f(x) = [x - (a + bi)][x - (a - bi)]$

$$= [(x - a) - bi][(x - a) + bi]$$
$$= (x - a)^2 - (bi)^2$$
$$= x^2 - 2ax + a^2 + b^2$$

Section 2.6 Rational Functions

1. rational functions

2. vertical asymptote

3. horizontal asymptote

4. slant asymptote

5. Because the denominator is zero when $x - 1 = 0$, the domain of *f* is all real numbers except $x = 1$.

x	0	0.5	0.9	0.99	$\to 1$
$f(x)$	-1	-2	-10	-100	$\to -\infty$

x	$1 \leftarrow$	1.01	1.1	1.5	2
$f(x)$	$\infty \leftarrow$	100	10	2	1

As *x* approaches 1 from the left, $f(x)$ decreases without bound towards $-\infty$. As *x* approaches 1 from the right, $f(x)$ increases without bound towards $+\infty$.

6. Because the denominator is zero when $x + 2 = 0$, the domain of *f* is all real numbers except $x = -2$.

x	-3	-2.5	-2.1	-2.01	$\to -2$
$f(x)$	15	25	105	1005	$\to \infty$

x	$-2 \leftarrow$	-1.99	-1.9	-1.5	-1
$f(x)$	$-\infty \leftarrow$	-955	-95	-15	-5

As *x* approaches -2 from the left, $f(x)$ increases without bound (∞). As *x* approaches -2 from the right, $f(x)$ decreases without bound $(-\infty)$.

7. Because the denominator is zero when $x^2 - 1 = 0$, the domain of *f* is all real numbers except $x = -1$ and $x = 1$.

x	-2	-1.5	-1.1	-1.01	$\to -1$
$f(x)$	4	5.4	17.3	152.3	$\to \infty$

x	$-1 \leftarrow$	-0.99	-0.9	-0.5	0
$f(x)$	$-\infty \leftarrow$	-147.8	-12.8	-1	0

As *x* approaches -1 from the left, $f(x)$ increases without bound (∞). As *x* approaches -1 from the right, $f(x)$ decreases without bound $(-\infty)$.

x	0	0.5	0.9	0.99	$\to 1$
$f(x)$	0	-1	-12.8	-147.8	$\to -\infty$

x	$1 \leftarrow$	1.01	1.1	1.5	2
$f(x)$	$\infty \leftarrow$	152.3	17.3	5.4	4

As *x* approaches 1 from the left, $f(x)$ decreases without bound $(-\infty)$. As *x* approaches 1 from the right, $f(x)$ increases without bound (∞).

8. Because the denominator is zero when $x^2 - 4 = 0$, the domain of *f* is all real numbers except $x = -2$ and $x = 2$.

x	-3	-2.5	-2.1	-2.01	$\to -2$
$f(x)$	-1.2	-2.2	-10.2	-100.2	$\to -\infty$

x	$-2 \leftarrow$	-1.99	-1.9	-1.5	-1
$f(x)$	$\infty \leftarrow$	99.7	9.7	1.7	0.7

As *x* approaches -2 from the left, $f(x)$ decreases without bound $(-\infty)$. As *x* approaches -2 from the right, $f(x)$ increases without bound (∞).

x	1	1.5	1.9	1.99	$\to 2$
$f(x)$	-0.7	-1.7	-9.7	-99.7	$\to -\infty$

x	$2\leftarrow$	2.01	2.1	2.5	3
$f(x)$	$\infty\leftarrow$	100.2	10.2	2.2	1.2

As *x* approaches 2 from the left, $f(x)$ decreases without bound $(-\infty)$. As *x* approaches 2 from the right, $f(x)$ increases without bound (∞).

9. $f(x) = \dfrac{4}{x^2}$

Domain: all real numbers except $x = 0$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

$\left[\text{Degree of } N(x) < \text{degree of } D(x)\right]$

10. $f(x) = \dfrac{1}{(x-2)^3}$

Domain: all real numbers except $x = 2$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

$\left[\text{Degree of } N(x) < \text{degree of } D(x)\right]$

11. $f(x) = \dfrac{5+x}{5-x} = \dfrac{x+5}{-x+5}$

Domain: all real numbers except $x = 5$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = -1$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

12. $f(x) = \dfrac{3-7x}{3+2x} = \dfrac{-7x+3}{2x+3}$

Domain: all real numbers except $x = -\dfrac{3}{2}$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = -\dfrac{7}{2}$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

13. $f(x) = \dfrac{x^3}{x^2-1}$

Domain: all real numbers except $x = \pm 1$

Vertical asymptotes: $x = \pm 1$

Horizontal asymptote: None

$\left[\text{Degree of } N(x) > \text{degree of } D(x)\right]$

14. $f(x) = \dfrac{4x^2}{x+2}$

Vertical asymptote: $x = -2$

Horizontal asymptote: None

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

15. $f(x) = \dfrac{x^2-3x-4}{2x^2+x-1}$

$= \dfrac{(x+1)(x-4)}{(2x-1)(x+1)}$

$= \dfrac{x-4}{2x-1}, x \neq -1$

Horizontal asymptote: $y = \dfrac{1}{2}$

$\left(\text{Degree of } N(x) = \text{degree of } D(x)\right)$

Vertical asymptote: $x = \dfrac{1}{2}$

(Because $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

16. $f(x) = \dfrac{-4x^2+1}{x^2+x+3}$

Domain: All real numbers. The denominator has no real zeros.

[Use the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = -4$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

17. $f(x) = \dfrac{1}{x+1}$

(a) Domain: all real numbers x except $x = -1$

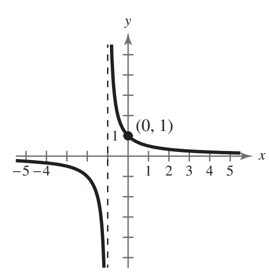
(b) y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 0$

(d)

x	-4	-3	0	1	2	3
$f(x)$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$



18. $f(x) = \dfrac{1}{x-3}$

(a) Domain: all real numbers x except $x = 3$

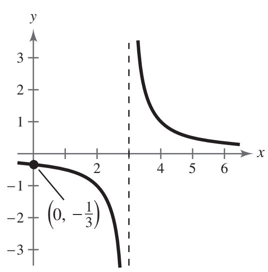
(b) y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	0	1	2	4	5	6
$f(x)$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$



19. $h(x) = \dfrac{-1}{x+4}$

(a) Domain: all real numbers x except $x = -4$

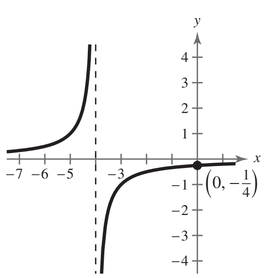
(b) y-intercept: $\left(0, -\dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = -4$

Horizontal asymptote: $y = 0$

(d)

x	-6	-5	-3	-2	-1	0
$h(x)$	$\frac{1}{2}$	1	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$



20. $g(x) = \dfrac{1}{6-x} = -\dfrac{1}{x-6}$

(a) Domain: all real numbers x except $x = 6$

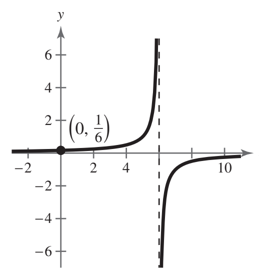
(b) y-intercept: $\left(0, \dfrac{1}{6}\right)$

(c) Vertical asymptote: $x = 6$

Horizontal asymptote: $y = 0$

(d)

x	-2	0	2	4	8
$g(x)$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$



21. $C(x) = \dfrac{2x+3}{x+2}$

(a) Domain: all real numbers x except $x = -2$

(b) x-intercept: $\left(-\dfrac{3}{2}, 0\right)$

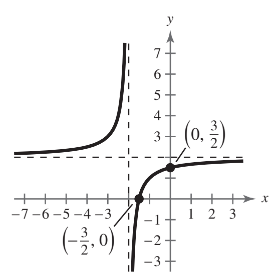
y-intercept: $\left(0, \dfrac{3}{2}\right)$

(c) Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 2$

(d)

x	-4	-3	-1	0	1	2
$C(x)$	$\frac{5}{2}$	3	1	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{7}{4}$



22. $P(x) = \dfrac{1 - 3x}{1 - x} = \dfrac{3x - 1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

(b) x-intercept: $\left(\dfrac{1}{3}, 0\right)$

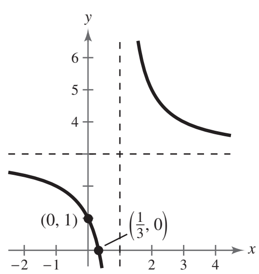
y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 3$

(d)

x	-1	0	2	3
$P(x)$	2	1	5	4



23. $f(x) = \dfrac{x^2}{x^2 + 9}$

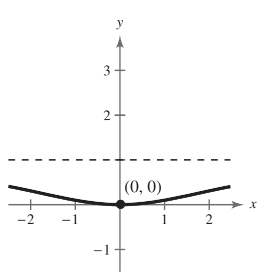
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 1$

(d)

x	± 1	± 2	± 3
$f(x)$	$\dfrac{1}{10}$	$\dfrac{4}{13}$	$\dfrac{1}{2}$



24. $f(t) = \dfrac{1 - 2t}{t} = -\dfrac{2t - 1}{t}$

(a) Domain: all real numbers t except $t = 0$

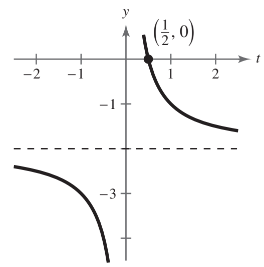
(b) t-intercept: $\left(\dfrac{1}{2}, 0\right)$

(c) Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$

(d)

t	-2	-1	$\dfrac{1}{2}$	1	2
$f(t)$	$-\dfrac{5}{2}$	-3	0	-1	$-\dfrac{3}{2}$



25. $g(s) = \dfrac{4s}{s^2 + 4}$

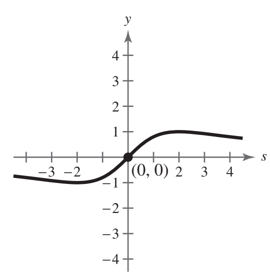
(a) Domain: all real numbers s

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

s	-2	-1	0	1	2
$g(s)$	-1	$-\dfrac{4}{5}$	0	$\dfrac{4}{5}$	1



26. $f(x) = -\dfrac{x}{(x-2)^2}$

(a) Domain: all real numbers x except $x = 2$

(b) x-intercept: $(0, 0)$

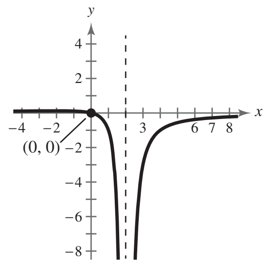
y-intercept: $\left(0, -\dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	3	4	5
$f(x)$	$\frac{1}{8}$	$\frac{1}{9}$	0	-1	-3	-1	$-\frac{5}{9}$



27. $f(x) = \dfrac{2x}{x^2 - 3x - 4} = \dfrac{2x}{(x-4)(x+1)}$

(a) Domain: all real numbers x except $x = 4$ and $x = -1$

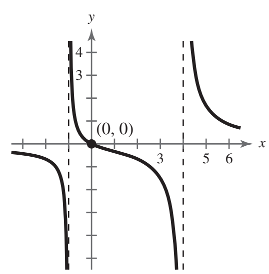
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = 4$, $x = -1$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	0	1	2	3	5
$f(x)$	$-\frac{3}{7}$	$-\frac{2}{3}$	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{3}{2}$	$\frac{5}{3}$



28. $f(x) = \dfrac{3x}{x^2 + 2x - 3} = \dfrac{3x}{(x+3)(x-1)}$

(a) Domain: all real numbers x except $x = -3$ and $x = 1$

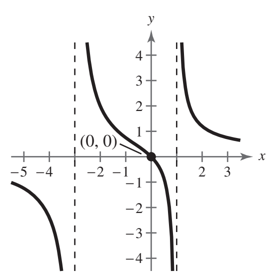
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = -3$, $x = 1$

Horizontal asymptote: $y = 0$

(d)

x	-5	-4	-2	-1	0	2	3
$f(x)$	$-\frac{5}{4}$	$-\frac{12}{5}$	2	$\frac{3}{4}$	0	$\frac{6}{5}$	$\frac{3}{4}$



29. $f(x) = \dfrac{x-4}{x^2 - 16} = \dfrac{x-4}{(x+4)(x-4)} = \dfrac{1}{x+4}, x \neq 4$

(a) Domain: all real numbers except $x = \pm 4$

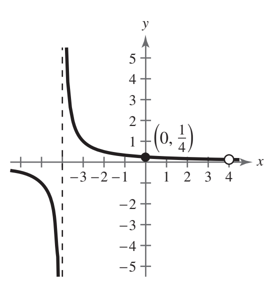
(b) y-intercept: $\left(0, \dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = -4$

Horizontal asymptote: $y = 0$

(d)

x	-5	-4	-3	-2	-1
$f(x)$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{3}$



30. $f(x) = \dfrac{x+1}{x^2-1} = \dfrac{x+1}{(x+1)(x-1)} = \dfrac{1}{x-1},\ x \neq -1$

(a) Domain: all real numbers except $x = \pm 1$

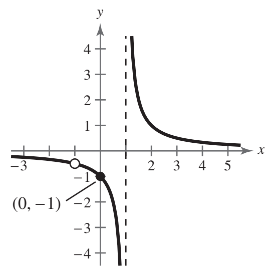
(b) y-intercept: $(0, -1)$

(c) Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 0$

(d)

x	-1	0	1	2	3
$f(x)$	Undef.	-1	Undef.	1	$\frac{1}{2}$



31. $f(t) = \dfrac{t^2-1}{t-1} = \dfrac{(t+1)(t-1)}{t-1} = t+1,\quad t \neq 1$

(a) Domain: all real numbers t except $t = 1$

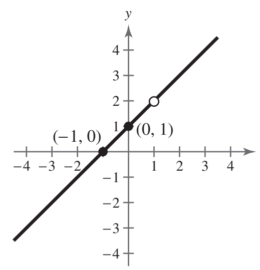
(b) t-intercept: $(-1, 0)$

y-intercept: $(0, 1)$

(c) No asymptotes

(d)

t	-3	-2	-1	0	1	2
$f(t)$	-2	-1	0	1	Undef.	3



32. $f(x) = \dfrac{x^2-36}{x+6} = \dfrac{(x+6)(x-6)}{x+6} = x-6,\ \ x \neq -6$

(a) Domain: all real numbers x except $x = -6$

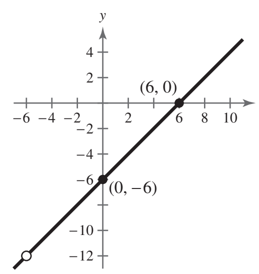
(b) x-intercept: $(6, 0)$

y-intercept: $(0, -6)$

(c) No asymptotes

(d)

x	-6	-4	-2	0	2	4	6	8
$f(x)$	Undef.	-10	-8	-6	-4	-2	0	2



33. $f(x) = \dfrac{x^2-25}{x^2-4x-5} = \dfrac{(x+5)(x-5)}{(x-5)(x+1)} = \dfrac{x+5}{x+1},\ x \neq 5$

(a) Domain: all real numbers x except $x = 5$ and $x = -1$

(b) x-intercept: $(-5, 0)$

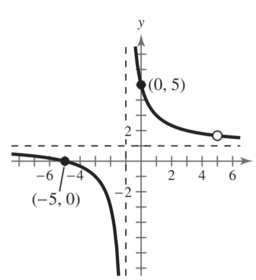
y-intercept: $(0, 5)$

(c) Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 1$

(d)

x	-5	-3	0	3	5
$f(x)$	0	-1	5	2	Undef.



34. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2}$

$= \dfrac{(x-2)(x+2)}{(x-2)(x-1)} = \dfrac{x+2}{x-1},\ x \neq 2$

(a) Domain: all real numbers x except $x = 1$ and $x = 2$

(b) x-intercept: $(-2, 0)$

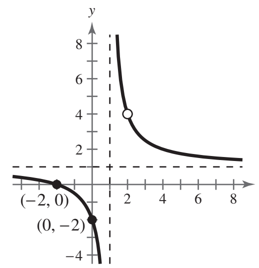
y-intercept: $(0, -2)$

(c) Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 1$

(d)

x	-4	-2	0	2	4
$f(x)$	$\frac{2}{5}$	0	-2	Undef..	2



35. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x+3)(x-2)} = \dfrac{x}{x-2},\quad x \neq -3$

(a) Domain: all real numbers x except $x = -3$ and $x = 2$

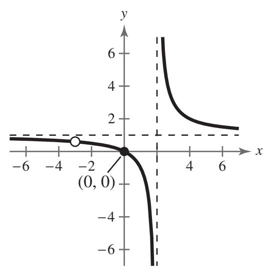
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

(d)

x	-1	0	1	3	4
$f(x)$	$\frac{1}{3}$	0	-1	3	2



36. $f(x) = \dfrac{5(x+4)}{x^2 + x - 12} = \dfrac{5(x+4)}{(x+4)(x-3)} = \dfrac{5}{x-3},\quad x \neq -4$

(a) Domain: all real numbers x except $x = -4$ and $x = 3$

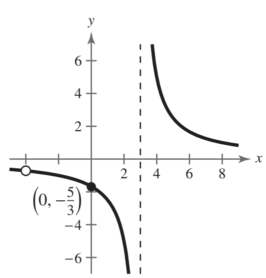
(b) y-intercept: $\left(0, -\dfrac{5}{3}\right)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	-2	0	2	5	7
$f(x)$	-1	$-\frac{5}{3}$	-5	$\frac{5}{2}$	$\frac{5}{4}$



37. $f(x) = \dfrac{2x^2 - 5x - 3}{x^3 - 2x^2 - x + 2} = \dfrac{(2x + 1)(x - 3)}{(x - 2)(x + 1)(x - 1)}$

(a) Domain: all real numbers x except $x = 2, x = \pm 1$

(b) x-intercepts: $\left(-\dfrac{1}{2}, 0\right), (3, 0)$

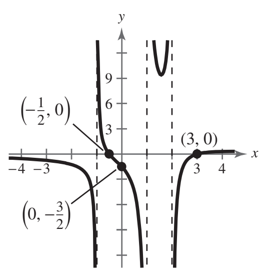
y-intercept: $\left(0, -\dfrac{3}{2}\right)$

(c) Vertical asymptotes: $x = 2, x = -1,$ and $x = 1$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	0	$\frac{3}{2}$	3	4
$f(x)$	$-\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{3}{2}$	$\frac{48}{5}$	0	$\frac{3}{10}$



38. $f(x) = \dfrac{x^2 - x - 2}{x^3 - 2x^2 - 5x + 6} = \dfrac{(x + 1)(x - 2)}{(x - 1)(x + 2)(x - 3)}$

(a) Domain: all real numbers x except $x = -2, x = 1,$ and $x = 3$

(b) x-intercepts: $(-1, 0), (2, 0)$

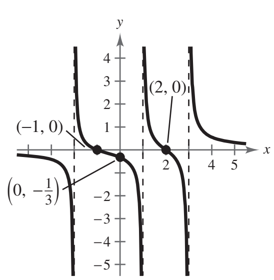
y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptotes: $x = -2, x = 1, x = 3$

Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0	2	4
$f(x)$	$-\frac{9}{35}$	$-\frac{5}{12}$	0	$-\frac{1}{3}$	0	$\frac{5}{9}$



39. Because the function has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 0$,

$f(x) = \dfrac{4}{x + 2}$ matches graph (d).

