C H A P T E R 2

Section 2.1

1. nonnegative integer, real

2. quadratic, parabola

3. Yes, $f(x) = (x - 2)^2 + 3$ is in the form $f(x) = a(x - h)^2 + k$. The vertex is $(2, 3)$.

4. No, the graph of $f(x) = -3x^2 + 5x + 2$ is a parabola opening downward, therefore there is a relative maximum at its vertex.

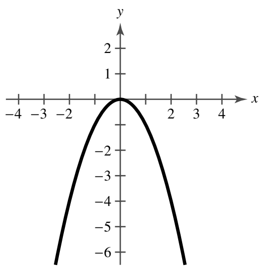
5. $f(x) = (x - 2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (c).

6. $f(x) = 3 - x^2$ opens downward and has vertex $(0, 3)$. Matches graph (d).

7. $f(x) = x^2 + 3$ opens upward and has vertex $(0, 3)$. Matches graph (b).

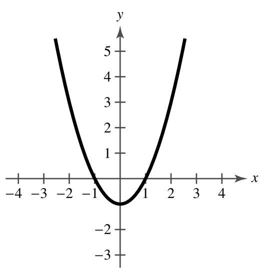
8. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (a).

9.



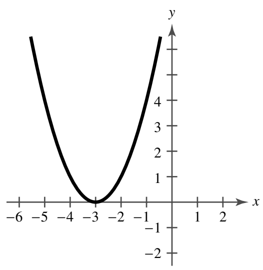
The graph of $y = -x^2$ is a reflection of $y = x^2$ in the x-axis.

10.



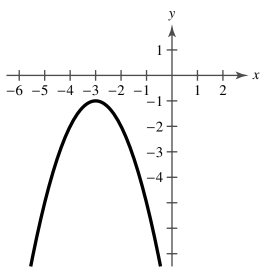
The graph of $y = x^2 - 1$ is a vertical shift downward one unit of $y = x^2$.

11.



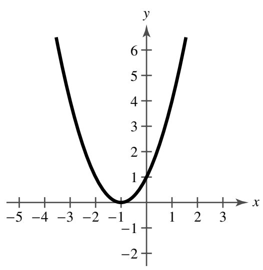
The graph of $y = (x + 3)^2$ is a horizontal shift three units to the left of $y = x^2$.

12.



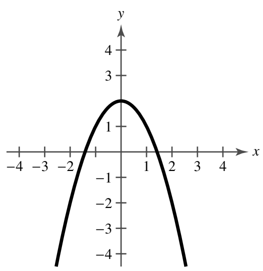
The graph of $y = -(x + 3)^2 - 1$ is a reflection in the x-axis, a horizontal shift three units to the left, and a vertical shift one unit downward of $y = x^2$.

13.



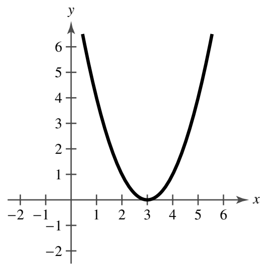
The graph of $y = (x + 1)^2$ is a horizontal shift one unit to the left of $y = x^2$.

14.



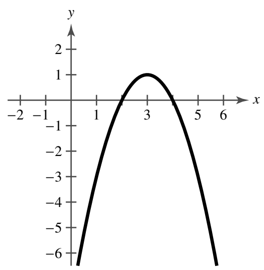
The graph of $y = -x^2 + 2$ is a reflection in the x-axis and a vertical shift two units upward of $y = x^2$.

15.



The graph of $y=(x-3)^2$ is a horizontal shift three units to the right of $y=x^2$.

16.



The graph of $y=-(x-3)^2+1$ is a horizontal shift three units to the right, a reflection in the x-axis, and a vertical shift one unit upward of $y=x^2$.

17. $f(x)=25-x^2$
$=-x^2+25$
A parabola opening downward with vertex $(0,\ 25)$

18. $f(x)=x^2-7$
A parabola opening upward with vertex $(0,\ -7)$

19. $f(x)=\frac{1}{2}x^2-4$
A parabola opening upward with vertex $(0,\ -4)$

20. $f(x)=16-\frac{1}{4}x^2$
$=-\frac{1}{4}x^2+16$
A parabola opening downward with vertex $(0,\ 16)$

21. $f(x)=(x+4)^2-3$
A parabola opening upward with vertex $(-4,\ -3)$

22. $f(x)=(x-6)^2+3$
A parabola opening upward with vertex $(6,\ 3)$

23. $h(x)=x^2-8x+16$
$=(x-4)^2$
A parabola opening upward with vertex $(4,\ 0)$

24. $g(x)=x^2+2x+1$
$=(x+1)^2$
A parabola opening upward with vertex $(-1,\ 0)$

25.
$$\begin{aligned} f(x)&=x^2-x+\frac{5}{4}\\ &=(x^2-x)+\frac{5}{4}\\ &=\left(x^2-x+\frac{1}{4}\right)+\frac{5}{4}-\frac{1}{4}\\ &=\left(x-\frac{1}{2}\right)^2+1 \end{aligned}$$
A parabola opening upward with vertex $\left(\frac{1}{2},\ 1\right)$

26.
$$\begin{aligned} f(x)&=x^2+3x+\frac{1}{4}\\ &=(x^2+3x)+\frac{1}{4}\\ &=\left(x^2+3x+\frac{9}{4}\right)+\frac{1}{4}-\frac{9}{4}\\ &=\left(x+\frac{3}{2}\right)^2-2 \end{aligned}$$
A parabola opening upward with vertex $\left(-\frac{3}{2},\ -2\right)$

27.
$$\begin{aligned} f(x)&=-x^2+2x+5\\ &=-(x^2-2x)+5\\ &=-(x^2-2x+1)+5+1\\ &=-(x-1)^2+6 \end{aligned}$$
A parabola opening downward with vertex $(1, 6)$

28.
$$\begin{aligned} f(x)&=-x^2-4x+1\\ &=-(x^2+4x)+1\\ &=-(x^2+4x+4)+1+4\\ &=-(x+2)^2+5 \end{aligned}$$
A parabola opening downward with vertex $(-2,\ 5)$

29.
$$\begin{aligned} h(x)&=4x^2-4x+21\\ &=4(x^2-x)+21\\ &=4\left(x^2-x+\frac{1}{4}\right)+21-4\left(\frac{1}{4}\right)\\ &=4\left(x-\frac{1}{2}\right)^2+20 \end{aligned}$$
A parabola opening upward with vertex $\left(\frac{1}{2},\ 20\right)$

30.
$$\begin{aligned} f(x)&=2x^2-x+1\\ &=2\left(x^2-\frac{1}{2}x\right)+1\\ &=2\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)+1-2\left(\frac{1}{16}\right)\\ &=2\left(x-\frac{1}{4}\right)^2+\frac{7}{8} \end{aligned}$$
A parabola opening upward with vertex $\left(\frac{1}{4},\ \frac{7}{8}\right)$

31. $$f(x) = -(x^2 + 2x - 3)$$
$$= -(x^2 + 2x) + 3$$
$$= -(x^2 + 2x + 1) + 3 + 1$$
$$= -(x + 1)^2 + 4$$

$$-(x^2 + 2x - 3) = 0$$
$$-(x + 3)(x - 1) = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x - 1 = 0 \Rightarrow x = 1$$

A parabola opening downward with vertex $(-1,\ 4)$ and x-intercepts $(-3,\ 0)$ and $(1,\ 0)$.

32. $$f(x) = -(x^2 + x - 30)$$
$$= -(x^2 + x) + 30$$
$$= -\left(x^2 + x + \frac{1}{4}\right) + 30 + \frac{1}{4}$$
$$= -\left(x + \frac{1}{2}\right)^2 + \frac{121}{4}$$

$$-(x^2 + x - 30) = 0$$
$$-(x - 5)(x + 6) = 0$$
$$x - 5 = 0 \Rightarrow x = 5$$
$$x + 6 = 0 \Rightarrow x = -6$$

A parabola opening downward with vertex $\left(-\frac{1}{2}, \frac{121}{4}\right)$ and x-intercepts $(5,\ 0)$ and $(-6,\ 0)$.

33. $$g(x) = x^2 + 8x + 11$$
$$= (x^2 + 8x) + 11$$
$$= (x^2 + 8x + 16) + 11 - 16$$
$$= (x + 4)^2 - 5$$

$$x^2 + 8x + 11 = 0$$
$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(11)}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{64 - 44}}{2}$$
$$= \frac{-8 \pm \sqrt{20}}{2}$$
$$= \frac{-8 \pm 2\sqrt{5}}{2}$$
$$= -4 \pm \sqrt{5}$$

A parabola opening upward with vertex $(-4,\ -5)$ and x-intercepts $(-4 \pm \sqrt{5},\ 0)$.

34. $$f(x) = x^2 + 10x + 14$$
$$= (x^2 + 10x) + 14$$
$$= (x^2 + 10x + 25) + 14 - 25$$
$$= (x + 5)^2 - 11$$

$$x^2 + 10x + 14 = 0$$
$$x = \frac{-10 \pm \sqrt{10^2 - 4(1)(14)}}{2(1)}$$
$$= \frac{-10 \pm \sqrt{100 - 56}}{2}$$
$$= \frac{-10 \pm \sqrt{44}}{2}$$
$$= \frac{-10 \pm 2\sqrt{11}}{2}$$
$$= -5 \pm \sqrt{11}$$

A parabola opening upward with vertex $(-5,\ -11)$ and x-intercepts $(-5 \pm \sqrt{11},\ 0)$

35. $$f(x) = -2x^2 + 16x - 31$$
$$= -2(x^2 - 8x) - 31$$
$$= -2(x^2 - 8x + 16) - 31 + 32$$
$$= -2(x - 4)^2 + 1$$

$$-2x^2 + 16x - 31 = 0$$
$$x = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-31)}}{2(-2)}$$
$$= \frac{-16 \pm \sqrt{256 - 248}}{-4}$$
$$= \frac{-16 \pm \sqrt{8}}{-4}$$
$$= \frac{-16 \pm 2\sqrt{2}}{-4}$$
$$= 4 \pm \frac{1}{2}\sqrt{2}$$

A parabola opening downward with vertex (4, 1) and x-intercepts $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$

36. $$f(x) = -4x^2 + 24x - 41$$
$$= -4(x^2 - 6x) - 41$$
$$= -4(x^2 - 6x + 9) - 41 + 36$$
$$= -4(x - 3)^2 - 5$$

$$-4x^2 + 24x - 41 = 0$$
$$x = \frac{-24 \pm \sqrt{24^2 - 4(-4)(-41)}}{2(-4)}$$
$$= \frac{-24 \pm \sqrt{576 - 656}}{-8}$$
$$= \frac{-24 \pm \sqrt{-80}}{-8}$$

No real solution

A parabola opening downward with vertex $(3, -5)$ and no x-intercepts

37. $(-1, 4)$ is the vertex.

$f(x) = a(x+1)^2 + 4$

Since the graph passes through the point $(1, 0)$, we have:

$$0 = a(1+1)^2 + 4$$
$$0 = 4a + 4$$
$$-1 = a$$

Thus, $f(x) = -(x+1)^2 + 4$. Note that $(-3, 0)$ is on the parabola.