40. Because the function has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 0$,

$f(x) = \dfrac{5}{x - 2}$ matches graph (a).

41. Because the function has vertical asymptotes at $x = \pm 2$ and a horizontal asymptote at $y = 2$,

$f(x) = \dfrac{2x^2}{x^2 - 4}$ matches graph (c).

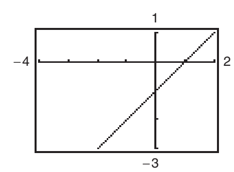
42. Because the function has a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 0$,

$f(x) = \dfrac{3x}{(x + 2)^2}$ matches graph (b).

43. (a) Domain of f: all real numbers x except $x = -1$

Domain of g: all real numbers x

(b)



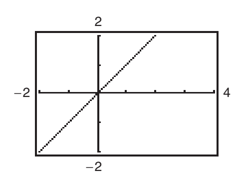
(c) Because there are only a finite number of pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

44. $f(x) = \dfrac{x^2(x - 2)}{x^2 - 2x}, g(x) = x$

(a) Domain of f: All real numbers x except $x = 0$ and $x = 2$

Domain of g: All real numbers x

(b)

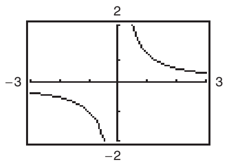


(c) Because there are only a finite number of pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

45. (a) Domain of f: all real numbers x except $x = 0, 2$

Domain of g: all real numbers $x = 0$

(b)



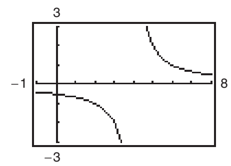
(c) Because there are only a finite number of pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

46. $f(x) = \dfrac{2x - 6}{x^2 - 7x + 12}, \; g(x) = \dfrac{2}{x - 4}$

(a) Domain of f: All real numbers x except $x = 3$ and $x = 4$

Domain of g: All real numbers x except $x = 4$

(b)



(c) Because there are only a finite number of pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

47. $h(x) = \dfrac{x^2 - 4}{x} = x - \dfrac{4}{x}$

(a) Domain: all real numbers x except $x = 0$

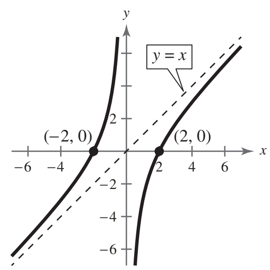
(b) x-intercepts: $(-2, 0), (2, 0)$

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-4	-3	-1	1	3	4
$h(x)$	-3	$\frac{-5}{3}$	3	-3	$\frac{5}{3}$	3



48. $g(x) = \dfrac{x^2 + 5}{x} = x + \dfrac{5}{x}$

(a) Domain: all real numbers x except $x = 0$

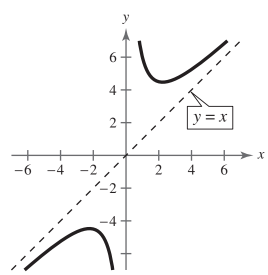
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-3	-2	-1	1	2	3
$g(x)$	$-\frac{14}{3}$	$-\frac{9}{2}$	-6	6	$\frac{9}{2}$	$\frac{14}{3}$



49. $f(x) = \dfrac{2x^2 + 1}{x} = 2x + \dfrac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

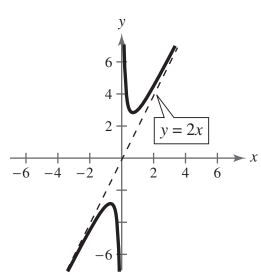
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = 2x$

(d)

x	-4	-2	2	4	6
$f(x)$	$-\frac{33}{4}$	$-\frac{9}{2}$	$\frac{9}{2}$	$\frac{33}{4}$	$\frac{73}{6}$



50. $f(x) = \dfrac{-x^2 - 2}{x} = -x - \dfrac{2}{x}$

(a) Domain: all real numbers x except $x = 0$

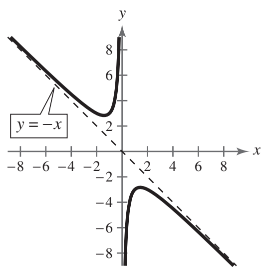
(b) x-intercepts: none

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = -x$

(d)

x	-3	-2	-1	1	2	3
$f(x)$	$\dfrac{11}{3}$	3	3	-3	-3	$-\dfrac{11}{3}$



51. $g(x) = \dfrac{x^2 + 1}{x} = x + \dfrac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

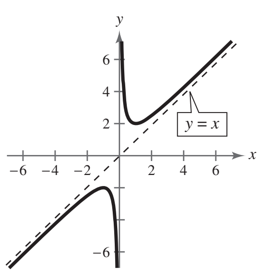
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-4	-2	2	4	6
$g(x)$	$-\dfrac{17}{4}$	$-\dfrac{5}{2}$	$\dfrac{5}{2}$	$\dfrac{17}{4}$	$\dfrac{37}{6}$



52. $h(x) = \dfrac{x^2}{x - 1} = x + 1 + \dfrac{1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

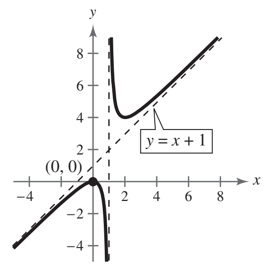
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = x + 1$

(d)

x	-4	-2	2	4	6
$h(x)$	$-\dfrac{16}{5}$	$-\dfrac{4}{3}$	4	$\dfrac{16}{3}$	$\dfrac{36}{5}$



53. $f(t) = \dfrac{t^2 + 1}{t + 5} = -t + 5 - \dfrac{26}{t + 5}$

(a) Domain: all real numbers t except $t = -5$

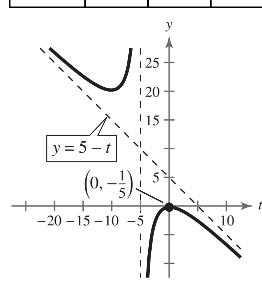
(b) Intercept: $\left(0, -\dfrac{1}{5}\right)$

(c) Vertical asymptote: $t = -5$

Slant asymptote: $y = -t + 5$

(d)

t	-7	-6	-4	-3	0
$f(t)$	25	37	-17	-5	$-\dfrac{1}{5}$



54. $f(x) = \dfrac{x^2 + 1}{x + 1} = x - 1 + \dfrac{2}{x + 1}$

(a) Domain: all real numbers x except $x = -1$

(b) x-intercept: none

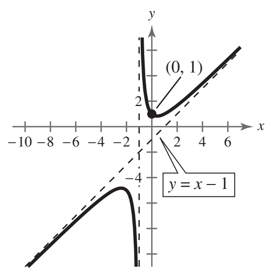
y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = -1$

Slant asymptote: $y = x - 1$

(d)

x	-3	-2	0	1	2	3
$f(x)$	-5	-5	1	1	$\frac{5}{3}$	$\frac{5}{2}$



55. $f(x) = \dfrac{x^3}{x^2 - 4} = x + \dfrac{4x}{x^2 - 4}$

(a) Domain: all real numbers x except $x = \pm 2$

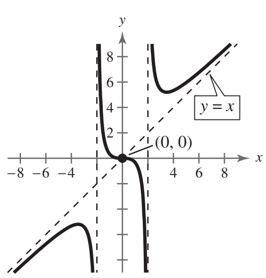
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = x$

(d)

x	-6	-4	-1	0	1	4	6
$f(x)$	$-\frac{27}{4}$	$-\frac{16}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{16}{3}$	$\frac{27}{4}$



56. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

(a) Domain: all real numbers x except $x = \pm 2$

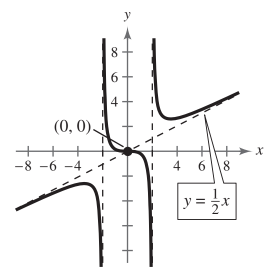
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

(d)

x	-6	-4	-1	1	4	6
$g(x)$	$-\frac{27}{8}$	$-\frac{8}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{8}{3}$	$\frac{27}{8}$



57. $f(x) = \dfrac{x^2 - x + 1}{x - 1} = x + \dfrac{1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

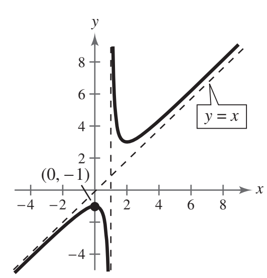
(b) y-intercept: $(0, -1)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = x$

(d)

x	-4	-2	0	2	4
$f(x)$	$-\frac{21}{5}$	$-\frac{7}{3}$	-1	3	$\frac{13}{3}$



58. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

(a) Domain: all real numbers x except $x = 2$

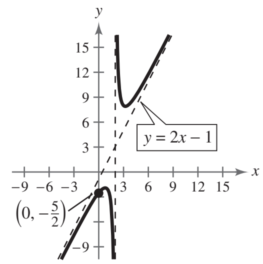
(b) y-intercept: $\left(0, -\dfrac{5}{2}\right)$

(c) Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x - 1$

(d)

x	-6	-3	1	3	6	7
$f(x)$	$-\dfrac{107}{8}$	$-\dfrac{38}{5}$	-2	8	$\dfrac{47}{4}$	$\dfrac{68}{5}$



59. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2} = \dfrac{(2x - 1)(x + 1)(x - 1)}{(x + 1)(x + 2)} = \dfrac{(2x - 1)(x - 1)}{x + 2}, \quad x \neq -1$

$= \dfrac{2x^2 - 3x + 1}{x + 2} = 2x - 7 + \dfrac{15}{x + 2}, \quad x \neq -1$

(a) Domain: all real numbers x except $x = -1$ and $x = -2$

(b) y-intercept: $\left(0, \dfrac{1}{2}\right)$

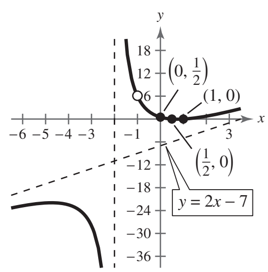
x-intercepts: $\left(\dfrac{1}{2}, 0\right), (1, 0)$

(c) Vertical asymptote: $x = -2$

Slant asymptote: $y = 2x - 7$

(d)

x	-4	-3	$-\dfrac{3}{2}$	0	1
$f(x)$	$-\dfrac{45}{2}$	-28	20	$\dfrac{1}{2}$	0



60. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2} = \dfrac{(x - 2)(x + 2)(2x + 1)}{(x - 2)(x - 1)}$

$= 2x + 7 + \dfrac{9}{x - 1}, \quad x \neq 2$

(a) Domain: all real numbers x except $x = 1$ and $x = 2$

(b) y-intercept: $(0, -2)$

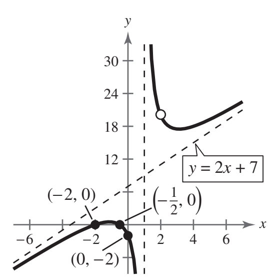
x-intercepts: $(-2, 0), \left(-\dfrac{1}{2}, 0\right)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = 2x + 7$

(d)

x	-3	-2	-1	0	$\dfrac{1}{2}$	$\dfrac{3}{2}$	3	4
$f(x)$	$-\dfrac{5}{4}$	0	$\dfrac{1}{2}$	-2	-10	28	$\dfrac{35}{2}$	18



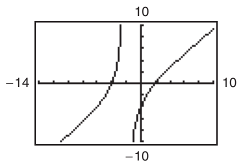
61. $f(x) = \dfrac{x^2 + 2x - 8}{x + 2} = x - \dfrac{8}{x + 2}$

Domain: all real numbers x except $x = -2$

Vertical asymptote: $x = -2$

Slant asymptote: $y = x$

Line: $y = x$



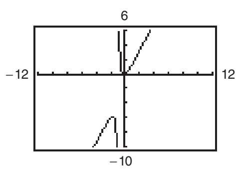
62. $f(x) = \dfrac{2x^2 + x}{x + 1} = 2x - 1 + \dfrac{1}{x + 1}$

Domain: all real numbers x except $x = -1$

Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x - 1$

Line: $y = 2x - 1$



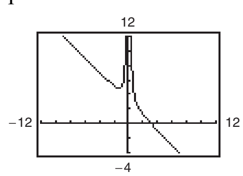
63. $g(x) = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers x except $x = 0$

Vertical asymptote: $x = 0$

Slant asymptote: $y = -x + 3$

Line: $y = -x + 3$



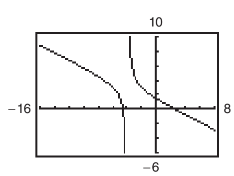
64. $h(x) = \dfrac{12 - 2x - x^2}{2(4 + x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4 + x}$

Domain: all real numbers x except $x = -4$

Vertical asymptote: $x = -4$

Slant asymptote: $y = -\dfrac{1}{2}x + 1$

Line: $y = -\dfrac{1}{2}x + 1$



65. $y = \dfrac{x + 1}{x - 3}$

(a) x-intercept: $(-1, 0)$

(b) $$\begin{aligned} 0 &= \frac{x + 1}{x - 3} \\ 0 &= x + 1 \\ -1 &= x \end{aligned}$$

66. $y = \dfrac{2x}{x - 3}$

(a) x-intercept: $(0, 0)$

(b) $$\begin{aligned} 0 &= \frac{2x}{x - 3} \\ 0 &= 2x \\ 0 &= x \end{aligned}$$

67. $y = \dfrac{1}{x} - x$

(a) x-intercepts: $(-1, 0), (1, 0)$

(b) $$\begin{aligned} 0 &= \frac{1}{x} - x \\ x &= \frac{1}{x} \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

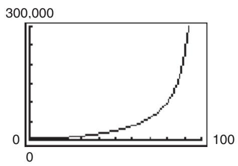
68. $y = x - 3 + \dfrac{2}{x}$

(a) x-intercepts: $(1, 0), (2, 0)$

(b) $$\begin{aligned} 0 &= x - 3 + \frac{2}{x} \\ 0 &= x^2 - 3x + 2 \\ 0 &= (x - 1)(x - 2) \\ x &= 1, x = 2 \end{aligned}$$

69. $C = \dfrac{25{,}000p}{100 - p}, \ 0 \le p < 100$

(a)



(b) $C = \dfrac{25{,}000(15)}{100 - 15} \approx \$4411.76$

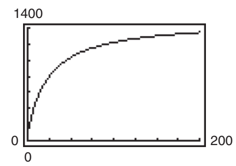
$C = \dfrac{25{,}000(50)}{100 - 50} = \$25{,}000$

$C = \dfrac{25{,}000(90)}{100 - 90} = \$225{,}000$

(c) $C \to \infty$ as $x \to 100$. No, it would not be possible to supply bins to 100% of the residents because the model is undefined for $p = 100$.

70. $N = \dfrac{20(5 + 3t)}{1 + 0.04t}, \ t \ge 0$

(a)



(b) $N(5) \approx 333$ deer

$N(10) = 500$ deer

$N(25) = 800$ deer

(c) The herd is limited by the horizontal asymptote:

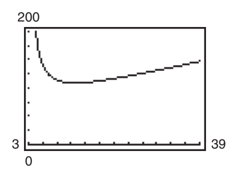
$N = \dfrac{60}{0.04} = 1500$ deer

71. $A = xy$ and

$(x - 3)(y - 2) = 64$

$y - 2 = \dfrac{64}{x - 3}$

$y = 2 + \dfrac{64}{x - 3} = \dfrac{2x + 58}{x - 3}$



Thus, $A = xy = x\left(\dfrac{2x + 58}{x - 3}\right) = \dfrac{2x(x + 29)}{x - 3}, \ x > 3.$

By graphing the area function, we see that A is minimum when $x \approx 12.8$ inches and $y \approx 8.5$ inches.

72. (a) $A = xy$ and

$(x - 4)(y - 2) = 30$

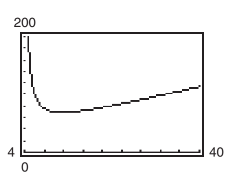
$y - 2 = \dfrac{30}{x - 4}$

$y = 2 + \dfrac{30}{x - 4} = \dfrac{2x + 22}{x - 4}$

Thus, $A = xy = x\left(\dfrac{2x + 22}{x - 4}\right) = \dfrac{2x(x + 11)}{x - 4}.$

(b) Domain: Since the margins on the left and right are each 2 inches, $x > 4$. In interval notation, the domain is $(4, \infty)$.

(c)



The area is minimum when $x \approx 11.75$ inches and $y \approx 5.87$ inches.

x	5	6	7	8	9	10	11	12	13	14	15
y_1 (Area)	160	102	84	76	72	70	69.143	69	69.333	70	70.909

The area is minimum when x is approximately 12.

73. (a) Let t_1 = time from Akron to Columbus and t_2 = time from Columbus back to Akron.

$$xt_1 = 100 \Rightarrow t_1 = \frac{100}{x}$$

$$yt_2 = 100 \Rightarrow t_2 = \frac{100}{y}$$

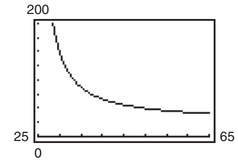
$$50(t_1 + t_2) = 200$$
$$t_1 + t_2 = 4$$
$$\frac{100}{x} + \frac{100}{y} = 4$$
$$100y + 100x = 4xy$$
$$25y + 25x = xy$$
$$25x = xy - 25y$$
$$25x = y(x - 25)$$

Thus, $y = \dfrac{25x}{x - 25}$.

(b) Vertical asymptote: $x = 25$

Horizontal asymptote: $y = 25$

(c)



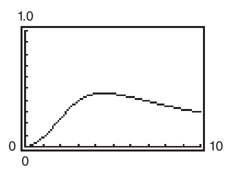
(d)

x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

(e) *Sample answer:* No. You might expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

(f) No. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

74. $C = \dfrac{3t^2 + t}{t^3 + 50}, t > 0$



The horizontal asymptote is the t-axis, or $C = 0$. This indicates that the chemical will eventually dissipate.

75. False. Polynomial functions do not have vertical asymptotes.

76. False. The graph of $f(x) = \dfrac{x}{x^2 + 1}$ crosses $y = 0$, which is a horizontal asymptote.

77. False. A graph can have a vertical asymptote and a horizontal asymptote or a vertical asymptote and a slant asymptote, but a graph cannot have both a horizontal asymptote and a slant asymptote.

A horizontal asymptote occurs when the degree of $N(x)$ is equal to the degree of $D(x)$ or when the degree of $N(x)$ is less than the degree of $D(x)$. A slant asymptote occurs when the degree of $N(x)$ is greater than the degree of $D(x)$ by one. Because the degree of a polynomial is constant, it is impossible to have both relationships at the same time.

78. (a) True. When $x = 1$ the graph of f has a vertical asymptote, therefore $D(1) = D$.

(b) True. Since the graph of f has a horizontal asymptote at $y = 2$, the degrees of $N(x)$ and $D(x)$ are equal.

(c) False. Since the horizontal asymptote is at $y = 2$, which shows that the ratio of the leading coefficients of $N(x)$ and $D(x)$ is 2, not 1.

79. Yes. No. Every rational function is the ratio of two polynomial functions of the form $f(x) = \dfrac{N(x)}{D(x)}$.

80. Vertical asymptote: None $\Rightarrow$ The denominator is not zero for any value of x (unless the numerator is also zero there).

Horizontal asymptote: $y = 2 \Rightarrow$ The degree of the numerator equals the degree of the denominator.

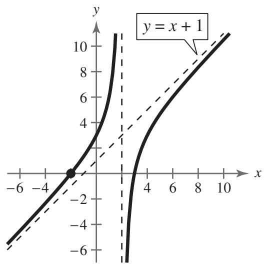
$f(x) = \dfrac{2x^2}{x^2 + 1}$ is one possible function. There are many correct answers.

81. Vertical asymptotes: $x = -2, x = 1 \Rightarrow (x + 2)(x - 1)$ are factors of the denominator.

Horizontal asymptotes: None $\Rightarrow$ The degree of the numerator is greater than the degree of the denominator.

$f(x) = \dfrac{x^3}{(x + 2)(x - 1)}$ is one possible function. There are many correct answers.

82. Answers will vary.



Sample answer:

$y = x + 1 + \dfrac{a}{x - 2}$ This has a slant asymptote of $x + 1$ and a vertical asymptote of $x = 2$.

$0 = -2 + 1 + \dfrac{a}{-2 - 2}$ Since $x = -2$ is a zero, $(-2, 0)$ is on the graph. Use this point to solve for a.

$1 = \dfrac{a}{-4}$

$-4 = a$

Thus, $y = x + 1 - \dfrac{4}{x - 2} = \dfrac{(x + 1)(x - 2) - 4}{x - 2} = \dfrac{x^2 - x - 6}{x - 2}$.

$f(x) = \dfrac{x^2 - x - 6}{x - 2}$

Section 2.7 Nonlinear Inequalities

1. positive; negative

2. key; test intervals

3. zeros; undefined values

4. $P = R - C$

5. $x^2 - 3 < 0$

(a) $x = 3$

$(3)^2 - 3 \overset{?}{<} 0$

$6 \not< 0$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$(0)^2 - 3 \overset{?}{<} 0$

$-3 < 0$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$\left(\frac{3}{2}\right)^2 - 3 \overset{?}{<} 0$

$-\frac{3}{4} < 0$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$(-5)^2 - 3 \overset{?}{<} 0$

$22 \not< 0$

No, $x = -5$ *is not* a solution

6. $x^2 - 2x - 8 \geq 0$

(a) $x = 5$

$(5)^2 - 2(5) - 8 \overset{?}{\geq} 0$

$7 \geq 0$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$(0)^2 - 2(0) - 8 \overset{?}{\geq} 0$

$-8 \ngeq 0$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$(-4)^2 - 2(-4) - 8 \overset{?}{\geq} 0$

$16 + 8 - 8 \overset{?}{\geq} 0$

$16 \geq 0$

Yes, $x = -4$ *is* a solution.

(d) $x = 1$

$(1)^2 - 2(1) - 8 \overset{?}{\geq} 0$

$1 - 2 - 8 \overset{?}{\geq} 0$

$-9 \ngeq 0$

No, $x = 1$ *is not* a solution

7. $\dfrac{x+2}{x-4} \ge 3$

(a) $x = 5$

$\dfrac{5+2}{5-4} \stackrel{?}{\ge} 3$

$7 \ge 3$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$\dfrac{4+2}{4-4} \stackrel{?}{\ge} 3$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$\dfrac{-\frac{9}{2}+2}{-\frac{9}{2}-4} \stackrel{?}{\ge} 3$

$\dfrac{5}{17} \not\ge 3$

No, $x = -\dfrac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$\dfrac{\frac{9}{2}+2}{\frac{9}{2}-4} \stackrel{?}{\ge} 3$

$13 \ge 3$

Yes, $x = \dfrac{9}{2}$ *is* a solution.

8. $\dfrac{3x^2}{x^2+4} < 1$

(a) $x = -2$

$\dfrac{3(-2)^2}{(-2)^2+4} \stackrel{?}{<} 1$

$\dfrac{12}{8} \not< 1$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$\dfrac{3(-1)^2}{(-1)^2+4} \stackrel{?}{<} 1$

$\dfrac{3}{5} < 1$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$\dfrac{3(0)^2}{(0)^2+4} \stackrel{?}{<} 1$

$0 < 1$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$\dfrac{3(3)^2}{(3)^2+4} \stackrel{?}{<} 1$

$\dfrac{27}{13} \not< 1$

No, $x = 3$ *is not* a solution.

9. $x^2 - 3x - 18 = (x+3)(x-6)$

$x + 3 = 0 \Rightarrow x = -3$

$x - 6 = 0 \Rightarrow x = 6$

The key numbers are -3 and 6.

10. $9x^3 - 25x^2 = 0$

$x^2(9x - 25) = 0$

$x^2 = 0 \Rightarrow x = 0$

$9x - 25 = 0 \Rightarrow x = \frac{25}{9}$

The key numbers are 0 and $\frac{25}{9}$.

11. $\dfrac{1}{x-5} + 1 = \dfrac{1 + 1(x-5)}{x-5}$

$= \dfrac{x-4}{x-5}$

$x - 4 = 0 \Rightarrow x = 4$

$x - 5 = 0 \Rightarrow x = 5$

The key numbers are 4 and 5.

12. $\dfrac{x}{x+2} - \dfrac{2}{x-1} = \dfrac{x(x-1) - 2(x+2)}{(x+2)(x-1)}$

$= \dfrac{x^2 - x - 2x - 4}{(x+2)(x-1)}$

$= \dfrac{(x-4)(x+1)}{(x+2)(x-1)}$

$(x-4)(x+1) = 0$

$x - 4 = 0 \Rightarrow x = 4$

$x + 1 = 0 \Rightarrow x = -1$

$(x+2)(x-1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-2, -1, 1,$ and 4.

13. $2x^2 + 4x < 0$

$2x(x + 2) < 0$

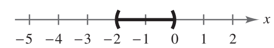
Key numbers: $x = 0, \ -2$

Test intervals: $(-\infty, -2), (-2, 0), (0, \infty)$

Test: Is $2x(x + 2) < 0$?

Interval	x-Value	Value of $2x(x + 2)$	Conclusion
$(-\infty, -2)$	-3	6	Positive
$(-2, 0)$	-1	-2	Negative
$(0, \infty)$	1	3	Positive

Solution set: $(-2, 0)$



14. $3x^2 - 9x \geq 0$

$3x(x - 3) \geq 0$

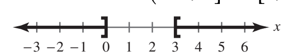
Key numbers: $x = 0, 3$

Test intervals: $(-\infty, 0), (0, 3), (3, \infty)$

Test: Is $3x(x - 3) > 0$?

Interval	x-Value	Value of $3x(x - 3)$	Conclusion
$(-\infty, 0)$	-1	12	Positive
$(0, 3)$	1	-6	Negative
$(3, \infty)$	4	12	Positive

Solution set: $(-\infty, 0] \cup [3, \infty)$



15. $x^2 < 9$

$x^2 - 9 < 0$

$(x + 3)(x - 3) < 0$

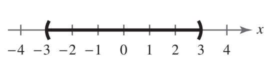
Key numbers: $x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 3), (3, \infty)$

Test: Is $(x + 3)(x - 3) < 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	–4	7	Positive
$(-3, 3)$	0	–9	Negative
$(3, \infty)$	4	7	Positive

Solution set: $(-3, 3)$



16. $x^2 \leq 25$

$x^2 - 25 \leq 0$

$(x + 5)(x - 5) \leq 0$

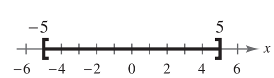
Key numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $(x + 5)(x - 5) \leq 0$?

Interval	x-Value	Value of $x^2 - 25$	Conclusion
$(-\infty, -5)$	–6	11	Positive
$(-5, 5)$	0	–25	Negative
$(5, \infty)$	6	11	Positive

Solution set: $[-5, 5]$



17. $(x+2)^2 \le 25$

$x^2 + 4x + 4 \le 25$

$x^2 + 4x - 21 \le 0$

$(x+7)(x-3) \le 0$

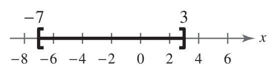
Key numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x+7)(x-3) \le 0$?

Interval	x-Value	Value of $(x+7)(x-3)$	Conclusion
$(-\infty, -7)$	-8	$(-1)(-11) = 11$	Positive
$(-7, 3)$	0	$(7)(-3) = -21$	Negative
$(3, \infty)$	4	$(11)(1) = 11$	Positive

Solution set: $[-7, 3]$



18. $(x-3)^2 \ge 1$

$x^2 - 6x + 8 \ge 0$

$(x-2)(x-4) \ge 0$

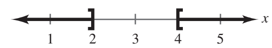
Key numbers: $x = 2, x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x-2)(x-4) > 0$

$(2, 4) \Rightarrow (x-2)(x-4) < 0$

$(4, \infty) \Rightarrow (x-2)(x-4) > 0$

Solution set: $(-\infty, 2] \cup [4, \infty)$



19. $x^2 + 6x + 1 \ge -7$

$x^2 + 6x + 8 \ge 0$

$(x+2)(x+4) \ge 0$

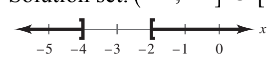
Key numbers: $x = -2, x = -4$

Test Intervals: $(-\infty, -4), (-4, -2), (-2, \infty)$

Test: Is $(x+2)(x+4) > 0$?

Interval	x-Value	Value of $(x+2)(x+4)$	Conclusion
$(-\infty, -4)$	-6	8	Positive
$(-4, -2)$	-3	-1	Negative
$(-2, \infty)$	0	8	Positive

Solution set: $(-\infty, -4] \cup [-2, \infty)$



20. $x^2 - 8x + 2 < 11$

$x^2 - 8x - 9 < 0$

$(x-9)(x+1) < 0$

Key numbers: $x = -1, x = 9$

Test intervals: $(-\infty, -1) \Rightarrow (x-9)(x+1) > 0$

$(-1, 9) \Rightarrow (x-9)(x+1) < 0$

$(9, \infty) \Rightarrow (x-9)(x+1) > 0$

Solution set: $(-1, 9)$

21. $x^2 + x < 6$

$x^2 + x - 6 < 0$

$(x+3)(x-2) < 0$

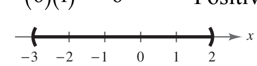
Key numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x+3)(x-2) < 0$?

Interval	x-Value	Value of $(x+3)(x-2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



22.
$$x^2 + 2x > 3$$
$$x^2 + 2x - 3 > 0$$
$$(x + 3)(x - 1) > 0$$

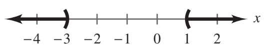
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 1) > 0$

$(-3, 1) \Rightarrow (x + 3)(x - 1) < 0$

$(1, \infty) \Rightarrow (x + 3)(x - 1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



23.
$$x^2 < 3 - 2x$$
$$x^2 + 2x - 3 < 0$$
$$(x + 3)(x - 1) < 0$$

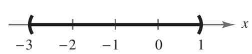
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x + 3)(x - 1) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



24.
$$x^2 > 2x + 8$$
$$x^2 - 2x - 8 > 0$$
$$(x - 4)(x + 2) > 0$$

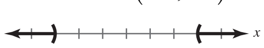
Key numbers: $x = -2, x = 4$

Test intervals: $(-\infty, -2), (-2, 4), (4, \infty)$

Test: Is $(x - 4)(x + 2) > 0$?

Interval	x-Value	Value of $(x - 4)(x + 2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



25.
$$3x^2 - 11x > 20$$
$$3x^2 - 11x - 20 > 0$$
$$(3x + 4)(x - 5) > 0$$

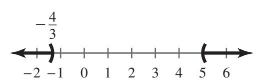
Key numbers: $x = 5, x = -\frac{4}{3}$

Test intervals: $\left(-\infty, -\frac{4}{5}\right), \left(-\frac{4}{3}, 5\right), (5, \infty)$

Test: Is $(3x + 4)(x - 5) > 0$?

Interval	x-Value	Value of $(3x + 4)(x - 5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	-3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$



26.
$$-2x^2 + 6x + 15 \le 0$$
$$2x^2 - 6x - 15 \ge 0$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$
$$= \frac{6 \pm \sqrt{156}}{4}$$
$$= \frac{6 \pm 2\sqrt{39}}{4}$$
$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

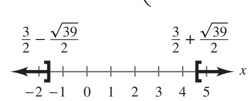
Test intervals:

$\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$

$\left(\frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$

$\left(\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$

Solution set: $\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right)$



27. $x^3 - 3x^2 - x + 3 > 0$

$x^2(x - 3) - (x - 3) > 0$

$(x - 3)(x^2 - 1) > 0$

$(x - 3)(x + 1)(x - 1) > 0$

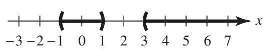
Key numbers: $x = -1, x = 1, x = 3$

Test intervals: $(-\infty -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1)(x - 1) > 0$?

Interval	x-Value	Value of $(x - 3)(x + 1)(x - 1)$	Conclusion
$(-\infty, -1)$	-2	$(-5)(-1)(-3) = -15$	Negative
$(-1, 1)$	0	$(-3)(1)(-1) = 3$	Positive
$(1, 3)$	2	$(-1)(3)(1) = -3$	Negative
$(3, \infty)$	4	$(1)(5)(3) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$



28. $x^3 + 2x^2 - 4x - 8 \le 0$

$x^2(x + 2) - 4(x + 2) \le 0$

$(x + 2)(x^2 - 4) \le 0$

$(x + 2)^2(x - 2) \le 0$

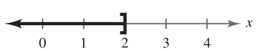
Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$



29.
$$-x^3 + 7x^2 + 9x > 63$$
$$x^3 - 7x^2 - 9x < -63$$
$$x^3 - 7x^2 - 9x + 63 < 0$$

$$x^2(x - 7) - 9(x - 7) < 0$$
$$(x - 7)(x^2 - 9) < 0$$
$$(x - 7)(x + 3)(x - 3) < 0$$

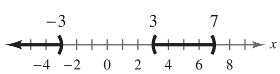
Key numbers: $x = -3, x = 3, x = 7$

Test intervals: $(-\infty, -3), (-3, 3), (3, 7), (7, \infty)$

Test: Is $(x - 7)(x + 3)(x - 3) < 0$?

Interval	x-Value	Value of $(x - 7)(x + 3)(x - 3)$	Conclusion
$(-\infty, -3)$	-4	$(-11)(-1)(-7) = -77$	Negative
$(-3, 3)$	0	$(-7)(3)(-3) = 63$	Positive
$(3, 7)$	4	$(-3)(7)(1) = -21$	Negative
$(7, \infty)$	8	$(1)(11)(5) = 55$	Positive

Solution set: $(-\infty, -3) \cup (3, 7)$



30.
$$2x^3 + 13x^2 - 8x - 46 \ge 6$$
$$2x^3 + 13x^2 - 8x - 52 \ge 0$$
$$x^2(2x + 13) - 4(2x + 13) \ge 0$$
$$(2x + 13)(x^2 - 4) \ge 0$$
$$(2x + 13)(x + 2)(x - 2) \ge 0$$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals:

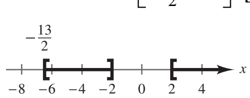
$\left(-\infty, -\frac{13}{2}\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



31. $4x^3 - 6x^2 < 0$

$2x^2(2x - 3) < 0$

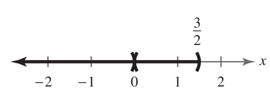
Key numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0) \Rightarrow 2x^2(2x - 3) < 0$

$\left(0, \frac{3}{2}\right) \Rightarrow 2 \Rightarrow 2x^2(2x - 3) < 0$

$\left(\frac{3}{2}, \infty\right) \Rightarrow 2x^2(2x - 3) > 0$

Solution set: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$



32. $4x^3 - 12x^2 > 0$

$4x^2(x - 3) > 0$

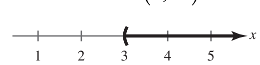
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow 4x^2(x - 3) < 0$

$(0, 3) \Rightarrow 4x^2(x - 3) < 0$

$(3, \infty) \Rightarrow 4x^2(x - 3) > 0$

Solution set: $(3, \infty)$



33. $x^3 - 4x \ge 0$

$x(x + 2)(x - 2) \ge 0$

Key numbers: $x = 0, x = \pm 2$

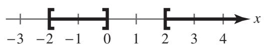
Test intervals: $(-\infty, -2) \Rightarrow x(x + 2)(x - 2) < 0$

$(-2, 0) \Rightarrow x(x + 2)(x - 2) > 0$

$(0, 2) \Rightarrow x(x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow x(x + 2)(x - 2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$



34. $2x^3 - x^4 \le 0$

$x^3(2 - x) \le 0$

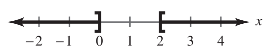
Key numbers: $x = 0, x = 2$

Test intervals: $(-\infty, 0) \Rightarrow x^3(2 - x) < 0$

$(0, 2) \Rightarrow x^3(2 - x) > 0$

$(2, \infty) \Rightarrow x^3(2 - x) < 0$

Solution set: $(-\infty, 0] \cup [2, \infty)$



35. $(x - 1)^2(x + 2)^3 \ge 0$

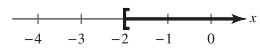
Key numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2) \Rightarrow (x - 1)^2(x + 2)^3 < 0$

$(-2, 1) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

$(1, \infty) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

Solution set: $[-2, \infty)$



36. $x^4(x - 3) \le 0$

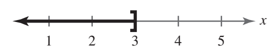
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow x^4(x - 3) < 0$

$(0, 3) \Rightarrow x^4(x - 3) < 0$

$(3, \infty) \Rightarrow x^4(x - 3) > 0$

Solution set: $(-\infty, 3]$



37. $4x^2 - 4x + 1 \le 0$

$(2x - 1)^2 \le 0$

Key number: $x = \frac{1}{2}$

Test Interval	x-Value	Polynomial Value	Conclusion
$\left(-\infty, \frac{1}{2}\right)$	$x = 0$	$[2(0) - 1]^2 = 1$	Positive
$\left(\frac{1}{2}, \infty\right)$	$x = 1$	$[2(1) - 1]^2 = 1$	Positive

The solution set consists of the single real number $\frac{1}{2}$.