38. $(-2, -1)$ is the vertex.

$f(x) = a(x+2)^2 - 1$

Since the graph passes through $(0, 3)$, we have:

$$3 = a(0+2)^2 - 1$$
$$3 = 4a - 1$$
$$4 = 4a$$
$$1 = a$$

Thus, $y = (x+2)^2 - 1$.

39. $(-2,5)$ is the vertex.

$f(x) = a(x+2)^2 + 5$

Since the graph passes through the point $(0, 9)$, we have:

$$9 = a(0+2)^2 + 5$$
$$4 = 4a$$
$$1 = a$$

Thus, $f(x) = (x+2)^2 + 5$.

40. $(4, 1)$ is the vertex.

$f(x) = a(x-4)^2 + 1$

Since the graph passes through the point $(6, -7)$, we have:

$$-7 = a(6-4)^2 + 1$$
$$-7 = 4a + 1$$
$$-8 = 4a$$
$$-2 = a$$

Thus, $f(x) = -2(x-4)^2 + 1$.

41. $(1, -2)$ is the vertex.

$f(x) = a(x-1)^2 - 2$

Since the graph passes through the point $(-1, 14)$, we have:

$$14 = a(-1-1)^2 - 2$$
$$14 = 4a - 2$$
$$16 = 4a$$
$$4 = a$$

Thus, $f(x) = 4(x-1)^2 - 2$.

42. $(-4, -1)$ is the vertex.

$f(x) = a(x+4)^2 - 1$

Since the graph passes through the point $(-2, 4)$, we have:

$$4 = a(-2+4)^2 - 1$$
$$5 = 4a$$
$$a = \frac{5}{4}$$

Thus, $f(x) = \frac{5}{4}(x+4)^2 - 1$.

43. $\left(\frac{1}{2}, 1\right)$ is the vertex.

$f(x) = a\left(x - \frac{1}{2}\right)^2 + 1$

Since the graph passes through the point $\left(-2, -\frac{21}{5}\right)$, we have:

$$-\frac{21}{5} = a\left(-2 - \frac{1}{2}\right)^2 + 1$$
$$-\frac{21}{5} = \frac{25}{4}a + 1$$
$$-\frac{26}{5} = \frac{25}{4}a$$
$$-\frac{104}{125} = a$$

Thus, $f(x) = -\frac{104}{125}\left(x - \frac{1}{2}\right)^2 + 1$.

44. $\left(-\frac{1}{4}, -1\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 - 1$

Since the graph passes through the point $\left(0, -\frac{17}{16}\right)$, we have:

$$-\frac{17}{16} = a\left(0 + \frac{1}{4}\right)^2 - 1$$
$$-\frac{17}{16} = \frac{1}{16}a - 1$$
$$-\frac{1}{16} = \frac{1}{16}a$$
$$a = -1$$

Thus, $f(x) = -\left(x + \frac{1}{4}\right)^2 - 1$.

45. $y = x^2 - 4x - 5$

x-intercepts: $(5, 0)$, $(-1, 0)$

$$0 = x^2 - 4x - 5$$
$$0 = (x-5)(x+1)$$
$$x = 5 \text{ or } x = -1$$

46. $y = 2x^2 + 5x - 3$

x-intercepts: $\left(\frac{1}{2},\ 0\right),\ (-3,\ 0)$

$0 = 2x^2 + 5x - 3$

$0 = (2x - 1)(x + 3)$

$x = \frac{1}{2},\ -3$

47. $y = x^2 + 8x + 16$

x-intercept: $(-4,\ 0)$

$0 = x^2 + 8x + 16$

$0 = (x + 4)^2$

$x = -4$

48. $y = x^2 - 6x + 9$

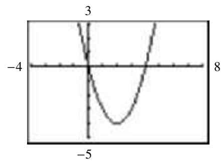
x-intercept: $(3,\ 0)$

$0 = x^2 - 6x + 9$

$0 = (x - 3)^2$

$x = 3$

49. $y = x^2 - 4x$



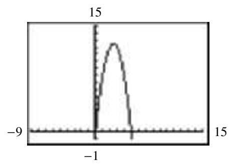
x-intercepts: $(0,\ 0),\ (4,\ 0)$

$0 = x^2 - 4x$

$0 = x(x - 4)$

$x = 0$ or $x = 4$

50. $y = -2x^2 + 10x$



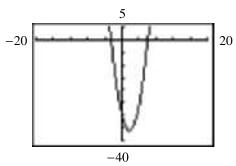
x-intercepts: $(0,\ 0),\ (5,\ 0)$

$0 = -2x^2 + 10x$

$0 = x(-2x + 10)$

$x = 0,\ x = 5$

51. $y = 2x^2 - 7x - 30$



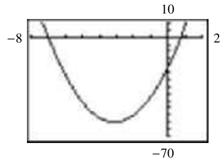
x-intercepts: $\left(-\frac{5}{2},\ 0\right),\ (6,\ 0)$

$0 = 2x^2 - 7x - 30$

$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2}$ or $x = 6$

52. $y = 4x^2 + 25x - 21$



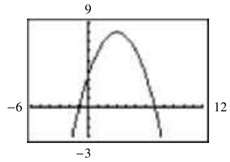
x-intercepts: $(-7,\ 0),\ (0.75,\ 0)$

$0 = 4x^2 + 25x - 21$

$= (x + 7)(4x - 3)$

$x = -7,\ \frac{3}{4}$

53. $y = -\frac{1}{2}(x^2 - 6x - 7)$



x-intercepts: $(-1,\ 0),\ (7,\ 0)$

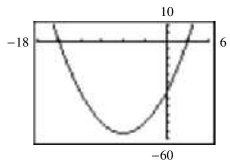
$0 = -\frac{1}{2}(x^2 - 6x - 7)$

$0 = x^2 - 6x - 7$

$0 = (x + 1)(x - 7)$

$x = -1,\ 7$

54. $y = \frac{7}{10}(x^2 + 12x - 45)$



x-intercepts: $(3,\ 0),\ (-15,\ 0)$

$0 = \frac{7}{10}(x^2 + 12x - 45)$

$0 = x^2 + 12x - 45$

$= (x - 3)(x + 15)$

$x = 3,\ -15$

55. $f(x) = [x - (-1)](x - 3)$, opens upward

$= (x + 1)(x - 3)$

$= x^2 - 2x - 3$

$g(x) = -[x - (-1)](x - 3)$, opens downward

$= -(x + 1)(x - 3)$

$= -(x^2 - 2x - 3)$

$= -x^2 + 2x + 3$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1,\ 0)$ and $(3,\ 0)$ for all real numbers $a \neq 0$.

56. $f(x) = x(x - 10) = x^2 - 10x$, opens upward.
$g(x) = -x(x - 10) = -x^2 + 10x$, opens downward.

Note: $f(x) = ax(x - 10)$ has x-intercepts $(0,\ 0)$ and $(10,\ 0)$ for all real numbers $a \neq 0$.

57. $f(x) = [x-(-3)]\left[x-\left(-\frac{1}{2}\right)\right](2)$, opens upward

$$= (x+3)\left(x+\frac{1}{2}\right)(2)$$
$$= (x+3)(2x+1)$$
$$= 2x^2+7x+3$$

$g(x) = -(2x^2+7x+3)$, opens downward

$$= -2x^2-7x-3$$

Note: $f(x) = a(x+3)(2x+1)$ has x-intercepts $(-3,\ 0)$ and $\left(-\frac{1}{2},\ 0\right)$ for all real numbers $a \neq 0$.

58. $f(x) = 2\left[x-\left(-\frac{5}{2}\right)\right](x-2)$

$$= 2\left(x+\frac{5}{2}\right)(x-2)$$
$$= 2x^2+x-10, \text{ opens upward}$$

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2-x+10$

Note: $f(x) = a\left(x+\frac{5}{2}\right)(x-2)$ has x-intercepts $\left(-\frac{5}{2},\ 0\right)$ and $(2,\ 0)$ for all real numbers $a \neq 0$.

59. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x+y=110 \Rightarrow y = 110-x$.

The product is

$$P(x) = xy = x(110-x) = 110x - x^2.$$
$$P(x) = -x^2+110x$$
$$= -(x^2-110x+3025-3025)$$
$$= -[(x-55)^2-3025]$$
$$= -(x-55)^2+3025$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

60. Let $x =$ first number and $y =$ second number. Then $x+y=66$ or $y = 66-x$. The product P is given by $P(x) = xy$.

$$P(x) = x(66-x)$$
$$= 66x - x^2$$
$$= -(x^2-66x)$$
$$= -(x^2-66x+33^2)+33^2$$
$$= -(x-33)^2+1089$$

The maximum P occurs at the vertex where $x = 33$, so $y = 66-33 = 33$. Therefore, the two numbers are 33 and 33.

61. Let x be the first number and y be the second number. Then $x+2y = 24 \Rightarrow x = 24-2y$. The product is

$P = xy = (24-2y)y = 24y - 2y^2$.

Completing the square,

$$P = -2y^2+24y$$
$$= -2(y^2-12y+36)+72$$
$$= -2(y-6)^2+72.$$

The maximum value of the product P occurs at the vertex of the parabola and equals 72. This happens when $y = 6$ and $x = 24-2(6) = 12$.

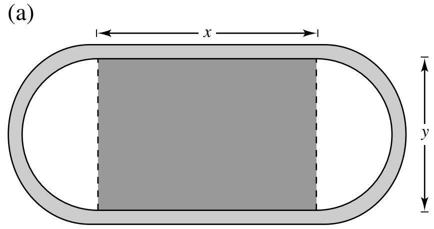
62. Let $x =$ first number and $y =$ second number.

Then $x+3y = 42$, $y = \frac{1}{3}(42-x)$. The product is

$$P(x) = xy = x\frac{1}{3}(42-x) = 14x - \frac{1}{3}x^2.$$
$$P(x) = -\frac{1}{3}x^2+14x$$
$$= -\frac{1}{3}(x^2-42x)$$
$$= -\frac{1}{3}(x^2-42x+441)+147$$
$$= -\frac{1}{3}(x-21)^2+147.$$

The maximum value of the product is 147, and occurs when $x = 21$ and $y = \frac{1}{3}(42-21) = 7$.

63. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track:

$$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$$

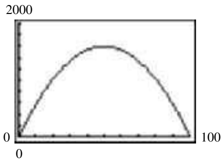
(c) Distance traveled around track in one lap:

$$d = \pi y + 2x = 200$$
$$\pi y = 200-2x$$
$$y = \frac{200-2x}{\pi}$$

(d) Area of rectangular region: $A = xy = x\left(\frac{200-2x}{\pi}\right)$

(e) The area is maximum when $x = 50$ and

$$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}.$$



64. (a) $4x + 3y = 200 \Rightarrow y = \frac{1}{3}(200 - 4x)$

$$\Rightarrow A = 2xy = 2x\frac{1}{3}(200 - 4x) = \frac{8x}{3}(50 - x)$$

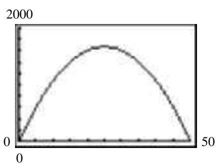
(b)

x	y	Area
2	$\frac{1}{3}[200 - 4(2)]$	$2xy = 256$
4	$\frac{1}{3}[200 - 4(4)]$	$2xy \approx 491$
6	$\frac{1}{3}[200 - 4(6)]$	$2xy = 704$
8	$\frac{1}{3}[200 - 4(8)]$	$2xy = 896$
10	$\frac{1}{3}[200 - 4(10)]$	$2xy \approx 1067$
12	$\frac{1}{3}[200 - 4(12)]$	$2xy = 1216$

x	y	Area
20	$\frac{1}{3}[200 - 4(20)]$	$2xy = 1600$
22	$\frac{1}{3}[200 - 4(22)]$	$2xy \approx 1643$
24	$\frac{1}{3}[200 - 4(24)]$	$2xy = 1664$
26	$\frac{1}{3}[200 - 4(26)]$	$2xy = 1664$
28	$\frac{1}{3}[200 - 4(28)]$	$2xy \approx 1643$
30	$\frac{1}{3}[200 - 4(30)]$	$2xy = 1600$

Maximum area when $x = 25,\ y = 33\frac{1}{3}$

(c) $A = \dfrac{8x(50 - x)}{3}$



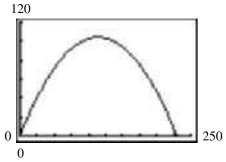
Maximum when $x = 25,\ y = 33\frac{1}{3}$

(d)
$$\begin{aligned} A &= \frac{8}{3}x(50 - x) \\ &= -\frac{8}{3}(x^2 - 50x) \\ &= -\frac{8}{3}(x^2 - 50x + 625 - 625) \\ &= -\frac{8}{3}[(x - 25)^2 - 625] \\ &= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3} \end{aligned}$$

The maximum area occurs at the vertex and is $\frac{5000}{3}$ square feet. This happens when $x = 25$ feet and $y = \frac{(200 - 4(25))}{3} = \frac{100}{3}$ feet. The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

(e) The result are the same.

65. (a)



(b) When $x = 0,\ y = \frac{3}{2}$ feet.

(c) The vertex occurs at

$$x = \frac{-b}{2a} = \frac{-9/5}{2(-16/2025)} = \frac{3645}{32} \approx 113.9.$$

The maximum height is

$$y = \frac{-16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + \frac{3}{2}$$
≈ 104.0 feet.

(d) Using a graphing utility, the zero of y occurs at $x \approx 228.6$, or 228.6 feet from the punter.

66. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

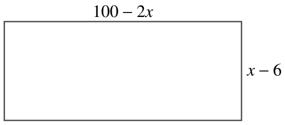
The maximum height of the dive occurs at the vertex,

$$x = \frac{-b}{2a} = -\frac{\frac{24}{9}}{2\left(\frac{-4}{9}\right)} = 3.$$

The height at $x = 3$ is $-\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16$.

The maximum height of the dive is 16 feet.

67. (a)



$A = lw$

$A = (100 - 2x)(x - 6)$

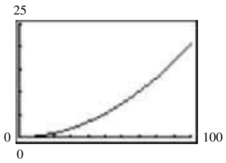
$A = -2x^2 + 112x - 600$

(b) $Y_1 = -2x^2 + 112x - 600$

X	Y
25	950
26	960
27	966
28	968
29	966
30	960

The area is maximum when $x = 28$ inches.

68. (a)



(b) The parabola intersects $y = 10$ at $s \approx 59.4$. Thus, the maximum speed is 59.4 mph. Analytically,

$$0.002s^2 + 0.05s - 0.029 = 10$$
$$2s^2 + 50s - 29 = 10{,}000$$
$$2s^2 + 50s - 10{,}029 = 0.$$

Using the Quadratic Formula,

$$s = \frac{-50 \pm \sqrt{50^2 - 4(2)(-10{,}029)}}{2(2)}$$
$$= \frac{-50 \pm \sqrt{82{,}732}}{4} \approx -84.4,\ 59.4.$$

The maximum speed is the positive root, 59.4 mph.

69. $R(p) = -10p^2 + 1580p$

(a) When $p = \$50$, $R(50) = \$54{,}000$.
When $p = \$70$, $R(70) = \$61{,}600$.
When $p = \$90$, $R(90) = \$61{,}200$.

(b) The maximum R occurs at the vertex,

$$p = \frac{-b}{2a}$$
$$p = \frac{-1580}{2(-10)} = \$79$$

(c) When $p = \$79$, $R(79) = \$62{,}410$.

(d) Answers will vary.

70. $R(p) = -12p^2 + 372p$

(a) When $p = \$12$, $R(12) = \$2736$.
When $p = \$16$, $R(16) = \$2880$.
When $p = \$20$, $R(20) = \$2640$.

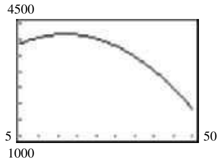
(b) The maximum R occurs at the vertex:

$$p = \frac{-b}{2a}$$
$$p = \frac{-372}{2(-12)} = \$15.50$$

(c) When $p = \$15.50$, $R(15.50) = \$2883$.

(d) Answers will vary.

71. (a)



(b) Using the graph, during 1966 the maximum average annual consumption of cigarettes appears to have occurred and was 4155 cigarettes per person.

Yes, the warning had an effect because the maximum consumption occurred in 1966 and consumption decreased from then on.

(c) In 2000, $C(50) = 1852$ cigarettes per person.

$$\frac{1852}{365} \approx 5 \text{ cigarettes per day}$$

72. (a) According to the model, $t = \dfrac{-b}{2a}$ or

$$t = \frac{-271.4}{2(-8.87)} \approx 15 \text{ or } 2005.$$

$P(15) \approx 82{,}437{,}000$

(b) When $t = 110$, $P(110) = 2{,}889{,}000$.

No, the population of Germany would not be expected to decrease this much

73. True.

$$-12x^2 - 1 = 0$$
$$12x^2 = -1, \text{ impossible}$$

74. True. For $f(x)$, $\dfrac{-b}{2a} = -\dfrac{-10}{2(-4)} = -\dfrac{10}{8} = -\dfrac{5}{4}$.

For $g(x)$, $\dfrac{-b}{2a} = \dfrac{-30}{2(12)} = \dfrac{-30}{24} = \dfrac{-5}{4}$.

In both cases, $x = -\dfrac{5}{4}$ is the axis of symmetry.

75. The parabola opens downward and the vertex is $(-2,\ -4)$. Matches (c) and (d).

76. The parabola opens upward and the vertex is $(1,\ 3)$. Matches (a).

77. The graph of $f(x) = (x - z)^2$ would be a horizontal shift z units to the right of $g(x) = x^2$.

78. The graph of $f(x)=x^2-z$ would be a vertical shift z units downward of $g(x)=x^2$.

79. The graph of $f(x)=z(x-3)^2$ would be a vertical stretch $(z>1)$ and horizontal shift three units to the right of $g(x)=x^2$. The graph of $f(x)=z(x-3)^2$ would be a vertical shrink $(0<z<1)$ and horizontal shift three units to the right of $g(x)=x^2$.

80. The graph of $f(x)=zx^2+4$ would be a vertical stretch $(z>1)$ and vertical shift four units upward of $g(x)=x^2$. The graph of $f(x)=zx^2+4$ would be a vertical shrink $(0<z<1)$ and vertical shift four units upward of $g(x)=x^2$.

81. For $a<0, f(x)=a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)$ is a maximum when $x=\frac{-b}{2a}$. In this case, the maximum value is $c-\frac{b^2}{4a}$. Hence,

$$\begin{aligned}25&=-75-\frac{b^2}{4(-1)}\\-100&=300-b^2\\400&=b^2\\b&=\pm 20.\end{aligned}$$

82. For $a<0, f(x)=a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)$ is a maximum when $x=\frac{-b}{2a}$. In this case, the maximum value is $c-\frac{b^2}{4a}$. Hence,

$$\begin{aligned}48&=-16-\frac{b^2}{4(-1)}\\-192&=64-b^2\\b^2&=256\\b&=\pm 16.\end{aligned}$$

83. For $a>0, f(x)=a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)$ is a minimum when $x=\frac{-b}{2a}$. In this case, the minimum value is $c-\frac{b^2}{4a}$. Hence,

$$\begin{aligned}10&=26-\frac{b^2}{4}\\40&=104-b^2\\b^2&=64\\b&=\pm 8.\end{aligned}$$

84. For $a>0, f(x)=a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)$ is a minimum when $x=\frac{-b}{2a}$. In this case, the minimum value is $c-\frac{b^2}{4a}$. Hence,

$$\begin{aligned}-50&=-25-\frac{b^2}{4}\\-200&=-100-b^2\\b^2&=100\\b&=\pm 10.\end{aligned}$$

85. Let $x=$ first number and $y=$ second number.
Then $x+y=s$ or $y=s-x$.
The product is given by $P=xy$ or $P=x(s-x)$.

$$\begin{aligned}P&=x(s-x)\\P&=sx-x^2\end{aligned}$$

The maximum P occurs at the vertex when $x=\frac{-b}{2a}$.

$$x=\frac{-s}{2(-1)}=\frac{s}{2}$$

When $x=\frac{s}{2}$, $y=s-\frac{s}{2}=\frac{s}{2}$.

So, the numbers x and y are both $\frac{s}{2}$.

86. If $f(x)=ax^2+bx+c$ has two real zeros, then by the Quadratic Formula they are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

The average of the zeros of f is

$$\frac{\frac{-b-\sqrt{b^2-4ac}}{2a}+\frac{-b+\sqrt{b^2-4ac}}{2a}}{2}=\frac{\frac{-2b}{2a}}{2}=-\frac{b}{2a}.$$

This is the x-coordinate of the vertex of the graph.

87. $y=ax^2+bx-4$
$(1,\ 0)$ on graph: $0=a+b-4$
$(4,\ 0)$ on graph: $0=16a+4b-4$
From the first equation, $b=4-a$.
Thus, $0=16a+4(4-a)-4=12a+12\Rightarrow a=-1$ and hence $b=5$, and $y=-x^2+5x-4$.

88. Model (a) is preferable. $a>0$ means the parabola opens upward and profits are increasing for t to the right of the vertex

$$t\ge-\frac{b}{(2a)}.$$

89. $x+y=8 \Rightarrow y=8-x$

$$-\frac{2}{3}x+8-x=6$$
$$-\frac{5}{3}x+8=6$$
$$-\frac{5}{3}x=-2$$
$$x=1.2$$
$$y=8-1.2=6.8$$

The point of intersection is $(1.2,\ 6.8)$.

90. $y=3x-10=\frac{1}{4}x+1$

$$12x-40=x+4$$
$$11x=44$$
$$x=4$$
$$y=3(4)-10$$
$$y=12-10=2$$

The graphs intersect at $(4,\ 2)$.

91. $y=x+3=9-x^2$

$$x^2+x-6=0$$
$$(x+3)(x-2)=0$$
$$x=-3,\quad x=2$$
$$y=-3+3=0$$
$$y=2+3=5$$

Thus, $(-3,\ 0)$ and $(2,\ 5)$ are the points of intersection.