38. $x^2 + 3x + 8 > 0$

Using the Quadratic Formula you can determine the key numbers are $x = -\dfrac{3}{2} \pm \dfrac{\sqrt{23}}{2}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 + 3(0) + 8 = 8$	Positive

The solution set is the set of all real numbers.

39. $x^2 - 6x + 12 \le 0$

Using the Quadratic Formula, you can determine that the key numbers are $x = 3 \pm \sqrt{3}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 - 6(0) + 12 = 12$	Positive

The solution set is empty, that is there are no real solutions.

40. $x^2 - 8x + 16 > 0$

$(x - 4)^2 > 0$

Key number: $x = 4$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, 4)$	$x = 0$	$(0 - 4)^2 = 16$	Positive
$(4, \infty)$	$x = 5$	$(5 - 4)^2 = 1$	Positive

The solution set consists of all real numbers except $x = 4$, or $(-\infty, 4) \cup (4, \infty)$.

41. $\dfrac{4x - 1}{x} > 0$

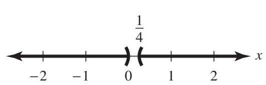
Key numbers: $x = 0, x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\dfrac{4x - 1}{x} > 0$?

Interval	x-Value	Value of $\dfrac{4x - 1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\dfrac{-5}{-1} = 5$	Positive
$\left(0, \dfrac{1}{4}\right)$	$\dfrac{1}{8}$	$\dfrac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\dfrac{1}{4}, \infty\right)$	1	$\dfrac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$



42. $\dfrac{x^2 - 1}{x} < 0$

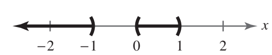
$\dfrac{(x - 1)(x + 1)}{x} < 0$

Key numbers: $x = -1, x = 0, x = 1$

Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Interval	x-Value	Value of $\dfrac{(x - 1)(x + 1)}{x}$	Conclusion
$(-\infty, -1)$	-2	$\dfrac{(-3)(-1)}{-2} = -\dfrac{3}{2}$	Negative
$(-1, 0)$	$-\dfrac{1}{2}$	$\dfrac{\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}} = \dfrac{3}{2}$	Positive
$(0, 1)$	$\dfrac{1}{2}$	$\dfrac{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}{\frac{1}{2}} = -\dfrac{3}{2}$	Negative
$(1, \infty)$	2	$\dfrac{(1)(3)}{2} = \dfrac{3}{2}$	Positive

Solution set: $(-\infty, -1) \cup (0, 1)$



43.

$$\frac{3x+5}{x-1} < 2$$

$$\frac{3x+5}{x-1} - 2 < 0$$

$$\frac{3x+5-2(x-1)}{x-1} < 0$$

$$\frac{x+7}{x-1} < 0$$

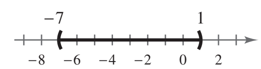
Key numbers: $x = -7, x = 1$

Test intervals: $(-\infty, -7), (-7, 1), (1, \infty)$

Test: Is $\dfrac{x+7}{x-1} < 0$?

Interval	x-Value	Value of $\dfrac{x+7}{x-1}$	Conclusion
$(-\infty, -7)$	-8	$\dfrac{-1}{-9} = \dfrac{1}{9}$	Positive
$(-7, 1)$	0	$\dfrac{0+7}{0-1} = -7$	Negative
$(1, \infty)$	2	$\dfrac{2+9}{2-1} = 11$	Positive

Solution set: $(-7, 1)$



44.

$$\frac{x+12}{x+2} \geq 3$$

$$\frac{x+12}{x+2} - 3 \geq 0$$

$$\frac{x+12-3(x+2)}{x+2} \geq 0$$

$$\frac{6-2x}{x+2} \geq 0$$

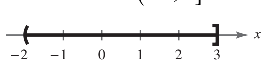
Key numbers: $x = -2, x = 3$

Test intervals: $(-\infty, -2), (-2, 3), (-3, \infty)$

Test: Is $\dfrac{6-2x}{x+2} > 0$?

Interval	x-Value	Value of $\dfrac{6-2x}{x+2}$	Conclusion
$(-\infty, -2)$	-3	$\dfrac{6-2(-3)}{(-3)+2} = -12$	Negative
$(-2, 3)$	0	$\dfrac{6-0}{0+2} = 3$	Positive
$(3, \infty)$	4	$\dfrac{6-8}{4+2} = \dfrac{1}{3}$	Negative

Solution set: $(-2, 3]$



45.

$$\frac{2}{x+5} > \frac{1}{x-3}$$

$$\frac{2}{x+5} - \frac{1}{x-3} > 0$$

$$\frac{2(x-3) - 1(x+5)}{(x+5)(x-3)} > 0$$

$$\frac{x-11}{(x+5)(x-3)} > 0$$

Key numbers: $x = -5, x = 3, x = 11$

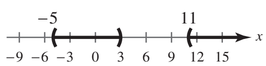
Test intervals: $(-\infty, -5) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(-5, 3) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

$(3, 11) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(11, \infty) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

Solution set: $(-5, 3) \cup (11, \infty)$



46.

$$\frac{5}{x-6} > \frac{3}{x+2}$$

$$\frac{5(x+2) - 3(x-6)}{(x-6)(x+2)} > 0$$

$$\frac{2x+28}{(x-6)(x+2)} > 0$$

Key numbers: $x = -14, x = -2, x = 6$

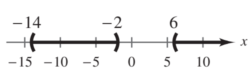
Test intervals: $(-\infty, -14) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



47.

$$\frac{1}{x-3} \le \frac{9}{4x+3}$$

$$\frac{1}{x-3} - \frac{9}{4x+3} \le 0$$

$$\frac{4x+3-9(x-3)}{(x-3)(4x+3)} \le 0$$

$$\frac{30-5x}{(x-3)(4x+3)} \le 0$$

Key numbers: $x = 3, x = -\frac{3}{4}, x = 6$

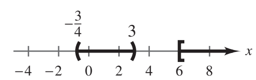
Test intervals: $\left(-\infty, -\frac{3}{4}\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$\left(-\frac{3}{4}, 3\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

$(3, 6) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$(6, \infty) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

Solution set: $\left(-\frac{3}{4}, 3\right) \cup [6, \infty)$



48.

$$\frac{1}{x} \ge \frac{1}{x+3}$$

$$\frac{1(x+3) - 1(x)}{x(x+3)} \ge 0$$

$$\frac{3}{x(x+3)} \ge 0$$

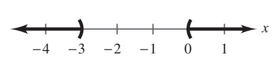
Key numbers: $x = -3, x = 0$

Test intervals: $(-\infty, -3) \Rightarrow \frac{3}{x(x+3)} > 0$

$(-3, 0) \Rightarrow \frac{3}{x(x+3)} < 0$

$(0, \infty) \Rightarrow \frac{3}{x(x+3)} > 0$

Solution intervals: $(-\infty, -3) \cup (0, \infty)$



49. $$\frac{x^2+2x}{x^2-9} \le 0$$

$$\frac{x(x+2)}{(x+3)(x-3)} \le 0$$

Key numbers: $x = 0, x = -2, x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} > 0$

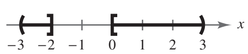
$(-3, -2) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} < 0$

$(-2, 0) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



50. $$\frac{x^2+x-6}{x} \ge 0$$

$$\frac{(x+3)(x-2)}{x} \ge 0$$

Key numbers: $x = -3, x = 0, x = 2$

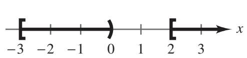
Test intervals: $(-\infty, -3) \Rightarrow \dfrac{(x+3)(x-2)}{x} < 0$

$(-3, 0) \Rightarrow \dfrac{(x+3)(x-2)}{x} > 0$

$(0, 2) \Rightarrow \dfrac{(x+3)(x-2)}{x} < 0$

$(2, \infty) \Rightarrow \dfrac{(x+3)(x-2)}{x} > 0$

Solution set: $[-3, 0) \cup [2, \infty)$



51. $$\frac{3}{x-1} + \frac{2x}{x+1} > -1$$

$$\frac{3(x+1) + 2x(x-1) + 1(x+1)(x-1)}{(x-1)(x+1)} > 0$$

$$\frac{3x^2+x+2}{(x-1)(x+1)} > 0$$

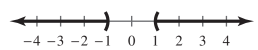
Key numbers: $x = -1, x = 1$

Test intervals: $(-\infty, -1) \Rightarrow \dfrac{3x^2+x+2}{(x-1)(x+1)} > 0$

$(-1, 1) \Rightarrow \dfrac{3x^2+x+2}{(x-1)(x+1)} < 0$

$(1, \infty) \Rightarrow \dfrac{3x^2+x+2}{(x-1)(x+1)} > 0$

Solution set: $(-\infty, -1) \cup (1, \infty)$



52. $$\frac{3x}{x-1} \le \frac{x}{x+4} + 3$$

$$\frac{3x(x+4) - x(x-1) - 3(x+4)(x-1)}{(x-1)(x+4)} \le 0$$

$$\frac{-x^2+4x+12}{(x-1)(x+4)} \le 0$$

$$\frac{-(x-6)(x+2)}{(x-1)(x+4)} \le 0$$

Key numbers: $x = -4, x = -2, x = 1, x = 6$

Test intervals: $(-\infty, -4) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

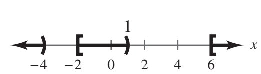
$(-4, -2) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(-2, 1) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

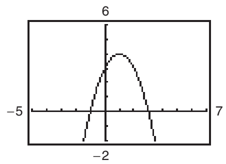
$(1, 6) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(6, \infty) \Rightarrow \dfrac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

Solution set: $(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$



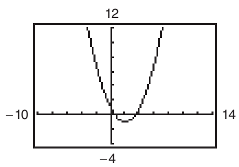
53. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

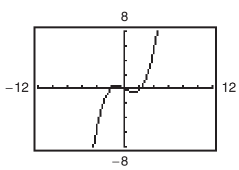
54. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 0$ when $2 - \sqrt{2} \le x \le 2 + \sqrt{2}$.

(b) $y \ge 7$ when $x \le -2$ or $x \ge 6$.

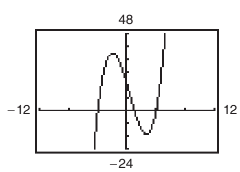
55. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \ge 0$ when $-2 \le x \le 0$ or $2 \le x < \infty$.

(b) $y \le 6$ when $x \le 4$.

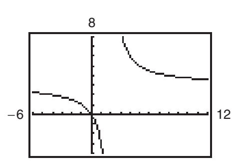
56. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$ when $-\infty < x \le -4$ or $1 \le x \le 4$.

(b) $y \ge 36$ when $x = -2$ or $5 \le x < \infty$.

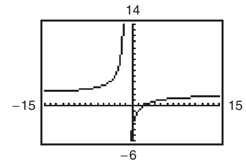
57. $y = \dfrac{3x}{x - 2}$



(a) $y \le 0$ when $0 \le x < 2$.

(b) $y \ge 6$ when $2 < x \le 4$.

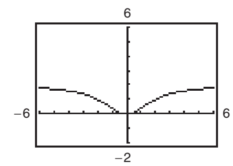
58. $y = \dfrac{2(x - 2)}{x + 1}$



(a) $y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$ when $-2 \le x < -1$.

59. $y = \dfrac{2x^2}{x^2 + 4}$



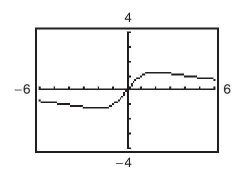
(a) $y \ge 1$ when $x \le -2$ or $x \ge 2$.

This can also be expressed as $|x| \ge 2$.

(b) $y \le 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

60. $y = \dfrac{5x}{x^2 + 4}$



(a) $y \ge 1$ when $1 \le x \le 4$.

(b) $y \le 0$ when $-\infty < x \le 0$.

61. $0.3x^2 + 6.26 < 10.8$

$0.3x^2 + 4.54 < 0$

Key numbers: $x \approx \pm 3.89$

Test intervals: $(-\infty, -3.89), (-3.89, 3.89), (3.89, \infty)$

Solution set: $(-3.89, 3.89)$

62. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

63. $-0.5x^2 + 12.5x + 1.6 > 0$

Key numbers: $x \approx -0.13, x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

Solution set: $(-0.13, 25.13)$

64. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Key numbers: $x \approx -4.42, x \approx 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

65.
$$\frac{1}{2.3x - 5.2} > 3.4$$
$$\frac{1}{2.3x - 5.2} - 3.4 > 0$$
$$\frac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$$
$$\frac{-7.82x + 18.68}{2.3x - 5.2} > 0$$

Key numbers: $x \approx 2.39, x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

66.
$$\frac{2}{3.1x - 3.7} > 5.8$$
$$\frac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$$
$$\frac{23.46 - 17.98x}{3.1x - 3.7} > 0$$

Key numbers: $x \approx 1.19, x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

67. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0, t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

Key numbers: $t = 4, t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds $< t <$ 6 seconds

68. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \Rightarrow t = 0$

$t - 8 = 0 \Rightarrow t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 - 8t + 8) < 0$

$t^2 - 8t + 8 > 0$

Key numbers: $t = 4 - 2\sqrt{2}, t = 4 + 2\sqrt{2}$

Test intervals:

$(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}),$

$(4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\le t < 4 - 2\sqrt{2}$ seconds
and
$4 + 2\sqrt{2}$ seconds $< t \le 8$ seconds

69. $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$

$$\begin{aligned} P &= R - C \\ &= (75x - 0.0005x^2) - (30x + 250{,}000) \\ &= -0.0005x^2 + 45x - 250{,}000 \end{aligned}$$

$$P \ge 750{,}000$$
$$-0.0005x^2 + 45x - 250{,}000 \ge 750{,}000$$
$$-0.0005x^2 + 45x - 1{,}000{,}000 \ge 0$$

Key numbers: $x = 40{,}000$, $x = 50{,}000$

(These were obtained by using the Quadratic Formula.)

Test intervals:

$(0, 40{,}000), (40{,}000, 50{,}000), (50{,}000, \infty)$

The solution set is $[40{,}000, 50{,}000]$ or $40{,}000 \le x \le 50{,}000$. The price per unit is

$$p = \frac{R}{x} = 75 - 0.0005x.$$

For $x = 40{,}000$, $p = \$55$. For $x = 50{,}000$, $p = \$50$. So, for $40{,}000 \le x \le 50{,}000$, $\$50.00 \le p \le \$55.00$.

70. $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$

$$\begin{aligned} P &= R - C \\ &= (50x - 0.0002x^2) - (12x + 150{,}000) \\ &= -0.0002x^2 + 38x - 150{,}000 \end{aligned}$$

$$P \ge 1{,}650{,}000$$
$$-0.0002x^2 + 38x - 150{,}000 \ge 1{,}650{,}000$$
$$-0.0002x^2 + 38x - 1{,}800{,}000 \ge 0$$

Key numbers: $x = 90{,}000$ and $x = 100{,}000$

Test intervals:

$(0, 90{,}000), (90{,}000, 100{,}000), (100{,}000, \infty)$

The solution set is $[90{,}000, 100{,}000]$ or $90{,}000 \le x \le 100{,}000$. The price per unit is

$$p = \frac{R}{x} = 50 - 0.0002x.$$

For $x = 90{,}000$, $p = \$32$. For $x = 100{,}000$, $p = \$30$. So, for $90{,}000 \le x \le 100{,}000$, $\$30 \le p \le \$32$.

71.
$$4 - x^2 \ge 0$$
$$(2 + x)(2 - x) \ge 0$$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

72. The domain of $\sqrt{x^2 - 9}$ can be found by solving the inequality:

$$x^2 - 9 \ge 0$$
$$(x + 3)(x - 3) \ge 0$$

Key numbers: $x = -3, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 3) > 0$

$(-3, 3) \Rightarrow (x + 3)(x - 3) < 0$

$(3, \infty) \Rightarrow (x + 3)(x - 3) > 0$

Domain: $(-\infty, -3] \cup [3, \infty)$

73.
$$x^2 - 9x + 20 \ge 0$$
$$(x - 4)(x - 5) \ge 0$$

Key numbers: $x = 4, x = 5$

Test intervals: $(-\infty, 4), (4, 5), (5, \infty)$

Interval	x-Value	Value of $(x - 4)(x - 5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

74. The domain of $\sqrt{49 - x^2}$ can be found by solving the inequality:

$$49 - x^2 \ge 0$$
$$x^2 - 49 \le 0$$
$$(x + 7)(x - 7) \le 0$$

Key numbers: $x = -7, x = 7$

Test intervals: $(-\infty, -7) \Rightarrow (x + 7)(x - 7) > 0$

$(-7, 7) \Rightarrow (x + 7)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 7)(x - 7) > 0$

Domain: $[-7, 7]$

75. $\dfrac{x}{x^2 - 2x - 35} \geq 0$

$\dfrac{x}{(x+5)(x-7)} \geq 0$

Key numbers: $x = 0, x = -5, x = 7$

Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x}{(x+5)(x-7)} < 0$

$(-5, 0) \Rightarrow \dfrac{x}{(x+5)(x-7)} > 0$

$(0, 7) \Rightarrow \dfrac{x}{(x+5)(x-7)} < 0$

$(7, \infty) \Rightarrow \dfrac{x}{(x+5)(x-7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

76. $\dfrac{x}{x^2 - 9} \geq 0$

$\dfrac{x}{(x+3)(x-3)} \geq 0$

Key numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$

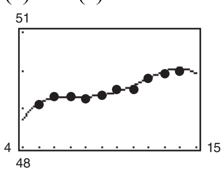
$(-3, 0) \Rightarrow \dfrac{x}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x+3)(x-3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

77. (a) and (c)



(b) $N = -0.001231t^4 + 0.04723t^3 - 0.6452t^2 + 3.783t + 41.21$

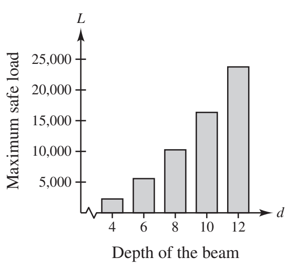
The model fits the data well.

(d) Using the zoom and trace features, the number of students enrolled in elementary and secondary schools fell below 48 million in the year 2017.

(e) No. The model can be used to predict enrollments for years close to those in its domain, $5 \leq t \leq 14$, but when you project too far into the future, the numbers predicted by the model decrease too rapidly to be considered reasonable.

78. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b) $2000 \leq 168.5d^2 - 472.1$

$2472.1 \leq 168.5d^2$

$14.67 \leq d^2$

$3.83 \leq d$

The minimum depth is 3.83 inches.

79. $2L + 2W = 100 \Rightarrow W = 50 - L$

$LW \geq 500$

$L(50 - L) \geq 500$

$-L^2 + 50L - 500 \geq 0$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \geq 0$?

Solution set: $25 - 5\sqrt{5} \leq L \leq 25 + 5\sqrt{5}$

13.8 meters $\leq L \leq$ 36.2 meters

80. $2L + 2W = 440 \Rightarrow W = 220 - L$

$$LW \geq 8000$$
$$L(220 - L) \geq 8000$$
$$-L^2 + 220L - 8000 \geq 0$$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \geq 0$?

Solution set: $110 - 10\sqrt{41} \leq L \leq 110 + 10\sqrt{41}$
$45.97 \text{ feet} \leq L \leq 174.03 \text{ feet}$

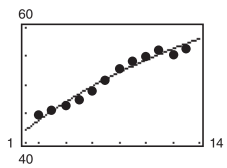
81. $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{2}$$
$$2R_1 = 2R + RR_1$$
$$2R_1 = R(2 + R_1)$$
$$\frac{2R_1}{2 + R_1} = R$$

Because $R \geq 1$,

$$\frac{2R_1}{2 + R_1} \geq 1$$
$$\frac{2R_1}{2 + R_1} - 1 \geq 0$$
$$\frac{R_1 - 2}{2 + R_1} \geq 0.$$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \geq 2$ ohms.

82. (a)



(b) The model fits the data well; each data value is close to the graph of the model.

(c) $$S = \frac{40.32 + 3.53t}{1 + 0.039t}$$
$$65 \leq \frac{40.32 + 3.53t}{1 + 0.039t}$$
$$0 \leq \frac{40.32 + 3.53t}{1 + 0.039t} - 65$$
$$0 \leq \frac{-24.68 + 0.995t}{1 + 0.039t}$$

Key numbers: $t \approx -25.6$ and $t \approx 24.8$. Use the domain of the model to create test intervals.

Test Intervals	t-Value	Expression Value	Conclusion
$(2, 24.8)$	$t = 5$	< 0	Negative
$(24.8, \infty)$	$t = 30$	> 0	Positive

So, the mean salary for classroom teachers will exceed $65,000 during the year 2024.

(d) Yes. The model yields a steady gradual increase in salaries for values of $t \geq 13$.

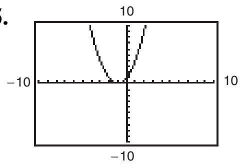
83. False.

There are four test intervals. The test intervals are $(-\infty, -3), (-3, 1), (1, 4),$ and $(4, \infty)$.

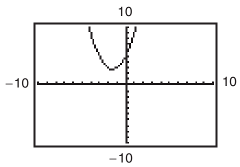
84. True.

The y-values are greater than zero for all values of x.

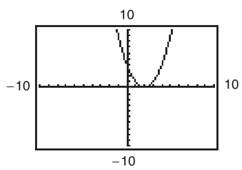
85.



For part (b), the y-values that are less than or equal to 0 occur only at $x = -1$.



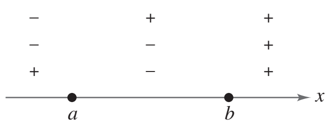
For part (c), there are no y-values that are less than 0.



For part (d), the y-values that are greater than 0 occur for all values of x except 2.

86. (a) $x = a, x = b$

(b)



(c) The real zeros of the polynomial.

87. $x^2 + bx + 9 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(1)(9) \geq 0$$
$$b^2 - 36 \geq 0$$

Key numbers: $b = -6, b = 6$

Test intervals: $(-\infty, -6) \Rightarrow b^2 - 36 > 0$

$(-6, 6) \Rightarrow b^2 - 36 < 0$

$(6, \infty) \Rightarrow b^2 - 36 > 0$

Solution set: $(-\infty, -6] \cup [6, \infty)$

(b) $b^2 - 4ac \geq 0$

Key numbers: $b = -2\sqrt{ac}, b = 2\sqrt{ac}$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

88. $x^2 + bx - 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(1)(-4) \geq 0$$
$$b^2 + 16 \geq 0$$

Key numbers: none

Test intervals: $(-\infty, \infty) \Rightarrow b^2 + 16 > 0$

Solution set: $(-\infty, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c < 0$, b can be any real number.

89. $3x^2 + bx + 10 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(3)(10) \geq 0$$
$$b^2 - 120 \geq 0$$

Key numbers: $b = -2\sqrt{30}, b = 2\sqrt{30}$

Test intervals: $\left(-\infty, -2\sqrt{30}\right) \Rightarrow b^2 - 120 > 0$

$\left(-2\sqrt{30}, 2\sqrt{30}\right) \Rightarrow b^2 - 120 < 0$

$\left(2\sqrt{30}, \infty\right) \Rightarrow b^2 - 120 > 0$

Solution set: $\left(-\infty, -2\sqrt{30}\,\right] \cup \left[2\sqrt{30}, \infty\right)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

90. $2x^2 + bx + 5 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(2)(5) \geq 0$$
$$b^2 - 40 \geq 0$$

Key numbers: $b = -2\sqrt{10}, b = 2\sqrt{10}$

Test intervals: $\left(-\infty, -2\sqrt{10}\right) \Rightarrow b^2 - 40 > 0$

$\left(-2\sqrt{10}, 2\sqrt{10}\right) \Rightarrow b^2 - 40 < 0$

$\left(2\sqrt{10}, \infty\right) \Rightarrow b^2 - 40 > 0$

Solution set: $\left(-\infty, -2\sqrt{10}\,\right] \cup \left[2\sqrt{10}, \infty\right)$

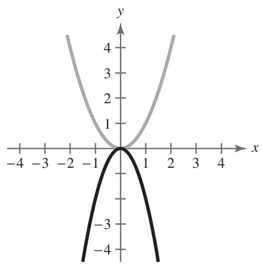
(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

Review Exercises for Chapter 2

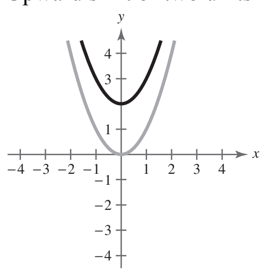
1. (a) $y = -2x^2$

Vertical stretch and a reflection in the x-axis



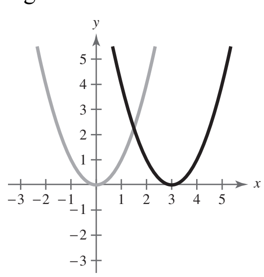
(b) $y = x^2 + 2$

Upward shift of two units



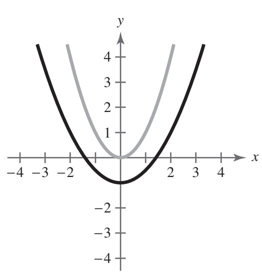
2. (a) $y = (x - 3)^2$

Right shift of three units



(b) $y = \frac{1}{2}x^2 - 1$

Vertical shrink and a downward shift of one unit



3. $g(x) = x^2 - 2x$

$= x^2 - 2x + 1 - 1$

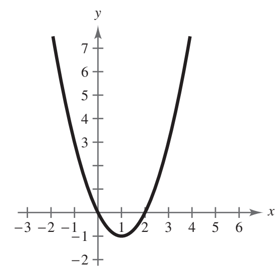
$= (x - 1)^2 - 1$

Vertex: $(1, -1)$

Axis of symmetry: $x = 1$

$0 = x^2 - 2x = x(x - 2)$

x-intercepts: $(0, 0), (2, 0)$



4. $f(x) = x^2 + 8x + 10$

$= x^2 + 8x + 16 - 16 + 10$

$= (x + 4)^2 - 6$

Vertex: $(-4, -6)$

Axis of symmetry: $x = -4$

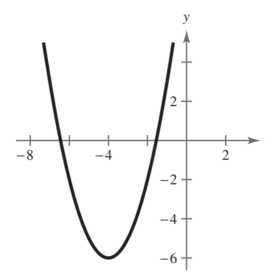
$0 = (x + 4)^2 - 6$

$(x + 4)^2 = 6$

$x + 4 = \pm\sqrt{6}$

$x = -4 \pm \sqrt{6}$

x-intercepts: $\left(-4 \pm \sqrt{6}, 0\right)$



5. $h(x) = 3 + 4x - x^2$

$= -(x^2 - 4x - 3)$

$= -(x^2 - 4x + 4 - 4 - 3)$

$= -\left[(x - 2)^2 - 7\right]$

$= -(x - 2)^2 + 7$

Vertex: $(2, 7)$

Axis of symmetry: $x = 2$

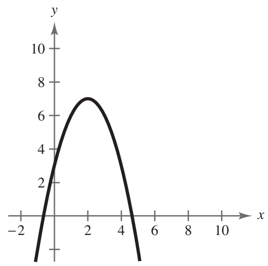
$0 = 3 + 4x - x^2$

$0 = x^2 - 4x - 3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}$$

x-intercepts: $\left(2 \pm \sqrt{7}, 0\right)$



6. $f(t) = -2t^2 + 4t + 1$

$= -2(t^2 - 2t + 1 - 1) + 1$

$= -2\left[(t - 1)^2 - 1\right] + 1$

$= -2(t - 1)^2 + 3$

Vertex: $(1, 3)$

Axis of symmetry: $t = 1$

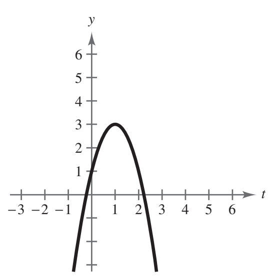
$0 = -2(t - 1)^2 + 3$

$2(t - 1)^2 = 3$

$t - 1 = \pm\sqrt{\dfrac{3}{2}}$

$t = 1 \pm \dfrac{\sqrt{6}}{2}$

t-intercepts: $\left(1 \pm \dfrac{\sqrt{6}}{2}, 0\right)$



7. $h(x) = 4x^2 + 4x + 13$

$= 4(x^2 + x) + 13$

$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 13$

$= 4\left(x^2 + x + \frac{1}{4}\right) - 1 + 13$

$= 4\left(x + \frac{1}{2}\right)^2 + 12$

Vertex: $\left(-\frac{1}{2}, 12\right)$

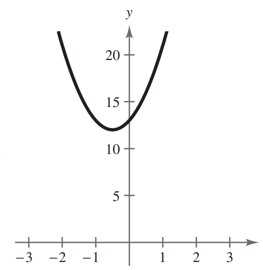
Axis of symmetry: $x = -\frac{1}{2}$

$0 = 4\left(x + \frac{1}{2}\right)^2 + 12$

$\left(x + \frac{1}{2}\right)^2 = -3$

No real zeros

x-intercepts: none



8. $f(x) = \dfrac{1}{3}(x^2 + 5x - 4)$

$= \dfrac{1}{3}\left(x^2 + 5x + \dfrac{25}{4} - \dfrac{25}{4} - 4\right)$

$= \dfrac{1}{3}\left[\left(x + \dfrac{5}{2}\right)^2 - \dfrac{41}{4}\right]$

$= \dfrac{1}{3}\left(x + \dfrac{5}{2}\right)^2 - \dfrac{41}{12}$

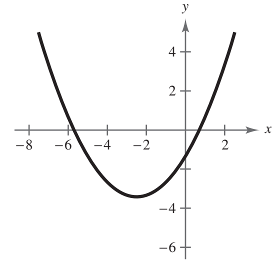
Vertex: $\left(-\dfrac{5}{2}, -\dfrac{41}{12}\right)$

Axis of symmetry: $x = -\dfrac{5}{2}$

$0 = x^2 + 5x - 4$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2(1)} = \frac{-5 \pm \sqrt{41}}{2}$$

x-intercepts: $\left(\dfrac{-5 \pm \sqrt{41}}{2}, 0\right)$



9. (a) $x + x + y + y = P$

$$2x + 2y = 1000$$
$$y = 500 - x$$

$$\begin{aligned} A &= xy \\ &= x(500 - x) \\ &= 500x - x^2 \end{aligned}$$

(b) $$\begin{aligned} A &= 500x - x^2 \\ &= -(x^2 - 500x + 62{,}500) + 62{,}500 \\ &= -(x - 250)^2 + 62{,}500 \end{aligned}$$

The maximum area occurs at the vertex when $x = 250$ and $y = 500 - 250 = 250$.

The dimensions with the maximum area are $x = 250$ meters and $y = 250$ meters.

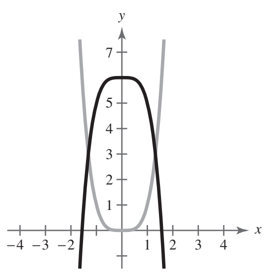
10. $C = 70{,}000 - 120x + 0.055x^2$

The minimum cost occurs at the vertex of the parabola.

Vertex: $-\dfrac{b}{2a} = -\dfrac{-120}{2(0.055)} \approx 1091$ units

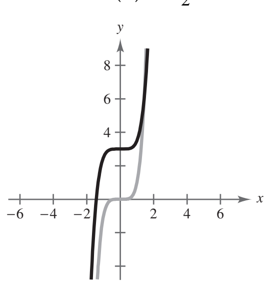
About 1091 units should be produced each day to yield a minimum cost.

11. $y = x^4, f(x) = 6 - x^4$



Transformation: Reflection in the x-axis and a vertical shift six units upward

12. $y = x^5, f(x) = \frac{1}{2}x^5 + 3$



Transformation: Vertical shrink and a vertical shift three units upward

13. $f(x) = -2x^2 - 5x + 12$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

14. $f(x) = 4x - \dfrac{1}{2}x^3$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

15. $g(x) = -3x^3 - 8x^4 + x^5$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

16. $h(x) = 5 + 9x^6 - 6x^5$

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

17. $g(x) = 2x^3 + 4x^2$

(a) The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

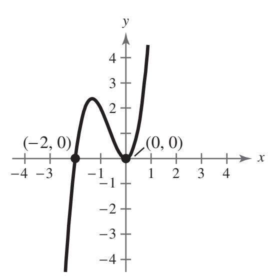
(b) $$\begin{aligned} g(x) &= 2x^3 + 4x^2 \\ 0 &= 2x^3 + 4x^2 \\ 0 &= 2x^2(x + 2) \\ 0 &= x^2(x + 2) \end{aligned}$$

Zeros: $x = -2, 0$

(c)

x	–3	–2	–1	0	1
$g(x)$	–18	0	2	0	6

(d)



18. $h(x) = 3x^2 - x^4$

(a) The degree is even and the leading coefficient, -1, is negative. The graph falls to the left and falls to the right.

(b) $g(x) = 3x^2 - x^4$

$0 = 3x^2 - x^4$

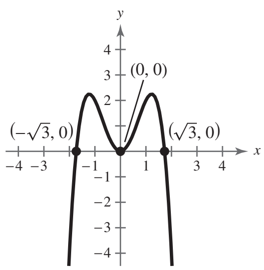
$0 = x^2(3 - x^2)$

Zeros: $x = 0, \pm\sqrt{3}$

(c)

x	-2	-1	0	1	2
$h(x)$	-4	2	0	2	-4

(d)



19. $f(x) = -x^3 + x^2 - 2$

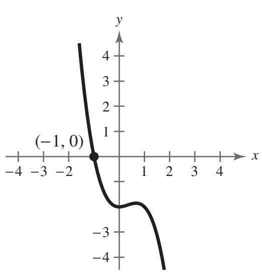
(a) The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

(b) Zero: $x = -1$

(c)

x	-3	-2	-1	0	1	2
$f(x)$	34	10	0	-2	-2	-6

(d)



20. $f(x) = x(x^3 + x^2 - 5x + 3)$

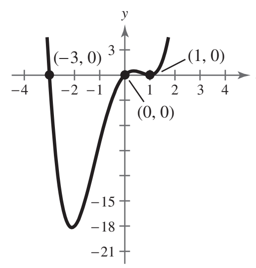
(a) The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

(b) Zeros: $x = 0, 1, -3$

(c)

x	-4	-3	-2	-1	0	1	2	3
$f(x)$	100	0	-18	-8	0	0	10	72

(d)



21. (a) $f(x) = 3x^3 - x^2 + 3$

x	-3	-2	-1	0	1	2	3
$f(x)$	-87	-25	-1	3	5	23	75

The zero is in the interval $[-1, 0]$.

(b) Zero: $x \approx -0.900$

22. (a) $f(x) = x^4 - 5x - 1$

x	-3	-2	-1	0	1	2	3
$f(x)$	95	25	5	-1	-5	5	65

There are zeros in the intervals $[-1, 0]$ and $[1, 2]$.