92. $y=x^3+2x-1=-2x+15$

$$x^3+4x-16=0$$
$$(x-2)\left(x^2+2x+8\right)=0$$
$$x=2$$
$$y=-2(2)+15=-4+15=11$$

The graphs intersect at $(2,\ 11)$.

93. Answers will vary. (Make a Decision)

Section 2.2

1. continuous

2. $n,\ n-1$

3. (a) solution
(b) $(x-a)$
(c) $(a,\ 0)$

4. touches, crosses

5. No. If f is an even-degree fourth-degree polynomial function, its left and right end behavior is either that it rises left and right or falls left and right.

6. No. Assuming f is an odd-degree polynomial function, if its leading coefficient is negative, it must rise to the left and fall to the right.

7. Because f is a polynomial, it is a continuous on $\left[x_1,\ x_2\right]$ and $f(x_1)<0$ and $f(x_2)>0$. Then $f(x)=0$ for some value of x in $\left[x_1,\ x_2\right]$.

8. The real zero in $\left[x_3,\ x_4\right]$ is of even multiplicity, since the graph touches the x-axis but does not cross the x-axis.

9. $f(x)=-2x+3$ is a line with y-intercept $(0,\ 3)$. Matches graph (f).

10. $f(x)=x^2-4x$ is a parabola with intercepts $(0,\ 0)$ and $(4,\ 0)$ and opens upward. Matches graph (h).

11. $f(x)=-2x^2-5x$ is a parabola with x-intercepts $(0,\ 0)$ and $\left(-\frac{5}{2},\ 0\right)$ and opens downward. Matches graph (c).

12. $f(x)=2x^3-3x+1$ has intercepts $(0,\ 1)$, $(1,\ 0)$, $\left(-\frac{1}{2}-\frac{1}{2}\sqrt{3},\ 0\right)$ and $\left(-\frac{1}{2}+\frac{1}{2}\sqrt{3},\ 0\right)$. Matches graph (a).

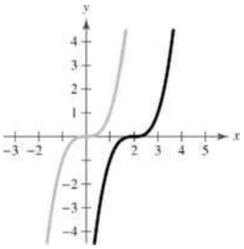
13. $f(x)=-\frac{1}{4}x^4+3x^2$ has intercepts $(0,\ 0)$ and $\left(\pm 2\sqrt{3},\ 0\right)$. Matches graph (e).

14. $f(x)=-\frac{1}{3}x^3+x^2-\frac{4}{3}$ has y-intercept $\left(0,\ -\frac{4}{3}\right)$. Matches graph (d).

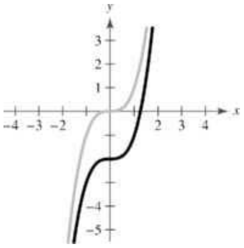
15. $f(x)=x^4+2x^3$ has intercepts $(0,\ 0)$ and $(-2,\ 0)$. Matches graph (g).

16. $f(x)=\frac{1}{5}x^5-2x^3+\frac{9}{5}x$ has intercepts $(0,\ 0)$, $(1,\ 0)$, $(-1,\ 0)$, $(3,\ 0)$, and $(-3,\ 0)$. Matches (b).

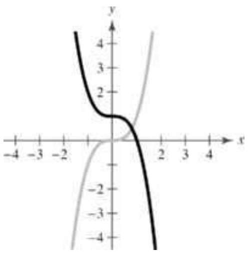
17. The graph of $f(x) = (x - 2)^3$ is a horizontal shift two units to the right of $y = x^3$.



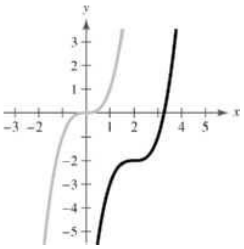
18. The graph of $f(x) = x^3 - 2$ is a vertical shift two units downward of $y = x^3$.



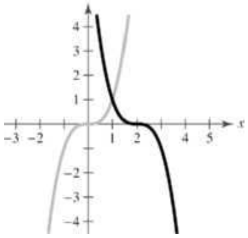
19. The graph of $f(x) = -x^3 + 1$ is a reflection in the x-axis and a vertical shift one unit upward of $y = x^3$.



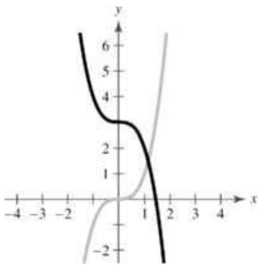
20. The graph of $f(x) = (x - 2)^3 - 2$ is a horizontal shift two units to the right and a vertical shift two units downward of $y = x^3$.



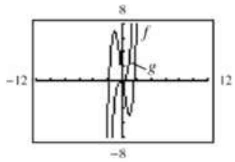
21. The graph of $f(x) = -(x - 2)^3$ is a horizontal shift two units to the right and a reflection in the x-axis of $y = x^3$.



22. The graph of $f(x) = -x^3 + 3$ is a reflection in the x-axis and a vertical shift three units upward of $y = x^3$.

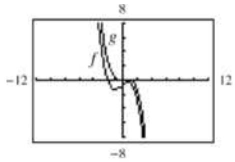


23.



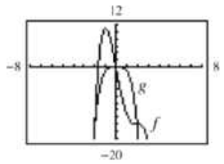
Yes, because both graphs have the same leading coefficient.

24.



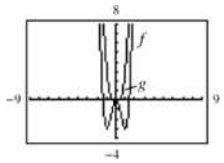
Yes, because both graphs have the same leading coefficient.

25.



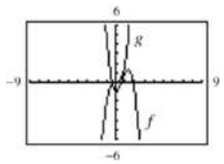
Yes, because both graphs have the same leading coefficient.

26.



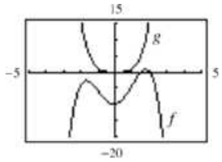
Yes, because both graphs have the same leading coefficient.

27.



No, because the graphs have different leading coefficients.

28.



No, because the graphs have different leading coefficients.

29. $f(x)=2x^4-3x+1$

Degree: 4

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

30. $h(x)=1-x^6$

Degree: 6

Leading coefficient: –1

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

31. $g(x)=5-\dfrac{7}{2}x-3x^2$

Degree: 2

Leading coefficient: –3

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

32. $f(x)=\dfrac{1}{3}x^3+5x$

Degree: 3

Leading coefficient: $\dfrac{1}{3}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

33. $f(x)=\dfrac{6x^5-2x^4+4x^2-5x}{3}$

Degree: 5

Leading coefficient: $\dfrac{6}{3}=2$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

34. $f(x)=\dfrac{3x^7-2x^5+5x^3+6x^2}{4}$

Degree: 7

Leading coefficient: $\dfrac{3}{4}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

35. $h(t)=-\dfrac{2}{3}\left(t^2-5t+3\right)$

Degree: 2

Leading coefficient: $-\dfrac{2}{3}$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

36. $f(s)=-\dfrac{7}{8}\left(s^3+5s^2-7s+1\right)$

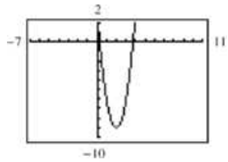
Degree: 3

Leading coefficient: $-\dfrac{7}{8}$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

37. (a) $f(x)=3x^2-12x+3$
$=3\left(x^2-4x+1\right)=0$
$x=\dfrac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$

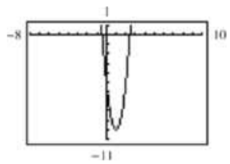
(b)



(c) $x\approx 3.732, 0.268$; the answers are approximately the same.

38. (a) $g(x)=5\left(x^2-2x-1\right)=0$
$x=\dfrac{2\pm\sqrt{4-4(-1)}}{2}=1\pm\sqrt{2}$

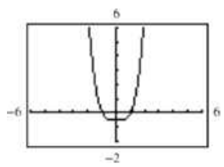
(b)



(c) $x\approx -0.414, 2.414$; the answers are approximately the same.

39. (a) $g(t)=\dfrac{1}{2}t^4-\dfrac{1}{2}$
$=\dfrac{1}{2}\left(t+1\right)(t-1)(t^2+1)=0$
$t=\pm 1$

(b)

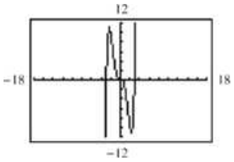


(c) $t=\pm 1$; the answers are the same.

40. (a) $0 = \frac{1}{4}x^3(x^2 - 9)$

$x = 0, \ \pm 3$

(b)



(c) $x = 0, \ \pm 3$; the answers are the same.

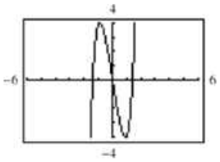
41. (a) $f(x) = x^5 + x^3 - 6x$

$= x\left(x^4 + x^2 - 6\right)$

$= x\left(x^2 + 3\right)(x^2 - 2) = 0$

$x = 0, \ \pm\sqrt{2}$

(b)



(c) $x = 0, 1.414, -1.414$; the answers are approximately the same.

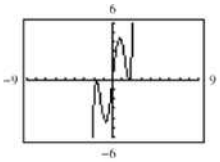
42. (a) $g(t) = t^5 - 6t^3 + 9t$

$= t\left(t^4 - 6t^2 + 9\right)$

$= t(t^2 - 3)^2 = 0$

$t = 0, \ \pm\sqrt{3}$

(b)



(c) $x = 0, \ \pm 1.732$; the answers are approximately the same.

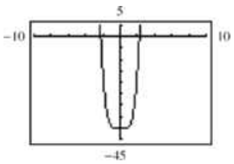
43. (a) $f(x) = 2x^4 - 2x^2 - 40$

$= 2\left(x^4 - x^2 - 20\right)$

$= 2\left(x^2 + 4\right)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right) = 0$

$x = \pm\sqrt{5}$

(b)



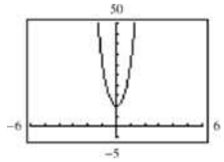
(c) $x = 2.236, -2.236$; the answers are approximately the same.

44. (a) $f(x) = 5(x^4 + 3x^2 + 2)$

$= 5(x^2 + 1)(x^2 + 2) > 0$

No real zeros

(b)



(c) No real zeros

45. (a) $f(x) = x^3 - 4x^2 - 25x + 100$

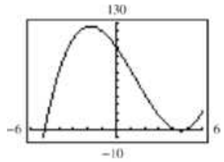
$= x^2(x - 4) - 25(x - 4)$

$= (x^2 - 25)(x - 4)$

$= (x - 5)(x + 5)(x - 4) = 0$

$x = \pm 5, \ 4$

(b)



(c) $x = 4, 5, -5$; the answers are the same.

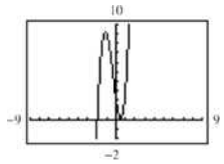
46. (a) $0 = 4x^3 + 4x^2 - 7x + 2$

$= (2x - 1)(2x^2 + 3x - 2)$

$= (2x - 1)(2x - 1)\,(x + 2) = 0$

$x = -2, \ \frac{1}{2}$

(b)



(c) $x = -2, \frac{1}{2}$; the answers are the same.