(b) Zeros: $x \approx -0.200$, $x \approx 1.772$

23.

$$\begin{array}{r} 6x + 3 \\ 5x - 3 \enclose{longdiv}{30x^2 - 3x + 8} \\ \underline{30x^2 - 18x} \\ 15x + 8 \\ \underline{15x - 9} \\ 17 \end{array}$$

$$\frac{30x^2 - 3x + 8}{5x - 3} = 6x + 3 + \frac{17}{5x - 3}$$

24. $$\begin{array}{r} 5x + 4 \\ x^2 - 5x - 1\overline{\smash{\big)}\,5x^3 - 21x^2 - 25x - 4} \\ \underline{5x^3 - 25x^2 - 5x} \\ 4x^2 - 20x - 4 \\ \underline{4x^2 - 20x - 4} \\ 0 \end{array}$$

$$\frac{5x^3 - 21x^2 - 25x - 4}{x^2 - 5x - 1} = 5x + 4,\ x \neq \frac{5}{2} \pm \frac{\sqrt{29}}{2}$$

25. $$\begin{array}{r|rrrr} 8 & 2 & -25 & 66 & 48 \\ & & 16 & -72 & -48 \\ \hline & 2 & -9 & -6 & 0 \end{array}$$

$$\frac{2x^3 - 25x^2 + 66x + 48}{x - 8} = 2x^2 - 9x - 6, \quad x \neq 8$$

26. $$\begin{array}{r|rrrrr} -3 & 1 & 0 & -2 & 9 & 0 \\ & & -3 & 9 & -21 & 36 \\ \hline & 1 & -3 & 7 & -12 & 36 \end{array}$$

$$\frac{x^4 - 2x^2 + 9x}{x + 3} = x^3 - 3x^2 + 7x - 12 + \frac{36}{x + 3}$$

27. $f(x) = 2x^3 + 11x^2 - 21x - 90$; Factor: $(x + 6)$

(a) $$\begin{array}{r|rrrr} -6 & 2 & 11 & -21 & -90 \\ & & -12 & 6 & 90 \\ \hline & 2 & -1 & -15 & 0 \end{array}$$

Yes, $(x + 6)$ is a factor of $f(x)$.

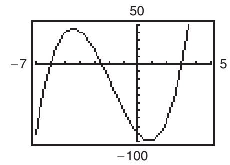
(b) $2x^2 - x - 15 = (2x + 5)(x - 3)$

The remaining factors are $(2x + 5)$ and $(x - 3)$.

(c) $f(x) = (2x + 5)(x - 3)(x + 6)$

(d) Zeros: $x = -\frac{5}{2}, 3, -6$

(e)



28. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

Factors: $(x + 2), (x - 3)$

(a) $$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

Yes, $(x + 2)$ and $(x - 3)$ are both factors of $f(x)$.

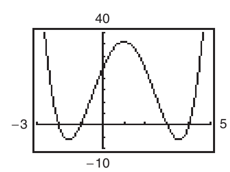
(b) $x^2 - 3x - 4 = (x + 1)(x - 4)$

The remaining factors are $(x + 1)$ and $(x - 4)$.

(c) $f(x) = (x + 1)(x - 4)(x + 2)(x - 3)$

(d) Zeros: $-2, -1, 3, 4$

(e)



29. $4 + \sqrt{-9} = 4 + 3i$

30. $-5i + i^2 = -1 - 5i$

31. $(6 - 4i) + (-9 + i) = (6 + (-9)) + (-4i + i) = -3 - 3i$

32. $(7 - 2i) - (3 - 8i) = (7 - 3) + (-2i + 8i) = 4 + 6i$

33. $$\begin{aligned} -3i(-2 + 5i) &= 6i - 15i^2 \\ &= 6i - 15(-1) \\ &= 15 + 6i \end{aligned}$$

34. $$\begin{aligned} (4 + i)(3 - 10i) &= 12 - 40i + 3i - 10i^2 \\ &= 12 - 37i - 10(-1) \\ &= 22 - 37i \end{aligned}$$

35. $$\begin{aligned} \frac{4}{1 - 2i} &= \frac{4}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} \\ &= \frac{4 + 8i}{1 - 4i^2} \\ &= \frac{4 + 8i}{5} \\ &= \frac{4}{5} + \frac{8}{5}i \end{aligned}$$

36. $\dfrac{3+2i}{5+i} = \dfrac{3+2i}{5+i}\cdot\dfrac{5-i}{5-i}$

$= \dfrac{15-3i+10i-2i^2}{25-i^2}$

$= \dfrac{17+7i}{26}$

$= \dfrac{17}{26}+\dfrac{7i}{26}$

37. $\dfrac{4}{2-3i}+\dfrac{2}{1+i} = \dfrac{4}{2-3i}\cdot\dfrac{2+3i}{2+3i}+\dfrac{2}{1+i}\cdot\dfrac{1-i}{1-i}$

$= \dfrac{8+12i}{4+9}+\dfrac{2-2i}{1+1}$

$= \dfrac{8}{13}+\dfrac{12}{13}i+1-i$

$= \left(\dfrac{8}{13}+1\right)+\left(\dfrac{12}{13}i-i\right)$

$= \dfrac{21}{13}-\dfrac{1}{13}i$

38. $\dfrac{1}{2+i}-\dfrac{5}{1+4i} = \dfrac{(1+4i)-5(2+i)}{(2+i)(1+4i)}$

$= \dfrac{1+4i-10-5i}{2+8i+i+4i^2}$

$= \dfrac{-9-i}{-2+9i}\cdot\dfrac{(-2-9i)}{(-2-9i)}$

$= \dfrac{18+81i+2i+9i^2}{4-81i^2}$

$= \dfrac{9+83i}{85} = \dfrac{9}{85}+\dfrac{83i}{85}$

39. $x^2-2x+10=0$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(10)}}{2(1)}$

$= \dfrac{2\pm\sqrt{-36}}{2}$

$= \dfrac{2\pm 6i}{2}$

$= 1\pm 3i$

40. $6x^2+3x+27=0$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-3\pm\sqrt{3^2-4(6)(27)}}{2(6)}$

$= \dfrac{-3\pm\sqrt{-639}}{12}$

$= \dfrac{-3\pm 3i\sqrt{71}}{12} = -\dfrac{1}{4}\pm\dfrac{\sqrt{71}}{4}i$

41. Since $g(x) = x^2-2x-8$ is a 2nd degree polynomial function, it has two zeros.

42. Since $h(t) = t^2-t^5$ is a 5th degree polynomial function, it has five zeros.

43. $f(x) = 4x^3-27x^2+11x+42$

Possible rational zeros: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm1, \pm\frac{3}{2}, \pm\frac{7}{4},$
$\pm2, \pm3, \pm\frac{7}{2}, \pm\frac{21}{4}, \pm6, \pm7, \pm\frac{21}{2}, \pm14, \pm21, \pm42$

$$\begin{array}{r|rrrr} -1 & 4 & -27 & 11 & 42 \\ & & -4 & 31 & -42 \\ \hline & 4 & -31 & 42 & 0 \end{array}$$

$4x^3-27x^2+11x+42 = (x+1)(4x^2-31x+42)$
$= (x+1)(x-6)(4x-7)$

The zeros of $f(x)$ are $x=-1$, $x=\frac{7}{4}$, and $x=6$.

44. $f(x) = x^4+x^3-11x^2+x-12$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -11 & 1 & -12 \\ & & 3 & 12 & 3 & 12 \\ \hline & 1 & 4 & 1 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 1 & 4 \\ & & -4 & 0 & -4 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$x^4+x^3-11x^2+x-12 = (x-3)(x+4)(x^2+1)$

The zeros of $f(x)$ are $x=3$ and $x=-4$.

45. $g(x) = x^3 - 7x^2 + 36$

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & 0 \end{array}$$

The zeros of $x^2 - 9x + 18 = (x - 3)(x - 6)$ are $x = 3, 6$. The zeros of $g(x)$ are $x = -2, 3, 6$.

$g(x) = (x + 2)(x - 3)(x - 6)$

46. $f(x) = x^4 + 8x^3 + 8x^2 - 72x - 153$

$$\begin{array}{r|rrrrr} 3 & 1 & 8 & 8 & -72 & -153 \\ & & 3 & 33 & 123 & 153 \\ \hline & 1 & 11 & 41 & 51 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 11 & 41 & 51 \\ & & -3 & -24 & -51 \\ \hline & 1 & 8 & 17 & \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 8x + 17$ are

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{-4}}{2} = -4 \pm i.$$

The zeros of $f(x)$ are $-3, 3, -4 - i, -4 + i$.

$f(x) = (x + 3)(x - 3)(x + 4 - i)(x + 4 + i)$

47. $h(x) = -2x^5 + 4x^3 - 2x^2 + 5$

$h(x)$ has three variations in sign, so h has either three or one positive real zeros.

$$\begin{aligned} h(-x) &= -2(-x)^5 + 4(-x)^3 - 2(-x)^2 + 5 \\ &= 2x^5 - 4x^3 - 2x^2 + 5 \end{aligned}$$

$h(-x)$ has two variations in sign, so h has either two or no negative real zeros.

48. $f(x) = 4x^3 - 3x^2 + 4x - 3$

(a)
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

Because the last row has all positive entries, $x = 1$ is an upper bound.

(b)
$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Because the last row entries alternate in sign, $x = -\frac{1}{4}$ is a lower bound.

49. Because the denominator is zero when $x + 10 = 0$, the domain of f is all real numbers except $x = -10$.

x	-11	-10.5	-10.1	-10.01	-10.001	$\to -10$
$f(x)$	33	63	303	3003	30,003	$\to \infty$

x	$-10 \leftarrow$	-9.999	-9.99	-9.9	-9.5	-9
$f(x)$	$-\infty \leftarrow$	$-29,997$	-2997	-297	-57	-27

As x approaches -10 from the left, $f(x)$ increases without bound.

As x approaches -10 from the right, $f(x)$ decreases without bound.

Vertical asymptote: $x = -10$

Horizontal asymptote: $y = 3$

50. Because the denominator is zero when $x^2 - 10x + 24 = (x - 4)(x - 6) = 0$, the domain of f is all real numbers except $x = 4$ and $x = 6$.

x	3	3.5	3.9	3.99	3.999	$\to 4$
$f(x)$	2.667	6.4	38.095	398.010	3998.001	$\to \infty$

x	$4 \leftarrow$	4.001	4.01	4.1	4.5	5
$f(x)$	$-\infty \leftarrow$	-4002.001	-402.010	-42.105	-10.67	-8

As x approaches 4 from the left, $f(x)$ increases without bound.

As x approaches 4 from the right, $f(x)$ decreases without bound.

x	5	5.5	5.9	5.99	5.999	$\to 6$
$f(x)$	-8	-10.67	-42.015	-402.010	-4002.001	$\to -\infty$

x	$6 \leftarrow$	6.001	6.01	6.1	6.5	7
$f(x)$	$\infty \leftarrow$	3998.001	398.010	38.095	6.4	2.667

As x approaches 6 from the left, $f(x)$ decreases without bound.

As x approaches 6 from the right, $f(x)$ increases without bound.

Vertical asymptotes: $x = 4$ and $x = 6$

Horizontal asymptote: $y = 0$

51. $f(x) = \dfrac{4}{x}$

(a) Domain: all real numbers x except $x = 0$

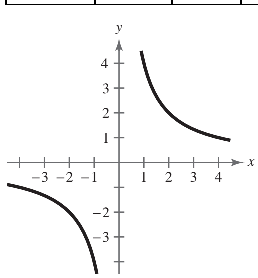
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	-1	1	2	3
$f(x)$	$-\frac{4}{3}$	-2	-4	4	2	$\frac{4}{3}$



52. $h(x) = \dfrac{x - 4}{x - 7}$

(a) Domain: all real numbers x except $x = 7$

(b) x-intercept: $(4, 0)$

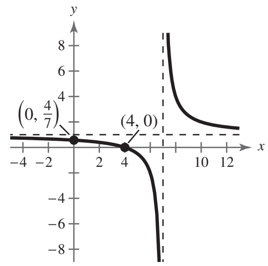
y-intercept: $\left(0, \dfrac{4}{7}\right)$

(c) Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 1$

(d)

x	-2	-1	0	1	2	3	4	5	6	8
$h(x)$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{4}{7}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	4



53. $f(x) = \dfrac{x}{x^2 - 16}$

(a) Domain: all real numbers x except $x \neq \pm 4$

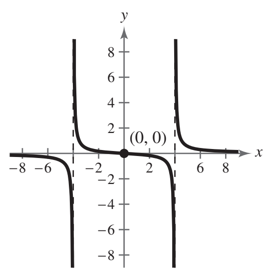
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 4$

Horizontal asymptote: $y = 0$

(d)

x	-5	-3	-2	-1	0	1	2	3	5
$f(x)$	$-\frac{5}{9}$	$\frac{3}{7}$	$\frac{1}{6}$	$\frac{1}{15}$	0	$-\frac{1}{15}$	$-\frac{1}{6}$	$-\frac{3}{7}$	$\frac{5}{9}$



54. $f(x) = \dfrac{-8x}{x^2 + 4}$

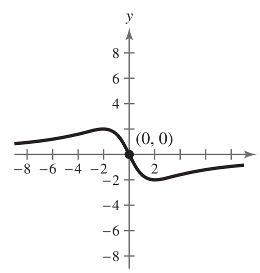
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	2
$f(x)$	2	$\frac{8}{5}$	0	$-\frac{8}{5}$	-2



55. $f(x) = \dfrac{6x^2 - 11x + 3}{3x^2 - x} = \dfrac{(3x - 1)(2x - 3)}{x(3x - 1)} = \dfrac{2x - 3}{x}, \quad x \neq \dfrac{1}{3}$

(a) Domain: all real numbers x except $x = 0$ and $x = \dfrac{1}{3}$

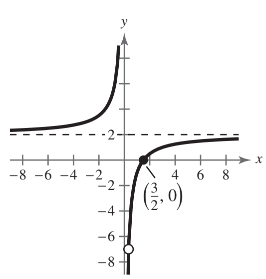
(b) x-intercept: $\left(\dfrac{3}{2}, 0\right)$

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 2$

(d)

x	-2	-1	1	2	3	4
$f(x)$	$\frac{7}{2}$	5	-1	$\frac{1}{2}$	1	$\frac{5}{4}$



56. $f(x) = \dfrac{6x^2 - 7x + 2}{4x^2 - 1}$

$= \dfrac{(2x - 1)(3x - 2)}{(2x - 1)(2x + 1)} = \dfrac{3x - 2}{2x + 1}, \; x \neq \dfrac{1}{2}$

(a) Domain: all real numbers x except $x = \pm\dfrac{1}{2}$

(b) y-intercept: $(0, -2)$

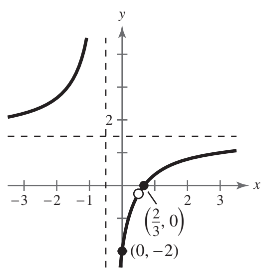
x-intercept: $\left(\dfrac{2}{3}, 0\right)$

(c) Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

(d)

x	-3	-2	-1	0	$\frac{2}{3}$	1	2
$f(x)$	$\frac{11}{5}$	$\frac{8}{3}$	5	-2	0	$\frac{1}{3}$	$\frac{4}{5}$



57. $f(x) = \dfrac{2x^3}{x^2 + 1} = 2x - \dfrac{2x}{x^2 + 1}$

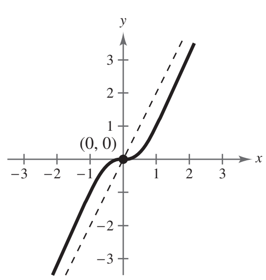
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Slant asymptote: $y = 2x$

(d)

x	-2	-1	0	1	2
$f(x)$	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



58. $f(x) = \dfrac{2x^2 + 2}{x + 1} = 2x - 2 + \dfrac{4}{x + 1}$

(a) Domain: all real numbers $x \neq -1$

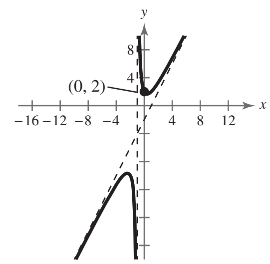
(b) y-intercept: $(0, 2)$

(c) Slant asymptote: $y = 2x - 2$

Vertical asymptote: $x = -1$

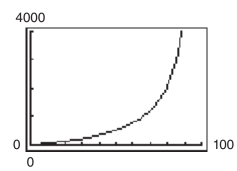
(d)

x	-3	-2	0	1	2	3
$f(x)$	10	-10	2	2	$\frac{10}{3}$	5



59. $C = \dfrac{528p}{100 - p}, \; 0 \leq p < 100$

(a)



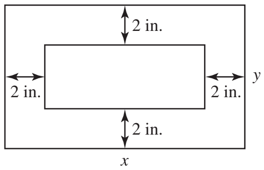
(b) When $p = 25$, $C = \dfrac{528(25)}{100 - 25} = \$176$ million.

When $p = 50$, $C = \dfrac{528(50)}{100 - 50} = \$528$ million.

When $p = 75$, $C = \dfrac{528(75)}{100 - 75} = \$1584$ million.

(c) As $p \to 100$, $C \to \infty$. No, the function is undefined when $p = 100$.

60.



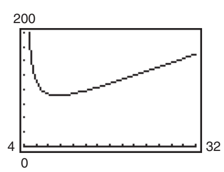
(a) The area of print is $(x-4)(y-4)$, which is 30 square inches.

$$(x-4)(y-4) = 30$$
$$y-4 = \frac{30}{x-4}$$
$$y = \frac{30}{x-4} + 4$$
$$y = \frac{30 + 4(x-4)}{x-4}$$
$$y = \frac{4x+14}{x-4}$$
$$y = \frac{2(2x+7)}{x-4}$$

$$\text{Total area} = xy = x\left[\frac{2(2x+7)}{x-4}\right] = \frac{2x(2x+7)}{x-4}$$

(b) Because the horizontal margins total 4 inches, x must be greater than 4 inches. The domain is $x > 4$.

(c)



The approximate dimensions are 9.48 in. $\times$ 9.48 in.

61.
$$12x^2 + 5x < 2$$
$$12x^2 + 5x - 2 < 0$$
$$(4x-1)(3x+2) < 0$$

Key numbers: $x = -\frac{2}{3}, x = \frac{1}{4}$

Test intervals: $\left(-\infty, -\frac{2}{3}\right), \left(-\frac{2}{3}, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $(4x-1)(3x+2) < 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $\left(-\frac{2}{3}, \frac{1}{4}\right)$.

62.
$$x^3 - 16x \geq 0$$
$$x(x+4)(x-4) \geq 0$$

Key numbers: $x = 0, x = \pm 4$

Test intervals: $(-\infty, -4), (-4, 0), (0, 4), (4, \infty)$

Test: Is $x(x+4)(x-4) \geq 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $[-4, 0] \cup [4, \infty)$.

63.
$$\frac{2}{x+1} \leq \frac{3}{x-1}$$
$$\frac{2(x-1) - 3(x+1)}{(x+1)(x-1)} \leq 0$$
$$\frac{2x - 2 - 3x - 3}{(x+1)(x-1)} \leq 0$$
$$\frac{-(x+5)}{(x+1)(x-1)} \leq 0$$

Key numbers: $x = -5, x = \pm 1$

Test intervals: $(-\infty, -5), (-5, -1), (-1, 1), (1, \infty)$

Test: Is $\frac{-(x+5)}{(x+1)(x-1)} \leq 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $[-5, -1) \cup (1, \infty)$.