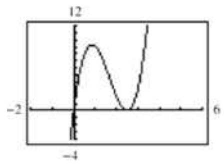
47. (a) $y = 4x^3 - 20x^2 + 25x$

$0 = 4x^3 - 20x^2 + 25x$

$0 = x(2x - 5)^2$

$x = 0, \ \frac{5}{2}$

(b)



(c) $x = 0, \ \frac{5}{2}$; the answers are the same.

48. (a) $y = x^5 - 5x^3 + 4x$

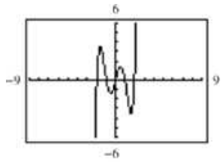
$= x(x^4 - 5x^2 + 4)$

$= x(x^2 - 4)(x^2 - 1)$

$= x(x-2)(x+2)(x-1)(x+1) = 0$

$x = 0,\ \pm 1,\ \pm 2$

(b)



(c) $x = 0,\ \pm 1,\ \pm 2$; the answers are the same.

49. $f(x) = x^2 - 25$

$= (x+5)(x-5)$

$x = \pm 5$ (multiplicity 1)

50. $f(x) = 49 - x^2$

$= (7-x)(7+x)$

$x = \pm 7$ (multiplicity 1)

51. $h(t) = t^2 - 6t + 9$

$= (t-3)^2$

$t = 3$ (multiplicity 2)

52. $f(x) = x^2 + 10x + 25$

$= (x+5)^2$

$x = -5$ (multiplicity 2)

53. $f(x) = x^2 + x - 2$

$= (x+2)(x-1)$

$x = -2, 1$ (multiplicity 1)

54. $f(x) = 2x^2 - 14x + 24$

$= 2(x^2 - 7x + 12)$

$= 2(x-3)(x-4)$

$x = 3, 4$ (multiplicity 1)

55. $f(t) = t^3 - 4t^2 + 4t$

$= t(t-2)^2$

$t = 0$ (multiplicity 1), 2 (multiplicity 2)

56. $f(x) = x^4 - x^3 - 20x^2$

$= x^2(x^2 - x - 20)$

$= x^2(x+4)(x-5)$

$x = -4$ (multiplicity 1), 5 (multiplicity 1), 0 (multiplicity 2)

57. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

$= \frac{1}{2}(x^2 + 5x - 3)$

$x = \dfrac{-5 \pm \sqrt{25 - 4(-3)}}{2} = -\dfrac{5}{2} \pm \dfrac{\sqrt{37}}{2}$

$\approx 0.5414,\ -5.5414$ (multiplicity 1)

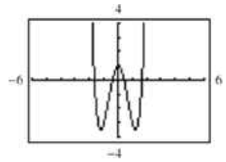
58. $f(x) = \frac{5}{3}x^2 + \frac{8}{3}x - \frac{4}{3}$

$= \frac{1}{3}(5x^2 + 8x - 4)$

$= \frac{1}{3}(5x - 2)(x + 2)$

$x = \frac{2}{5},\ -2$ (multiplicity 1)

59. $f(x) = 2x^4 - 6x^2 + 1$

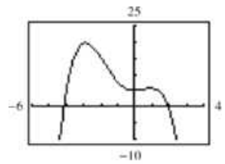


Zeros: $x \approx \pm 0.421, \pm 1.680$

Relative maximum: (0, 1)

Relative minima: (1.225, −3.5), (−1.225, −3.5)

60. $f(x) = -\frac{3}{8}x^4 - x^3 + 2x^2 + 5$

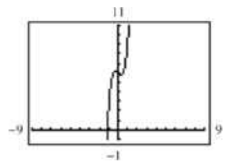


Zeros: −4.142, 1.934

Relative maxima: (0.915, 5.646), (−2.915, 19.688)

Relative minimum: (0, 5)

61. $f(x) = x^5 + 3x^3 - x + 6$

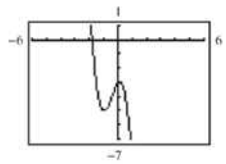


Zero: $x \approx -1.178$

Relative maximum: (−0.324, 6.218)

Relative minimum: (0.324, 5.782)

62. $f(x) = -3x^3 - 4x^2 + x - 3$

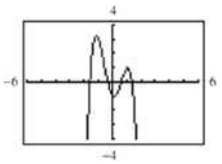


Zero: −1.819

Relative maximum: (0.111, −2.942)

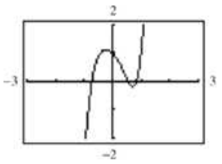
Relative minimum: (−1, −5)

63. $f(x) = -2x^4 + 5x^2 - x - 1$



Zeros: $-1.618, \ -0.366, \ 0.618, 1.366$
Relative minimum: $(0.101, \ -1.050)$
Relative maxima: $(-1.165, \ 3.267), \ (1.064, \ 1.033)$

64. $f(x) = 3x^5 - 2x^2 - x + 1$



Zeros: $-0.737, 0.548, 0.839$
Relative minimum: $(0.712, \ -0.177)$
Relative maximum: $(-0.238, \ 1.122)$

65. $f(x) = (x - 0)(x - 4) = x^2 - 4x$

Note: $f(x) = a(x - 0)(x - 4) = ax(x - 4)$ has zeros 0 and 4 for all nonzero real numbers a.

66. $f(x) = (x + 7)(x - 2) = x^2 + 5x - 14$

Note: $f(x) = a(x + 7)(x - 2)$ has zeros -7 and 2 for all nonzero real numbers a.

67. $f(x) = (x - 0)(x + 2)(x + 3) = x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x + 2)(x + 3)$ has zeros $0, \ -2,$ and -3 for all nonzero real numbers a.

68. $f(x) = (x - 0)(x - 2)(x - 5) = x^3 - 7x^2 + 10x$

Note: $f(x) = ax(x - 2)(x - 5)$ has zeros $0, \ 2,$ and 5 for all nonzero real numbers a.

69.
$$\begin{aligned} f(x) &= (x - 4)(x + 3)(x - 3)(x - 0) \\ &= (x - 4)(x^2 - 9)x \\ &= x^4 - 4x^3 - 9x^2 + 36x \end{aligned}$$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, \ -3, \ 3,$ and 0 for all nonzero real numbers a.

70.
$$\begin{aligned} f(x) &= (x - (-2))(x - (-1))(x - 0)(x - 1)(x - 2) \\ &= x(x + 2)(x + 1)(x - 1)(x - 2) \\ &= x(x^2 - 4)(x^2 - 1) \\ &= x(x^4 - 5x^2 + 4) \\ &= x^5 - 5x^3 + 4x \end{aligned}$$

Note: $f(x) = ax(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, \ -1, \ 0, \ 1, \ 2,$ for all nonzero real numbers a.

71.
$$\begin{aligned} f(x) &= \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right] \\ &= \left[\left(x - 1\right) - \sqrt{3}\right]\left[\left(x - 1\right) + \sqrt{3}\right] \\ &= (x - 1)^2 - \left(\sqrt{3}\right)^2 \\ &= x^2 - 2x + 1 - 3 \\ &= x^2 - 2x - 2 \end{aligned}$$

Note: $f(x) = a(x^2 - 2x - 2)$ has zeros $1 + \sqrt{3}$ and $1 - \sqrt{3}$ for all nonzero real numbers a.

72.
$$\begin{aligned} f(x) &= \left(x - \left(6 + \sqrt{3}\right)\right)\left(x - \left(6 - \sqrt{3}\right)\right) \\ &= \left(\left(x - 6\right) - \sqrt{3}\right)\left(\left(x - 6\right) + \sqrt{3}\right) \\ &= (x - 6)^2 - 3 \\ &= x^2 - 12x + 36 - 3 \\ &= x^2 - 12x + 33 \end{aligned}$$

Note: $f(x) = a(x^2 - 12x + 33)$ has zeros $6 + \sqrt{3}$ and $6 - \sqrt{3}$ for all nonzero real numbers a.

73.
$$\begin{aligned} f(x) &= \left(x - 2\right)\left[x - \left(4 + \sqrt{5}\right)\right]\left[x - \left(4 - \sqrt{5}\right)\right] \\ &= (x - 2)\left[\left(x - 4\right) - \sqrt{5}\right]\left[\left(x - 4\right) + \sqrt{5}\right] \\ &= (x - 2)[(x - 4)^2 - 5] \\ &= x^3 - 10x^2 + 27x - 22 \end{aligned}$$

Note: $f(x) = a(x - 2)[(x - 4)^2 - 5]$ has zeros $2, \ 4 + \sqrt{5},$ and $4 - \sqrt{5}$ for all nonzero real numbers a.

74.
$$\begin{aligned} f(x) &= (x - 4)\left(x - \left(2 + \sqrt{7}\right)\right)\left(x - \left(2 - \sqrt{7}\right)\right) \\ &= (x - 4)\left(\left(x - 2\right) - \sqrt{7}\right)\left(\left(x - 2\right) + \sqrt{7}\right) \\ &= (x - 4)((x - 2)^2 - 7) \\ &= (x - 4)(x^2 - 4x - 3) \\ &= x^3 - 8x^2 + 13x + 12 \end{aligned}$$

Note: $f(x) = a(x - 4)(x^2 - 4x - 3)$ has zeros $4, \ 2 \pm \sqrt{7}$ for all nonzero real numbers a.

75. $f(x) = (x + 2)^2(x + 1) = x^3 + 5x^2 + 8x + 4$

Note: $f(x) = a(x + 2)^2(x + 1)$ has zeros $-2, \ -2,$ and -1 for all nonzero real numbers a.

76.
$$\begin{aligned} f(x) &= (x - 3)(x - 2)^3 \\ &= x^4 - 9x^3 + 30x^2 - 44x + 24 \end{aligned}$$

Note: $f(x) = a(x - 3)(x - 2)^3$ has zeros $3, \ 2, \ 2, \ 2$ for all nonzero real numbers a.

77.
$$\begin{aligned} f(x) &= (x + 4)^2(x - 3)^2 \\ &= x^4 + 2x^3 - 23x^2 - 24x + 144 \end{aligned}$$

Note: $f(x) = a(x + 4)^2(x - 3)^2$ has zeros $-4, \ -4, \ 3, \ 3$ for all nonzero real numbers a

78. $f(x) = (x-5)^3(x-0)^2$
$= x^5 - 15x^4 + 75x^3 - 125x^2$

Note: $f(x) = a(x-5)^3x^2$ has zeros $5, 5, 5, 0, 0$ for all nonzero real numbers a.

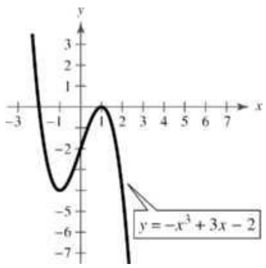
79. $f(x) = -(x+1)^2(x+2)$
$= -x^3 - 4x^2 - 5x - 2$

Note: $f(x) = a(x+1)^2(x+2)^2,\ a < 0,$ has zeros $-1, -1, -2,$ rises to the left, and falls to the right.

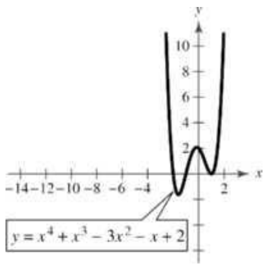
80. $f(x) = -(x-1)^2(x-4)^2$
$= -x^4 + 10x^3 - 33x^2 + 40x - 16$

Note: $f(x) = a(x-1)^2(x-4)^2,\ a < 0,$ has zeros $1, 1, 4, 4,$ falls to the left, and falls to the right.

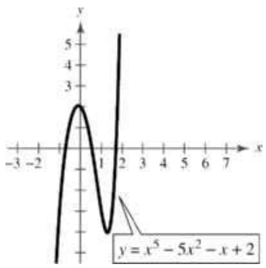
81.



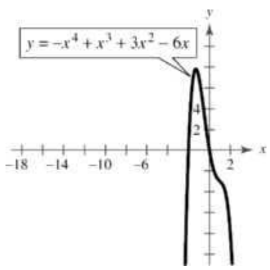
82.



83.



84.

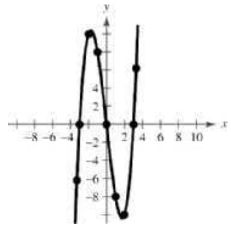


85. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 9x = x(x^2 - 9) = x(x-3)(x+3)$

Zeros: $0, 3, -3$

(c) and (d)

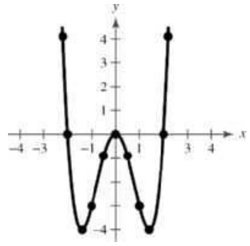


86. (a) The degree of g is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b) $g(x) = x^4 - 4x^2 = x^2(x^2 - 4)$
$= x^2(x-2)(x+2)$

Zeros: $0, 2, -2$

(c) and (d)

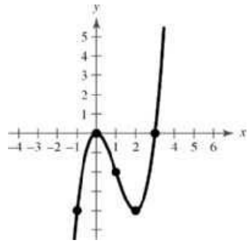


87. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 - 3x^2 = x^2(x-3)$

Zeros: $0, 3$

(c) and (d)

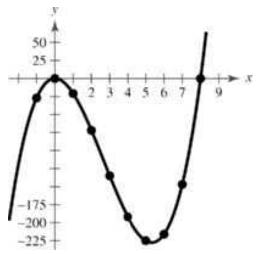


88. (a) The degree of f is odd and the leading coefficient is 3. The graph falls to the left and rises to the right.

(b) $f(x) = 3x^3 - 24x^2 = 3x^2(x-8)$

Zeros: $0, 8$

(c) and (d)

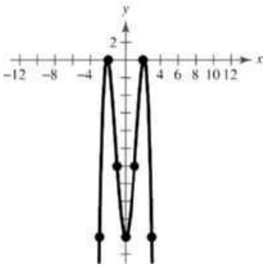


89. (a) The degree of f is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^4 + 9x^2 - 20 = -(x^2 - 4)(x^2 - 5)$

Zeros: ± 2, $\pm\sqrt{5}$

(c) and (d)

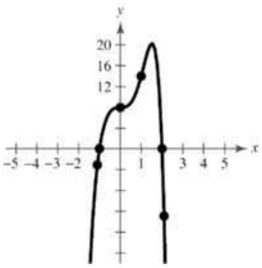


90. (a) The degree of f is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f(x) = -x^6 + 7x^3 + 8 = -(x^3 + 1)(x^3 - 8)$

Zeros: -1, 2

(c) and (d)

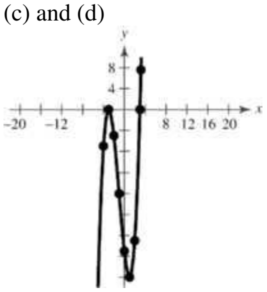


91. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $$f(x) = x^3 + 3x^2 - 9x - 27 = x^2(x+3) - 9(x+3)$$
$$= (x^2 - 9)(x+3)$$
$$= (x-3)(x+3)^2$$

Zeros: 3, -3

(c) and (d)

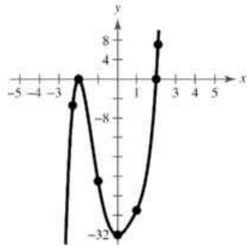


92. (a) The degree of h is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $$h(x) = x^5 - 4x^3 + 8x^2 - 32 = x^3(x^2 - 4) + 8(x^2 - 4)$$
$$= (x^3 + 8)(x^2 - 4)$$
$$= (x+2)(x^2 - 2x + 4)(x-2)(x+2)$$

Zeros: -2, 2

(c) and (d)

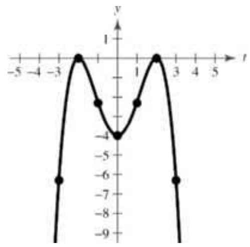


93. (a) The degree of g is even and the leading coefficient is $-\frac{1}{4}$. The graph falls to the left and falls to the right.

(b) $g(t) = -\frac{1}{4}(t^4 - 8t^2 + 16) = -\frac{1}{4}(t^2 - 4)^2$

Zeros: -2, -2, 2, 2

(c) and (d)

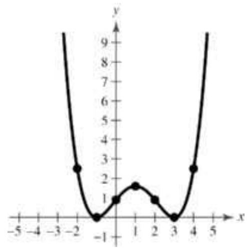


94. (a) The degree of g is even and the leading coefficient is $\frac{1}{10}$. The graph rises to the right and rises to the left.

(b) $g(x) = \frac{1}{10}(x+1)^2(x-3)^2$

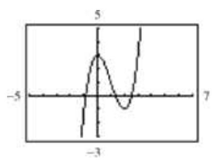
Zeros: -1, 3

(c) and (d)



95. $f(x) = x^3 - 3x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-1, 0)$, $(1, 2)$ and $(2, 3)$.

(b) Zeros: $-0.879, 1.347, 2.532$

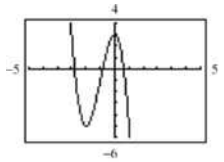
x	y
–0.9	–0.159
–0.89	–0.0813
–0.88	–0.0047
–0.87	0.0708
–0.86	0.14514
–0.85	0.21838
–0.84	0.2905

x	y
1.3	0.127
1.31	0.09979
1.32	0.07277
1.33	0.04594
1.34	0.0193
1.35	–0.0071
1.36	–0.0333

x	y
2.5	–0.125
2.51	–0.087
2.52	–0.0482
2.53	–0.0084
2.54	0.03226
2.55	0.07388
2.56	0.11642

96. $f(x) = -2x^3 - 6x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-3, -2)$, $(-1, 0)$, and $(0, 1)$.

(b) Zeros: $-2.810, -0.832, 0.642$

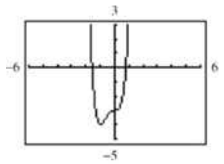
x	y_1
–2.83	–0.277
–2.82	–0.137
–2.81	≈ 0
–2.80	–0.136
–2.79	–0.269

x	y_1
–0.86	–0.166
–0.85	–0.0107
–0.84	–0.048
–0.83	0.010
–0.82	0.068

x	y_1
0.62	0.217
0.63	0.119
0.64	0.018
0.65	–0.084
0.66	–0.189

97. $g(x) = 3x^4 + 4x^3 - 3$

(a)



The function has two zeros. They are in the intervals $(-2, -1)$ and $(0, 1)$.

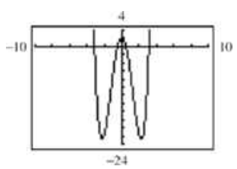
(b) Zeros: $-1.585, 0.779$

x	y_1
–1.6	0.2768
–1.59	0.09515
–1.58	–0.0812
–1.57	–0.2524
–1.56	–0.4184
–1.55	–0.5795
–1.54	–0.7356

x	y_1
0.75	–0.3633
0.76	–0.2432
0.77	–0.1193
0.78	0.00866
0.79	0.14066
0.80	0.2768
0.81	0.41717

98. $h(x) = x^4 - 10x^2 + 2$

(a)



The function has four zeros. They are in the intervals $(0, 1)$, $(3, 4)$, $(-1, 0)$, and $(-4, -3)$.

(b) Notice that h is even. Hence, the zeros come in symmetric pairs. Zeros: ± 0.452, ± 3.130.

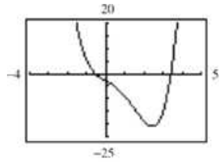
Because the function is even, we only need to verify the positive zeros.

x	y_1
0.42	0.26712
0.43	0.18519
0.44	0.10148
0.45	0.01601
0.46	–0.0712
0.47	–0.1602
0.48	–0.2509

x	y_1
3.09	–2.315
3.10	–1.748
3.11	–1.171
3.12	–0.5855
3.13	0.01025
3.14	0.61571
3.15	1.231

99. $f(x) = x^4 - 3x^3 - 4x - 3$

(a)



The function has two zeros. They are in the intervals $(-1, 0)$ and $(3, 4)$.

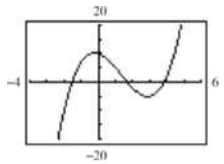
(b) Zeros: $-0.578, 3.418$

x	y_1
–0.61	0.2594
–0.60	0.1776
–0.59	0.09731
–0.58	0.0185
–0.57	–0.0589
–0.56	–0.1348
–0.55	–0.2094

x	y_1
3.39	–1.366
3.40	–0.8784
3.41	–0.3828
3.42	0.12071
3.43	0.63205
3.44	1.1513
3.45	1.6786

100. $f(x) = x^3 - 4x^2 - 2x + 10$

(a)



The function has three zeros. They are in the intervals $(-2, -1)$, $(1, 2)$, and $(3, 4)$

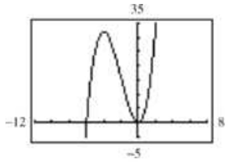
(b) Zeros: $-1.537, 1.693, 3.843$

x	y_1
–1.56	–0.4108
–1.55	–0.2339
–1.54	–0.0587
–1.53	0.11482
–1.52	0.28659
–1.51	0.45665
–1.50	0.625

x	y_1
1.66	0.2319
1.67	0.16186
1.68	0.09203
1.69	0.02241
1.70	–0.047
1.71	–0.1162
1.72	–0.1852

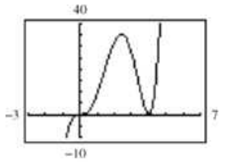
x	y_1
3.82	–0.2666
3.83	–0.1537
3.84	–0.0393
3.85	0.07663
3.86	0.19406
3.87	0.313
3.88	0.43347

101. $f(x) = x^2(x+6)$



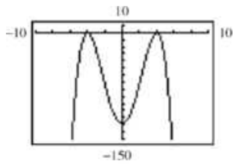
No symmetry
Two x-intercepts

102. $h(x) = x^3(x-4)^2$



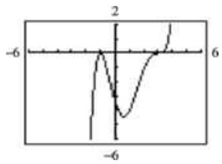
No symmetry
Two x-intercepts

103. $g(t) = -\frac{1}{2}(t-4)^2(t+4)^2$



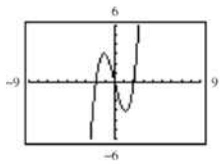
Symmetric with respect to the y-axis
Two x-intercepts

104. $g(x) = \frac{1}{8}(x+1)^2(x-3)^3$



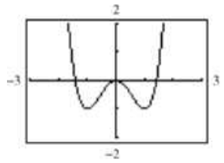
No symmetry
Two x-intercepts

105. $f(x) = x^3 - 4x = x(x+2)(x-2)$



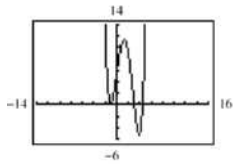
Symmetric with respect to the origin
Three x-intercepts

106. $f(x) = x^4 - 2x^2$



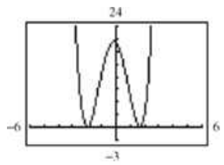
Symmetric with respect to the y-axis
Three x-intercepts

107. $g(x) = \frac{1}{5}(x+1)^2(x-3)(2x-9)$



No symmetry
Three x-intercepts

108. $h(x) = \frac{1}{5}(x+2)^2(3x-5)^2$



No symmetry
Two x-intercepts

109. (a) Volume = length $\times$ width $\times$ height

Because the box is made from a square, length = width.