64.
$$\frac{x^2 - 9x + 20}{x} < 0$$
$$\frac{(x-4)(x-5)}{x} < 0$$

Key numbers: $x = 0, x = 4, x = 5$

Test intervals: $(-\infty, 0), (0, 4), (4, 5), (5, \infty)$

Test: Is $\frac{(x-4)(x-5)}{x} < 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $(-\infty, 0) \cup (4, 5)$.

65.
$$P = \frac{1000(1+3t)}{5+t}$$
$$2000 \leq \frac{1000(1+3t)}{5+t}$$
$$2000(5+t) \leq 1000(1+3t)$$
$$10{,}000 + 2000t \leq 1000 + 3000t$$
$$-1000t \leq -9000$$
$$t \geq 9 \text{ days}$$

66. False. A fourth-degree polynomial can have at most four zeros, and complex zeros occur in conjugate pairs.

67. False. The domain of $f(x) = \dfrac{1}{x^2 + 1}$ is the set of all real numbers x.

68. An asymptote of a graph is a line to which the graph becomes arbitrarily close as x increases or decreases without bound.

Problem Solving for Chapter 2

1. $f(x) = ax^3 + bx^2 + cx + d$

$$\begin{array}{r|l}
 & ax^2 + (ak + b)x + (ak^2 + bk + c) \\
\hline
x - k & ax^3 + bx^2 \qquad + cx \qquad + d \\
 & \underline{ax^3 - akx^2} \\
 & (ak + b)x^2 + cx \\
 & \underline{(ak + b)x^2 - (ak^2 + bk)x} \\
 & (ak^2 + bk + c)x + d \\
 & \underline{(ak^2 + bk + c)x - (ak^3 + bk^2 + ck)} \\
 & (ak^3 + bk^2 + ck + d)
\end{array}$$

So, $f(x) = ax^3 + bx^2 + cx + d = (x - k)\left[ax^2 + (ak + b)x + (ak^2 + bx + c)\right] + ak^3 + bk^2 + ck + d$ and $f(x) = ak^3 + bk^2 + ck + d$. Because the remainder is $r = ak^3 + bk^2 + ck + d$, $f(k) = r$.

2. (a)

y	$y^3 + y^2$
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100

(b) (i) $x^3 + x^2 = 252 \Rightarrow x = 6$

(ii) $x^3 + 2x^2 = 288;\ a = 1, b = 2 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{1}{8}$

$$\frac{1}{8}x^3 + \frac{1}{8}(2x^2) = \frac{1}{8}(288)$$

$$\left(\frac{x}{2}\right)^3 + \left(\frac{x}{2}\right)^2 = 36 \Rightarrow \frac{x}{2} = 3 \Rightarrow x = 6$$

(iii) $3x^3 + x^2 = 90;\ a = 3, b = 1 \Rightarrow \dfrac{a^2}{b^3} = 9$

$$9(3x^3) + 9x^2 = 9(90)$$

$$(3x)^3 + (3x)^2 = 810 \Rightarrow 3x = 9 \Rightarrow x = 3$$

(iv) $2x^3 + 5x^2 = 2500;\ a = 2, b = 5 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{4}{125}$

$$\frac{4}{125}(2x^3) + \frac{4}{125}(5x^2) = \frac{4}{125}(2500)$$

$$\left(\frac{2x}{5}\right)^3 + \left(\frac{2x}{5}\right)^2 = 80 \Rightarrow \frac{2x}{5} = 4 \Rightarrow x = 10$$

(v) $7x^3 + 6x^2 = 1728;$

$a = 7, b = 6 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{49}{216}$

$$\frac{49}{216}(7x^3) + \frac{49}{216}(6x^2) = \frac{49}{216}(1728)$$

$$\left(\frac{7x}{6}\right)^3 + \left(\frac{7x}{6}\right)^2 = 392 \Rightarrow \frac{7x}{6} = 7 \Rightarrow x = 6$$

(vi) $10x^3 + 3x^2 = 297;$

$a = 10, b = 3 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{100}{27}$

$$\frac{100}{27}(10x^3) + \frac{100}{27}(3x^2) = \frac{100}{27}(297)$$

$$\left(\frac{10x}{3}\right)^3 + \left(\frac{10x}{3}\right)^2 = 1100 \Rightarrow \frac{10x}{3} = 10 \Rightarrow x = 3$$

(c) Answers will vary.

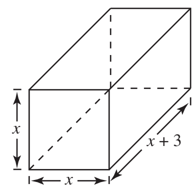
3. $V = l \cdot w \cdot h = x^2(x + 3)$

$x^2(x + 3) = 20$

$x^3 + 3x^2 - 20 = 0$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & 0 & -20 \\ & & 2 & 10 & 20 \\ \hline & 1 & 5 & 10 & 0 \end{array}$$



$(x - 2)(x^2 + 5x + 10) = 0$

$x = 2$ or $x = \dfrac{-5 \pm \sqrt{15}i}{2}$

Choosing the real positive value for x we have:
$x = 2$ and $x + 3 = 5$.

The dimensions of the mold are
2 inches × 2 inches × 5 inches.

4. False. Since $f(x) = d(x)q(x) + r(x)$, we have

$\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

The statement should be corrected to read $f(-1) = 2$

since $\dfrac{f(x)}{x + 1} = q(x) + \dfrac{f(-1)}{x + 1}$.

5. (a) $y = ax^2 + bx + c$

$(0, -4)$: $-4 = a(0)^2 + b(0) + c$

$-4 = c$

$(4, 0)$: $0 = a(4)^2 + b(4) - 4$

$0 = 16a + 4b - 4 = 4(4a + b - 1)$

$0 = 4a + b - 1$ or $b = 1 - 4a$

$(1, 0)$: $0 = a(1)^2 + b(1) - 4$

$4 = a + b$

$4 = a + (1 - 4a)$

$4 = 1 - 3a$

$3 = -3a$

$a = -1$

$b = 1 - 4(-1) = 5$

$y = -x^2 + 5x - 4$

(b) Enter the data points $(0, -4)$, $(1, 0)$, $(2, 2)$, $(4, 0)$, $(6, -10)$ and use the regression feature to obtain $y = -x^2 + 5x - 4$.

6. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$

Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$

Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$

Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2 + h) - f(2)}{(2 + h) - 2}$

$= \dfrac{(2 + h)^2 - 4}{h}$

$= \dfrac{4h + h^2}{h}$

$= 4 + h,\ h \neq 0$

(e) Slope $= 4 + h, \quad h \neq 0$

$4 + (-1) = 3$

$4 + 1 = 5$

$4 + 0.1 = 4.1$

The results are the same as in (a)–(c).

(f) Letting h get closer and closer to 0, the slope approaches 4. So, the slope at $(2, 4)$ is 4.

7. $f(x) = (x - k)q(x) + r$

(a) Cubic, passes through $(2, 5)$, rises to the right

One possibility:

$f(x) = (x - 2)x^2 + 5$

$= x^3 - 2x^2 + 5$

(b) Cubic, passes through $(-3, 1)$, falls to the right

One possibility:

$f(x) = -(x + 3)x^2 + 1$

$= -x^3 - 3x^2 + 1$

8. (a) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{1+i} = \dfrac{1}{1+i} \cdot \dfrac{1-i}{1-i}$

$= \dfrac{1-i}{2} = \dfrac{1}{2} - \dfrac{1}{2}i$

(b) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{3-i} = \dfrac{1}{3-i} \cdot \dfrac{3+i}{3+i}$

$= \dfrac{3+i}{10} = \dfrac{3}{10} + \dfrac{1}{10}i$

(c) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{-2+8i}$

$= \dfrac{1}{-2+8i} \cdot \dfrac{-2-8i}{-2-8i}$

$= \dfrac{-2-8i}{68} = -\dfrac{1}{34} - \dfrac{2}{17}i$

9. $(a+bi)(a-bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$

Since a and b are real numbers, $a^2 + b^2$ is also a real number.

10. $f(x) = \dfrac{ax+b}{cx+d}$

Vertical asymptote: $x = -\dfrac{d}{c}$

Horizontal asymptote: $y = \dfrac{a}{c}$

(i) $a > 0, b < 0, c > 0, d < 0$

Both the vertical asymptote and the horizontal asymptote are positive. Matches graph (d).

(ii) $a > 0, b > 0, c < 0, d < 0$

Both the vertical asymptote and the horizontal asymptote are negative. Matches graph (b).

(iii) $a < 0, b > 0, c > 0, d < 0$

The vertical asymptote is positive and the horizontal asymptote is negative. Matches graph (a).

(iv) $a > 0, b < 0, c > 0, d > 0$

The vertical asymptote is negative and the horizontal asymptote is positive. Matches graph (c).

11. $f(x) = \dfrac{ax}{(x-b)^2}$

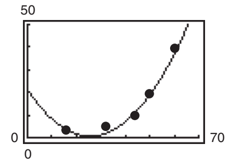
(a) $b \neq 0 \Rightarrow x = b$ is a vertical asymptote.

a causes a vertical stretch if $|a| > 1$ and a vertical shrink if $0 < |a| < 1$. For $|a| > 1$, the graph becomes wider as $|a|$ increases. When a is negative, the graph is reflected about the x-axis.

(b) $a \neq 0$. Varying the value of b varies the vertical asymptote of the graph of f. For $b > 0$, the graph is translated to the right. For $b < 0$, the graph is reflected in the x-axis and is translated to the left.

12. (a)

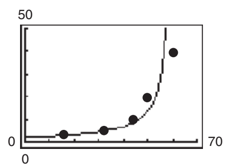
Age, x	Near point, y
16	3.0
32	4.7
44	9.8
50	19.7
60	39.4



$y \approx 0.031x^2 - 1.59x + 21.0$

(b) $\frac{1}{y} \approx -0.007x + 0.44$

$y \approx \frac{1}{-0.007x + 0.44}$



(c)

Age, x	Near point, y	Quadratic Model	Rational Model
16	3.0	3.66	3.05
32	4.7	2.32	4.63
44	9.8	11.83	7.58
50	19.7	19.97	11.11
60	39.4	38.54	50.00

The models are fairly good fits to the data. The quadratic model seems to be a better fit for older ages and the rational model a better fit for younger ages.

(d) For $x = 25$, the quadratic model yields $y \approx 0.625$ inch and the rational model yields $y \approx 3.774$ inches.

(e) The reciprocal model cannot be used to predict the near point for a person who is 70 years old because it results in a negative value $(y \approx -20)$. The quadratic model yields $y \approx 63.37$ inches.

13. Because complex zeros always occur in conjugate pairs, and a cubic function has three zeros and not four, a cubic function with real coefficients cannot have two real zeros and one complex zero.

Practice Test for Chapter 2

1. Sketch the graph of $f(x) = x^2 - 6x + 5$ and identify the vertex and the intercepts.

2. Find the number of units x that produce a minimum cost C if
$C = 0.01x^2 - 90x + 15{,}000$.

3. Find the quadratic function that has a maximum at $(1, 7)$ and passes through the point $(2, 5)$.

4. Find two quadratic functions that have x-intercepts $(2, 0)$ and $\left(\frac{4}{3}, 0\right)$.

5. Use the leading coefficient test to determine the right and left end behavior of the graph of the polynomial function $f(x) = -3x^5 + 2x^3 - 17$.

6. Find all the real zeros of $f(x) = x^5 - 5x^3 + 4x$.

7. Find a polynomial function with 0, 3, and -2 as zeros.

8. Sketch $f(x) = x^3 - 12x$.

9. Divide $3x^4 - 7x^2 + 2x - 10$ by $x - 3$ using long division.

10. Divide $x^3 - 11$ by $x^2 + 2x - 1$.

11. Use synthetic division to divide $3x^5 + 13x^4 + 12x - 1$ by $x + 5$.

12. Use synthetic division to find $f(-6)$ given $f(x) = 7x^3 + 40x^2 - 12x + 15$.

13. Find the real zeros of $f(x) = x^3 - 19x - 30$.

14. Find the real zeros of $f(x) = x^4 + x^3 - 8x^2 - 9x - 9$.

15. List all possible rational zeros of the function $f(x) = 6x^3 - 5x^2 + 4x - 15$.

16. Find the rational zeros of the polynomial $f(x) = x^3 - \frac{20}{3}x^2 + 9x - \frac{10}{3}$.

17. Write $f(x) = x^4 + x^3 + 5x - 10$ as a product of linear factors.

18. Find a polynomial with real coefficients that has $2, 3 + i,$ and $3 - 2i$ as zeros.

19. Use synthetic division to show that $3i$ is a zero of $f(x) = x^3 + 4x^2 + 9x + 36$.

20. Sketch the graph of $f(x) = \dfrac{x - 1}{2x}$ and label all intercepts and asymptotes.

21. Find all the asymptotes of $f(x) = \dfrac{8x^2 - 9}{x^2 + 1}$.

22. Find all the asymptotes of $f(x) = \dfrac{4x^2 - 2x + 7}{x - 1}$.

23. Given $z_1 = 4 - 3i$ and $z_2 = -2 + i$, find the following:

(a) $z_1 - z_2$

(b) $z_1 z_2$

(c) z_1/z_2

24. Solve the inequality: $x^2 - 49 \le 0$

25. Solve the inequality: $\dfrac{x+3}{x-7} \ge 0$

1 Functions and Their Graphs

1.2 Graphs of Equations

Objectives

- Sketch graphs of equations.
- Identify *x*- and *y*-intercepts of graphs of equations.
- Use symmetry to sketch graphs of equations.
- Write equations of and sketch graphs of circles.
- Use graphs of equations in solving real-life problems.

The Graph of an Equation

The Graph of an Equation

You have used a coordinate system to graphically represent the relationship between two quantities.

There, the graphical picture consisted of a collection of points in a coordinate plane.

Frequently, a relationship between two quantities is expressed as an **equation in two variables.**

For instance, $y = 7 - 3x$ is an equation in x and y. An ordered pair (a, b) is a **solution** or **solution point** of an equation in x and y when the substitutions $x = a$ and $y = b$ result in a true statement.

The Graph of an Equation

For instance, (1, 4) is a solution of $y = 7 - 3x$ because $4 = 7 - 3(1)$ is a true statement.

In this section you will review some basic procedures for sketching the graph of an equation in two variables.

The **graph of an equation** is the set of all points that are solutions of the equation.

The Graph of an Equation

The basic technique used for sketching the graph of an equation is the **point-plotting method**.

The Point-Plotting Method of Graphing

1. When possible, isolate one of the variables.
2. Construct a table of values showing several solution points.
3. Plot these points in a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

It is important to use negative values, zero, and positive values for *x* when constructing a table.

Example 2 – *Sketching the Graph of an Equation*

Sketch the graph of $y = -3x + 7$.

Solution:

Because the equation is already solved for y, construct a table of values that consists of several solution points of the equation.

For instance, when $x = -1$,

$$y = -3(-1) + 7$$

$$= 10$$

which implies that $(-1, 10)$ is a solution point of the equation.

Example 2 – *Solution*

cont'd

x	$y = -3x + 7$	(x, y)
-1	10	$(-1, 10)$
0	7	$(0, 7)$
1	4	$(1, 4)$
2	1	$(2, 1)$
3	-2	$(3, -2)$
4	-5	$(4, -5)$

From the table, it follows that

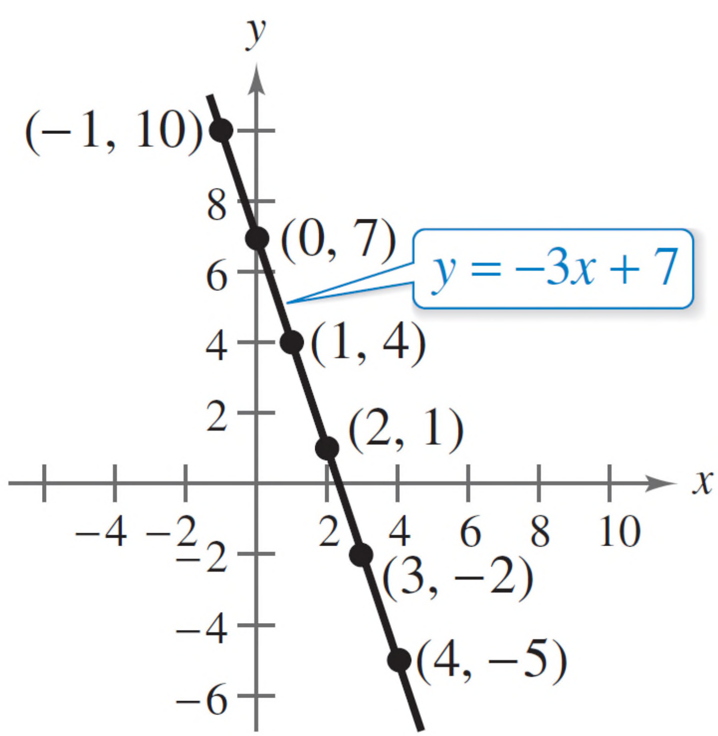
$$(-1, 10), (0, 7), (1, 4), (2, 1), (3, -2), \text{ and } (4, -5)$$

are solution points of the equation.

Example 2 – *Solution*

cont'd

After plotting these points and connecting them, you can see that they appear to lie on a line, as shown below.



Intercepts of a Graph

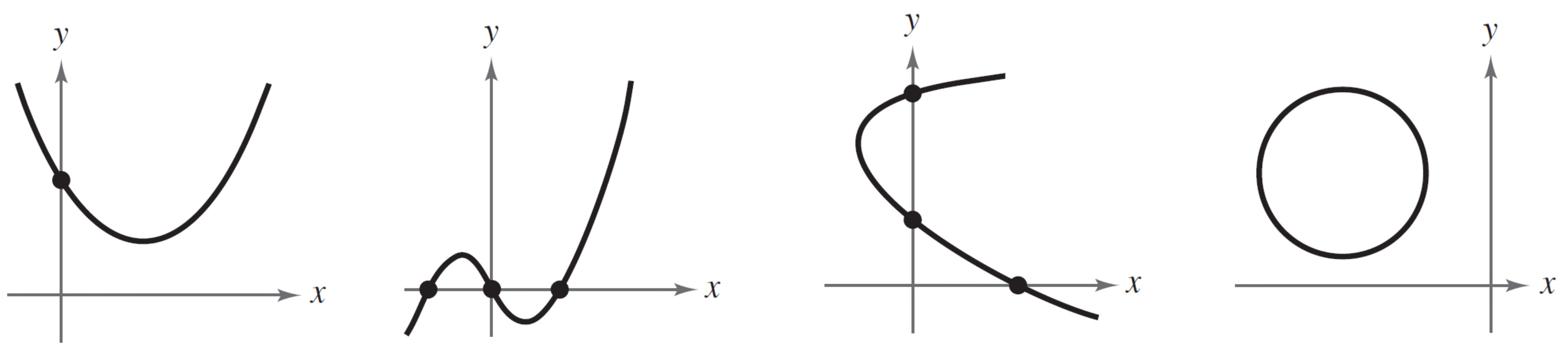
Intercepts of a Graph

It is often easy to determine the solution points that have zero as either the x–coordinate or the y–coordinate.