Thus: Volume = $(\text{length})^2 \times$ height = $(36-2x)^2 x$

(b) Domain: $0 < 36 - 2x < 36$

$$-36 < -2x < 0$$

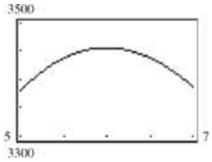
$$18 > x > 0$$

(c)

Height, x	Length and Width	Volume, V
1	$36-2(1)$	$1[36-2(1)]^2 = 1156$
2	$36-2(2)$	$2[36-2(2)]^2 = 2048$
3	$36-2(3)$	$3[36-2(3)]^2 = 2700$
4	$36-2(4)$	$4[36-2(4)]^2 = 3136$
5	$36-2(5)$	$5[36-2(5)]^2 = 3380$
6	$36-2(6)$	$6[36-2(6)]^2 = 3456$
7	$36-2(7)$	$7[36-2(7)]^2 = 3388$

Maximum volume is in the interval $5 < x < 7$.

(d)



$x = 6$ when $V(x)$ is maximum.

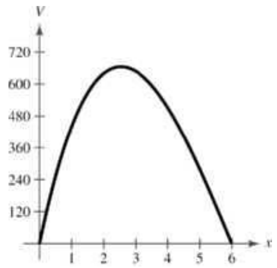
110. (a) $V(x) = \text{length} \times \text{width} \times \text{height}$

$= (24 - 2x)(24 - 4x)x$

$= 8x(12 - x)(6 - x)$

(b) Domain: $0 < x < 6$

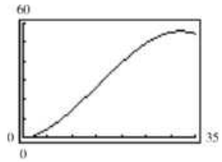
(c)



Maximum occurs at $x = 2.54$.

111. The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$ The point is (200, 160) which corresponds to spending $2,000,000 on advertising to obtain a revenue of $160 million.

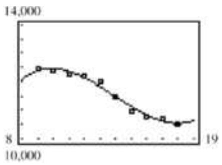
112.



Point of diminishing returns: $(15.2,\ 27.3)$

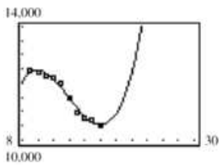
15.2 years

113. (a)



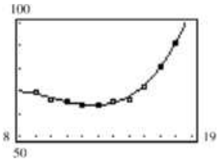
The model fits the data well.

(b)

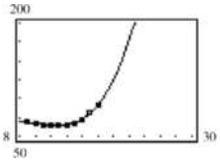


Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

(c)



The model fits the data well.



Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

114. True. $f(x) = x^6$ has only one zero, 0.

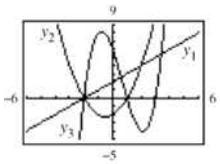
115. True. The degree is odd and the leading coefficient is -1.

116. False. The graph touches at $x = 1$, but does not cross the x-axis there.

117. False. The graph crosses the x-axis at $x = -3$ and $x = 0$.

118. False. The graph rises to the left, and rises to the right.

119.



The graph of y_3 will fall to the left and rise to the right. It will have another x-intercept at $(3,\ 0)$ of odd multiplicity (crossing the x-axis).

120. (a) Degree: 3
Leading coefficient: Positive

(b) Degree: 2
Leading coefficient: Positive

(c) Degree: 4
Leading coefficient: Positive

(d) Degree: 5
Leading coefficient: Positive

121. $(f + g)(-4) = f(-4) + g(-4)$

$= -59 + 128 = 69$

122. $(g - f)(3) = g(3) - f(3) = 8(3)^2 - [14(3) - 3]$

$= 72 - 39 = 33$

123. $(f \circ g)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right) g\left(-\frac{4}{7}\right) = (-11)\left(\frac{8 \cdot 16}{49}\right)$

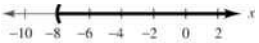
$= -\frac{1408}{49} \approx -28.7347$

124. $\left(\frac{f}{g}\right)(-1.5) = \frac{f(-1.5)}{g(-1.5)} = \frac{-24}{18} = -\frac{4}{3}$

125. $(f \circ g)(-1) = f(g(-1)) = f(8) = 109$

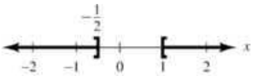
126. $(g \circ f)(0) = g(f(0)) = g(-3) = 8(-3)^2 = 72$

127. $3(x-5) < 4x - 7$



$$\begin{aligned} 3x - 15 &< 4x - 7 \\ -8 &< x \end{aligned}$$

128. $2x^2 - x \geq 1$



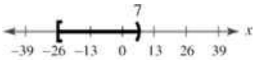
$$\begin{aligned} 2x^2 - x - 1 &\geq 0 \\ (2x+1)(x-1) &\geq 0 \end{aligned}$$

$[2x+1 \geq 0 \text{ and } x - 1 \geq 0]$ or $[2x+1 \leq 0 \text{ and } x - 1 \leq 0]$

$\left[x \geq -\frac{1}{2} \text{ and } x \geq 1\right]$ or $\left[x \leq -\frac{1}{2} \text{ and } x \leq 1\right]$

$x \geq 1$ or $x \leq -\frac{1}{2}$

129. $\frac{5x-2}{x-7} \leq 4$



$$\begin{aligned} \frac{5x-2}{x-7} - 4 &\leq 0 \\ \frac{5x - 2 - 4(x-7)}{x-7} &\leq 0 \\ \frac{x+26}{x-7} &\leq 0 \end{aligned}$$

$[x + 26 \geq 0 \text{ and } x - 7 < 0]$ or $[x + 26 \leq 0 \text{ and } x - 7 > 0]$

$[x \geq -26 \text{ and } x < 7]$ or $[x \leq -26 \text{ and } x > 7]$

$-26 \leq x < 7$ impossible

130. $|x+8| - 1 \geq 15$

$|x+8| \geq 16$

$x + 8 \geq 16$ or $x + 8 \leq -16$

$x \geq 8$ or $x \leq -24$

Section 2.3

1. $f(x)$ is the dividend, $d(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder.
2. improper
3. constant term, leading coefficient
4. Descartes's Rule, Signs
5. upper, lower
6. According to Descartes's Rule of Signs, given that $f(-x)$ has 5 variations in sign, there are either 5, 3, or 1 negative real zeros.
7. According to the Remainder Theorem, if you divide $f(x)$ by $x-4$ and the remainder is 7, then $f(4) = 7$.
8. To check whether $x-3$ is a factor of $f(x) = x^3 - 2x^2 + 3x + 4$, the synthetic division format should appear as $3\,\underline{|\,1\ -2\ 3\ 4}$ and if it is a factor, the remainder should be 0.

9.
$$\begin{array}{r} 2x+4 \\ x+3\overline{)2x^2+10x+12} \\ \underline{2x^2+6x} \\ 4x+12 \\ \underline{4x+12} \\ 0 \end{array}$$

$\frac{2x^2+10x+12}{x+3} = 2x+4,\ x \neq -3$

10.
$$\begin{array}{r} 5x+3 \\ x-4\overline{)5x^2-17x-12} \\ \underline{5x^2-20x} \\ 3x-12 \\ \underline{3x-12} \\ 0 \end{array}$$

$\frac{5x^2-17x-12}{x-4} = 5x+3,\ x \neq 4$

11.

$$\begin{array}{r} x^3+3x^2-1 \\ x+2\overline{\big)x^4+5x^3+6x^2-x-2} \\ \underline{x^4+2x^3} \\ 3x^3+6x^2 \\ \underline{3x^3+6x^2} \\ -x-2 \\ \underline{-x-2} \\ 0 \end{array}$$

$$\frac{x^4+5x^3+6x^2-x-2}{x+2}=x^3+3x^2-1,\ x\neq-2$$

12.

$$\begin{array}{r} x^2-x-20 \\ x-3\overline{\big)x^3-4x^2-17x+6} \\ \underline{x^3-3x^2} \\ -x^2-17x \\ \underline{-x^2+3x} \\ -20x+6 \\ \underline{-20x+60} \\ -54 \end{array}$$

$$\frac{x^3-4x^2-17x+6}{x-3}=x^2-x-20-\frac{54}{x-3}$$

13.

$$\begin{array}{r} x^2-3x+1 \\ 4x+5\overline{\big)4x^3-7x^2-11x+5} \\ \underline{-4x^3+5x^2} \\ -12x^2-11x \\ \underline{-12x^2-15x} \\ 4x+5 \\ -\underline{4x+5} \\ 0 \end{array}$$

$$\frac{4x^3-7x^2-11x+5}{4x+5}=x^2-3x+1,\ x\neq-\frac{5}{4}$$

14.

$$\begin{array}{r} x^2-25 \\ 2x-3\overline{\big)2x^3-3x^2-50x+75} \\ \underline{2x^3-3x^2} \\ -50x+75 \\ \underline{-50x+75} \\ 0 \end{array}$$

$$\frac{2x^3-3x^2-50x+75}{2x-3}=x^2-25,\ x\neq\frac{3}{2}$$

15.

$$\begin{array}{r} 7x^2-14x+28 \\ x+2\overline{\big)7x^3+0x^2+0x+3} \\ \underline{7x^3+14x^2} \\ -14x^2 \\ \underline{-14x^2-28x} \\ 28x+3 \\ \underline{28x+56} \\ -53 \end{array}$$

$$\frac{7x^3+3}{x+2}=7x^2-14x+28-\frac{53}{x+2}$$

16.

$$\begin{array}{r} 4x^3-2x^2+x-\frac{1}{2} \\ 2x+1\overline{\big)8x^4+0x^3+0x^2+0x-5} \\ \underline{8x^4+4x^3} \\ -4x^3 \\ \underline{-4x^3-2x^2} \\ 2x^2 \\ \underline{2x^2+x} \\ -x-5 \\ \underline{-x-\frac{1}{2}} \\ -\frac{9}{2} \end{array}$$

$$\frac{8x^4-5}{2x+1}=4x^3-2x^2+x-\frac{1}{2}-\frac{\frac{9}{2}}{2x+1}$$

17.