These points are called **intercepts** because they are the points at which the graph intersects or touches the x- or y-axis.

Intercepts of a Graph

It is possible for a graph to have no intercepts, one intercept, or several intercepts, as shown in Figure 1.14.



No *x*-intercepts; one *y*-intercept

Three *x*-intercepts; one *y*-intercept

One *x*-intercept; two *y*-intercepts

No intercepts

Figure 1.14

Note that an *x*-intercept can be written as the ordered pair (a, 0) and a *y*-intercept can be written as the ordered pair (0, b).

Intercepts of a Graph

Some texts denote the x-intercept as the x-coordinate of the point $(a, 0)$ [and the y-intercept as the y-coordinate of the point $(0, b)$] rather than the point itself.

Unless it is necessary to make a distinction, the term *intercept* will refer to either the point or the coordinate.

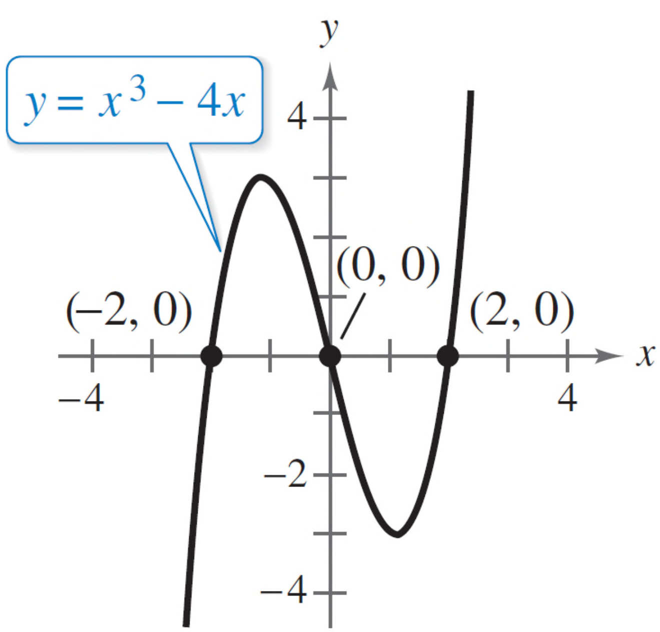
Finding Intercepts

1. To find x-intercepts, let y be zero and solve the equation for x.
2. To find y-intercepts, let x be zero and solve the equation for y.

Example 4 – *Finding x- and y-Intercepts*

To find the x-intercepts of the graph of $y = x^3 - 4x$, let $y = 0$.
Then $0 = x^3 - 4x = x(x^2 - 4)$ has solutions $x = 0$ and $x = \pm 2$.

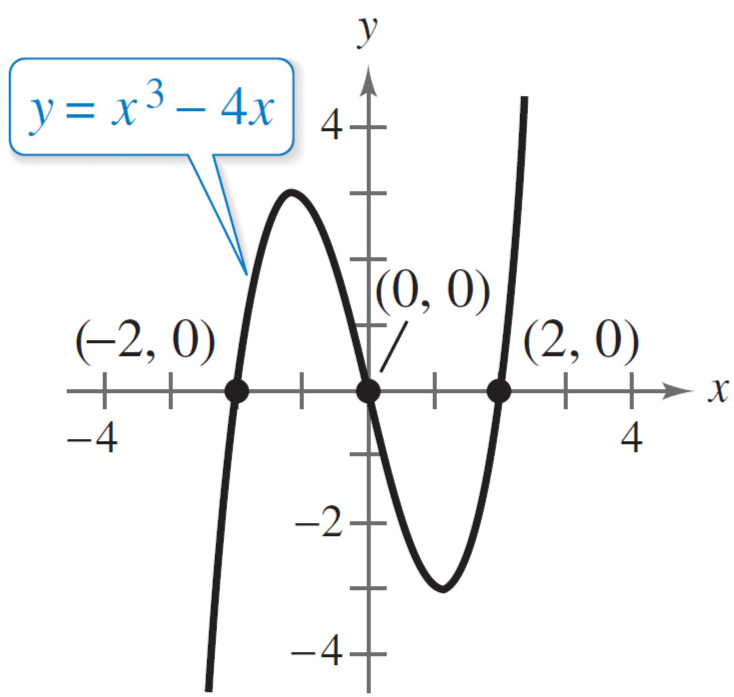
x-intercepts: (0, 0), (2, 0), (–2, 0) See figure.



Example 4 – *Finding x- and y-Intercepts* cont'd

To find the y-intercept of the graph of $y = x^3 - 4x$, let $x = 0$.
Then $y = (0)^3 - 4(0)$ has one solution, $y = 0$.

y-intercept: $(0, 0)$ See figure.

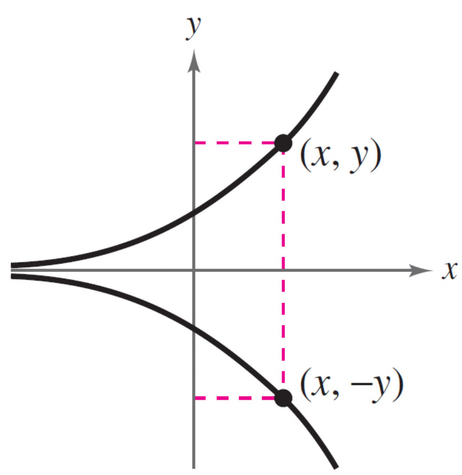


Symmetry

Symmetry

Graphs of equations can have **symmetry** with respect to one of the coordinate axes or with respect to the origin.

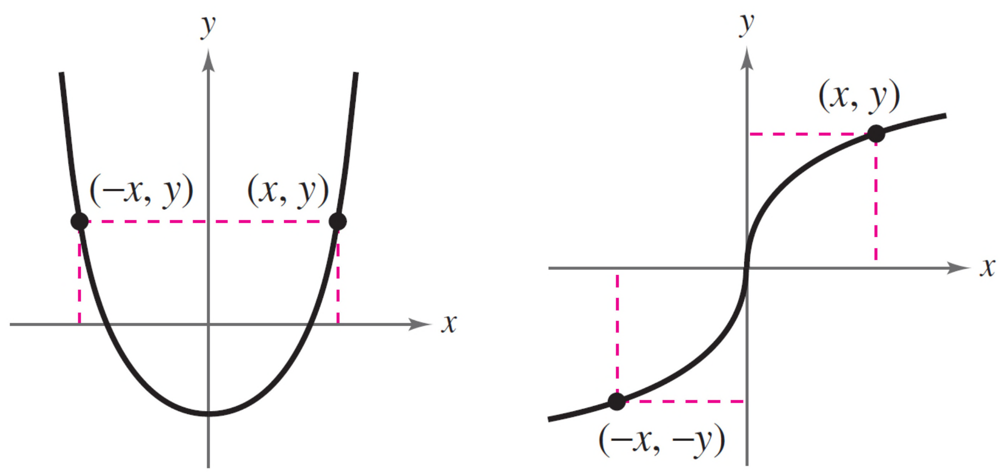
Symmetry with respect to the *x*-axis means that when the Cartesian plane were folded along the *x*-axis, the portion of the graph above the *x*-axis coincides with the portion below the *x*-axis.



x-Axis symmetry

Symmetry

Symmetry with respect to the *y*-axis or the origin can be described in a similar manner, as shown below.



y-Axis symmetry   Origin symmetry

Knowing the symmetry of a graph *before* attempting to sketch it is helpful, because then you need only half as many solution points to sketch the graph.

Symmetry

There are three basic types of symmetry, described as follows.

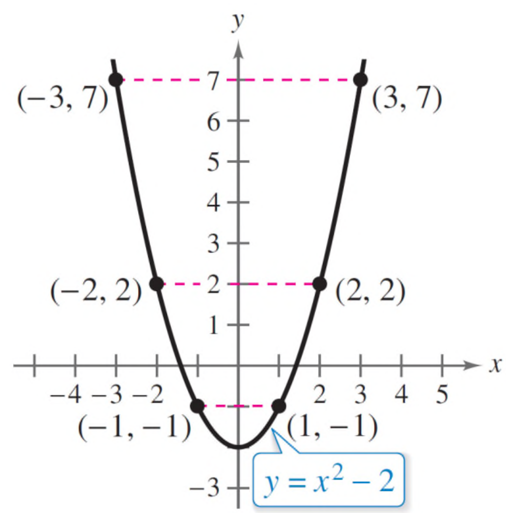
Graphical Tests for Symmetry

1. A graph is **symmetric with respect to the *x*-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the *y*-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

Symmetry

You can conclude that the graph of $y = x^2 - 2$ is symmetric with respect to the y-axis because the point $(-x, y)$ is also on the graph of $y = x^2 - 2$. (See the table below and Figure 1.15.)

x	-3	-2	-1	1	2	3
y	7	2	-1	-1	2	7
(x, y)	$(-3, 7)$	$(-2, 2)$	$(-1, -1)$	$(1, -1)$	$(2, 2)$	$(3, 7)$



y-Axis symmetry

Figure 1.15

Symmetry

Algebraic Tests for Symmetry

1. The graph of an equation is symmetric with respect to the x-axis when replacing y with $-y$ yields an equivalent equation.
2. The graph of an equation is symmetric with respect to the y-axis when replacing x with $-x$ yields an equivalent equation.
3. The graph of an equation is symmetric with respect to the origin when replacing x with $-x$ and y with $-y$ yields an equivalent equation.

Example 7 – *Sketching the Graph of an Equation*

Sketch the graph of $y = |x - 1|$.

Solution:

This equation fails all three tests for symmetry, and consequently its graph is not symmetric with respect to either axis or to the origin.

The absolute value bars indicates that y is always nonnegative.

Example 7 – *Solution*

cont'd

Construct a table of values. Then plot and connect the points, as shown in Figure 1.18.

x	-2	-1	0	1	2	3	4
$y = \lvert x - 1 \rvert$	3	2	1	0	1	2	3
(x, y)	$(-2, 3)$	$(-1, 2)$	$(0, 1)$	$(1, 0)$	$(2, 1)$	$(3, 2)$	$(4, 3)$

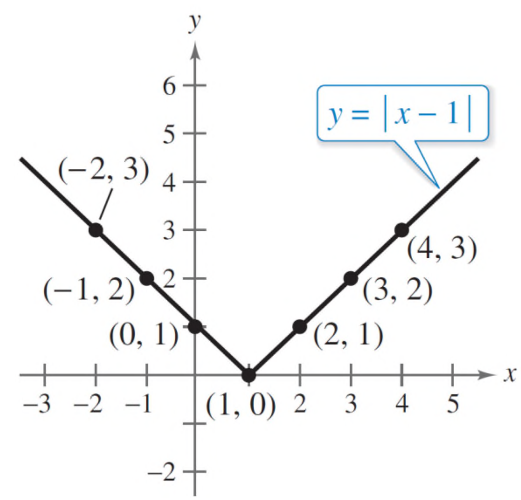


Figure 1.18

From the table, you can see that $x = 0$ when $y = 1$. So, the y-intercept is (0, 1). Similarly, $y = 0$ when $x = 1$. So, the x-intercept is (1, 0).

Circles

Circles

Consider the circle shown in Figure 1.19. A point (x, y) lies on the circle if and only if its distance from the center (h, k) is r. By the Distance Formula, $\sqrt{(x-h)^2+(y-k)^2}=r.$

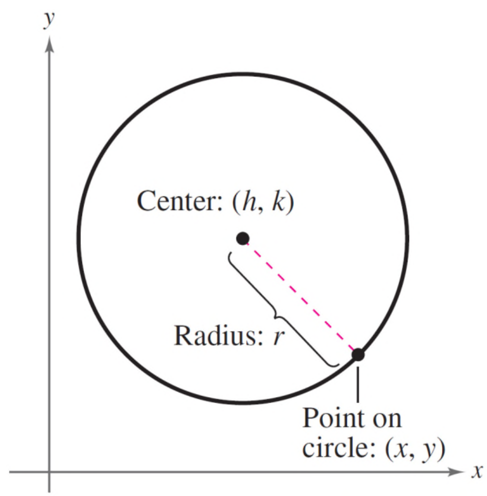


Figure 1.19

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

Circles

Standard Form of the Equation of a Circle

A point (x, y) lies on the circle of **radius** r and **center** (h, k) if and only if

$$(x - h)^2 + (y - k)^2 = r^2.$$

From this result, you can see that the standard form of the equation of a circle *with its center at the origin*, $(h, k) = (0, 0)$, is simply

$$x^2 + y^2 = r^2. \qquad \text{Circle with center at origin}$$

Example 8 – *Writing the Equation of a Circle*

The point (3, 4) lies on a circle whose center is at (–1, 2), as shown in Figure 1.20. Write the standard form of the equation of this circle.

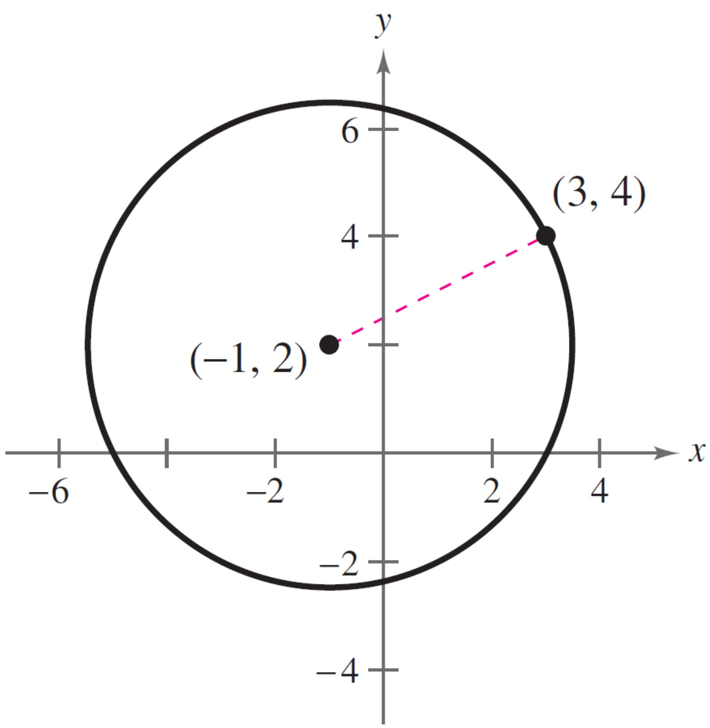


Figure 1.20

Example 8 – *Solution*

The radius of the circle is the distance between (–1, 2) and (3, 4).

$$
\begin{aligned}
r &= \sqrt{(x-h)^2 + (y-k)^2} && \text{Distance Formula} \\
&= \sqrt{[3-(-1)]^2 + (4-2)^2} && \text{Substitute for } x, y, h, \text{ and } k. \\
&= \sqrt{4^2 + 2^2} && \text{Simplify.} \\
&= \sqrt{16+4} && \text{Simplify.} \\
&= \sqrt{20} && \text{Radius}
\end{aligned}
$$

Example 8 – *Solution*

cont'd

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$$(x - h)^2 + (y - k)^2 = r^2$$ Equation of circle

$$[x - (-1)]^2 + (y - 2)^2 = (\sqrt{20})^2$$ Substitute for *h*, *k*, and *r*.

$$(x + 1)^2 + (y - 2)^2 = 20.$$ Standard form

Application

Application

You will learn that there are many ways to approach a problem. Three common approaches are illustrated in Example 9.

A Numerical Approach: Construct and use a table.

A Graphical Approach: Draw and use a graph.

An Algebraic Approach: Use the rules of algebra.

Example 9 – *Recommended Weight*

The median recommended weight y (in pounds) for men of medium frame who are 25 to 59 years old can be approximated by the mathematical model

$$y = 0.073x^2 - 6.99x + 289.0, \qquad 62 \le x \le 76$$

where x is the man's height (in inches).

a. Construct a table of values that shows the median recommended weights for men with heights of 62, 64, 66, 68, 70, 72, 74, and 76 inches.

Example 9 – *Recommended Weight* cont'd

b. Use the table of values to sketch a graph of the model. Then use the graph to estimate *graphically* the median recommended weight for a man whose height is 71 inches.

c. Use the model to confirm *algebraically* the estimate you found in part (b).

Example 9(a) – *Solution*

You can use a calculator to complete the table, as shown below.

Height, x	Weight, y
62	136.2
64	140.6
66	145.6
68	151.2
70	157.4
72	164.2
74	171.5
76	179.4

Example 9(b) – *Solution* cont'd

The table of values can be used to sketch the graph of the equation, as shown in Figure 1.21.

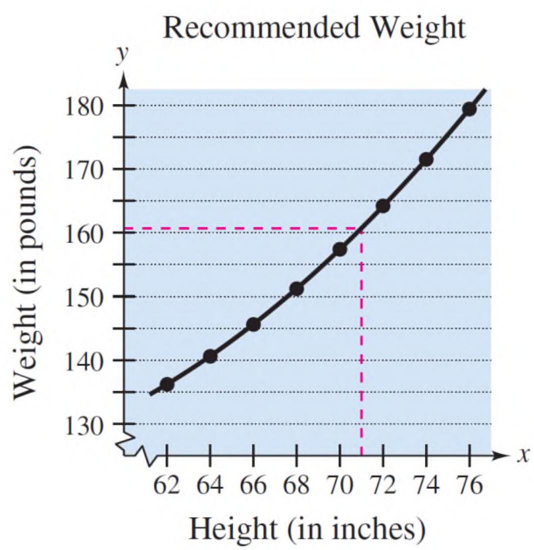


Figure 1.21

From the graph, you can estimate that a height of 71 inches corresponds to a weight of about 161 pounds.

Example 9(c) – *Solution*

cont'd

To confirm algebraically the estimate found in part (b), you can substitute 71 for x in the model.

$$y = 0.073(71)^2 - 6.99(71) + 289.0$$

$$\approx 160.70$$

So, the graphical estimate of 161 pounds is fairly good.