$$\begin{array}{r} 3x+5 \\ 2x^2+0x+1\overline{\big)6x^3+10x^2+x+8} \\ \underline{6x^3+0x^2+3x} \\ 10x^2-2x+8 \\ \underline{10x^2+0x+5} \\ -2x+3 \end{array}$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1}=3x+5-\frac{2x-3}{2x^2+1}$$

18.

$$\begin{array}{r} x^2+2x+4 \\ x^2-2x+3\overline{\big)x^4+0x^3+3x^2+0x+1} \\ \underline{x^4-2x^3+3x^2} \\ 2x^3+0x \\ \underline{2x^3-4x^2+6x} \\ 4x^2-6x+1 \\ \underline{4x^2-8x+12} \\ 2x-11 \end{array}$$

$$\frac{x^4+3x^2+1}{x^2-2x+3}=x^2+2x+4+\frac{2x-11}{x^2-2x+3}$$

19.

$$\begin{array}{r} x \\ x^2+1\overline{\big)x^3+0x^2+0x-9} \\ \underline{x^3+x} \\ -x-9 \end{array}$$

$$\frac{x^3-9}{x^2+1}=x-\frac{x+9}{x^2+1}$$

20.

$$\begin{array}{r} x^2 \\ x^3-1\overline{\big)x^5+0x^4+0x^3+0x^2+0x+7} \\ \underline{x^5-x^2} \\ x^2+7 \end{array}$$

$$\frac{x^5+7}{x^3-1}=x^2+\frac{x^2+7}{x^3-1}$$

21. $$\begin{array}{r} 2x \\ x^2-2x+1\overline{\smash{)}2x^3-4x^2-15x+5} \\ \underline{2x^3-4x^2+\ 2x} \\ -17x+5 \end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2}=2x-\frac{17x-5}{(x-1)^2}$$

22. $(x-1)^3=x^3-3x^2+3x-1$

$$\begin{array}{r} x+3 \\ x^3-3x^2+3x-1\overline{\smash{)}x^4} \\ \underline{x^4-3x^3+3x^2-x} \\ 3x^3-3x^2+x \\ \underline{3x^3-9x^2+9x-3} \\ 6x^2+8x+3 \end{array}$$

$$\frac{x^4}{(x-1)^3}=x+3+\frac{6x^2-8x+3}{(x-1)^3}$$

23. $$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$\frac{3x^3-17x^2+15x-25}{x-5}=3x^2-2x+5,\ x\neq 5$$

24. $$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$

$$\frac{5x^3+18x^2+7x-6}{x+3}=5x^2+3x-2,\ x\neq -3$$

25. $$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3+7x^2-x+26}{x-3}=6x^2+25x+74+\frac{248}{x-3}$$

26. $$\begin{array}{r|rrrr} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$

$$\frac{2x^3+14x^2-20x+7}{x+6}=2x^2+2x-32+\frac{199}{x+6}$$

27. $$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3-18x^2-16x+32}{x-2}=9x^2-16,\ x\neq 2$$

28. $$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3+6x+8}{x+2}=5x^2-10x+26-\frac{44}{x+2}$$

29. $$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3+512}{x+8}=x^2-8x+64,\ x\neq -8$$

30. $$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$

$$\frac{x^3-729}{x-9}=x^2+9x+81,\ x\neq 9$$

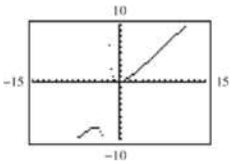
31. $$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3+16x^2-23x-15}{x+\frac{1}{2}}=4x^2+14x-30,\ x\neq -\frac{1}{2}$$

32. $$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$

$$\frac{3x^3-4x^2+5}{x-\frac{3}{2}}=3x^2+\frac{1}{2}x+\frac{3}{4}+\frac{49}{8x-12}$$

33. $$\begin{aligned} y_2&=x-2+\frac{4}{x+2} \\ &=\frac{(x-2)(x+2)+4}{x+2} \\ &=\frac{x^2-4+4}{x+2} \\ &=\frac{x^2}{x+2} \\ &=y_1 \end{aligned}$$



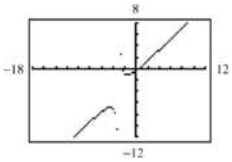
34. $y_2 = x - 1 + \dfrac{2}{x+3}$

$$= \frac{(x-1)(x+3)+2}{x+3}$$

$$= \frac{x^2+2x-3+2}{x+3}$$

$$= \frac{x^2+2x-1}{x+3}$$

$$= y_1$$



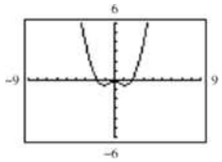
35. $y_2 = x^2 - 8 + \dfrac{39}{x^2+5}$

$$= \frac{(x^2-8)(x^2+5)+39}{x^2+5}$$

$$= \frac{x^4-8x^2+5x^2-40+39}{x^2+5}$$

$$= \frac{x^4-3x^2-1}{x^2+5}$$

$$= y_1$$

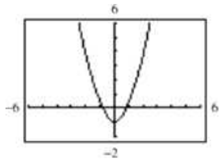


36. $y_2 = x^2 - \dfrac{1}{x^2+1}$

$$= \frac{x^2(x^2+1)-1}{x^2+1}$$

$$= \frac{x^4+x^2-1}{x^2+1}$$

$$= y_1$$



37. $f(x) = x^3 - x^2 - 14x + 11,\ k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2+3x-2)+3$

$f(4) = (0)(26)+3 = 3$

38. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14,\ k = -\dfrac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$f(x) = \left(x + \dfrac{2}{3}\right)(15x^3 - 6x + 4) + \dfrac{34}{3}$

$f\left(-\dfrac{2}{3}\right) = \dfrac{34}{3}$

39.

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2+3\sqrt{2} & 6 \\ \hline & 1 & 3+\sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = (x-\sqrt{2})\left(x^2 + (3+\sqrt{2})x + 3\sqrt{2}\right) - 8$

$f(\sqrt{2}) = 0(4+6\sqrt{2}) - 8 = -8$

40.

$$\begin{array}{r|rrrr} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & 5-2\sqrt{5} & 10 \\ \hline & 1 & 2-\sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$f(x) = (x+\sqrt{5})(x^2 + (2-\sqrt{5})x - 2\sqrt{5}) + 6$

$f(-\sqrt{5}) = 6$

41.

$$\begin{array}{r|rrrr} 1-\sqrt{3} & 4 & -6 & -12 & -4 \\ & & 4-4\sqrt{3} & 10-2\sqrt{3} & 4 \\ \hline & 4 & -2-4\sqrt{3} & -2-2\sqrt{3} & 0 \end{array}$$

$f(x) = (x-1+\sqrt{3})[4x^2 - (2+4\sqrt{3})x - (2+2\sqrt{3})]$

$f(1-\sqrt{3}) = 0$

42.

$$\begin{array}{r|rrrr} 2+\sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6-3\sqrt{2} & -2-4\sqrt{2} & 8 \\ \hline & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & 0 \end{array}$$

$f(x) = [x - (2+\sqrt{2})][-3x^2 + (2-3\sqrt{2})x + 8 - 4\sqrt{2}]$

$f(2+\sqrt{2}) = 0$

43. $f(x) = 2x^3 - 7x + 3$

(a)

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array} \quad = f(1)$$

(b)

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array} \quad = f(-2)$$

(c)

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array} \quad = f\left(\frac{1}{2}\right)$$

(d)

$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array} \quad = f(2)$$

44. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a) $$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array} = g(2)$$

(b) $$\begin{array}{r|rrrrrrr} 1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 2 & 2 & 5 & 5 & 4 & 4 \\ \hline & 2 & 2 & 5 & 5 & 4 & 4 & 7 \end{array} = g(1)$$

(c) $$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & 564 & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & 564 & 1695 \end{array} = g(3)$$

(d) $$\begin{array}{r|rrrrrrr} -1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & -2 & 2 & -5 & 5 & -4 & 4 \\ \hline & 2 & -2 & 5 & -5 & 4 & -4 & 7 \end{array} = g(-1)$$

45. $h(x) = x^3 - 5x^2 - 7x + 4$

(a) $$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array} = h(3)$$

(b) $$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array} = h(2)$$

(c) $$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array} = h(-2)$$

(d) $$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array} = h(-5)$$

46. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) $$\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array} = f(1)$$

(b) $$\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array} = f(-2)$$

(c) $$\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array} = f(5)$$

(d) $$\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56{,}700 \\ \hline & 4 & -56 & 567 & -5670 & 56{,}720 \end{array} = f(-10)$$

47. $$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} x^3 - 7x + 6 &= (x-2)(x^2 + 2x - 3) \\ &= (x-2)(x+3)(x-1) \end{aligned}$$

Zeros: $2, -3, 1$

48. $$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$$\begin{aligned} x^3 - 28x - 48 &= (x+4)(x^2 - 4x - 12) \\ &= (x+4)(x-6)(x+2) \end{aligned}$$

Zeros: $-4, -2, 6$

49. $$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$$\begin{aligned} &2x^3 - 15x^2 + 27x - 10 \\ &\quad = (x - \tfrac{1}{2})(2x^2 - 14x + 20) \\ &\quad = (2x-1)(x-2)(x-5) \end{aligned}$$

Zeros: $\frac{1}{2}, 2, 5$

50. $$\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

$$\begin{aligned} 48x^3 - 80x^2 + 41x - 6 &= (x - \tfrac{2}{3})(48x^2 - 48x + 9) \\ &= (3x-2)(4x-3)(4x-1) \end{aligned}$$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

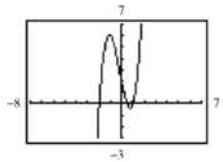
51. (a) $$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

(b) $2x^2 - 3x + 1 = (2x-1)(x-1)$

Remaining factors: $(2x-1), (x-1)$

(c) $f(x) = (x+2)(2x-1)(x-1)$

(d) Real zeros: $-2, \frac{1}{2}, 1$



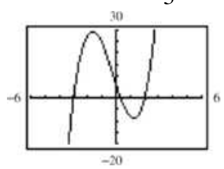
52. (a) $$\begin{array}{r|rrrr} -3 & 3 & 2 & -19 & 6 \\ & & -9 & 21 & -6 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

(b) $3x^2 - 7x + 2 = (3x-1)(x-2)$

Remaining factors: $(3x-1), (x-2)$

(c) $f(x) = (x+3)(3x-1)(x-2)$

(d) Real zeros: $-3, \frac{1}{3}, 2$



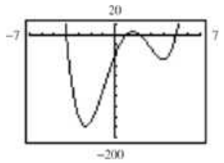
53. (a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$$
$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

(b) $x^2 - 3x + 2 = (x-2)(x-1)$

Remaining factors: $(x-2)$, $(x-1)$

(c) $f(x) = (x-5)(x+4)(x-2)(x-1)$

(d) Real zeros: $5, -4, 2, 1$



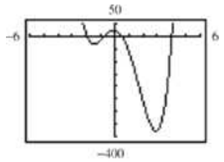
54. (a)
$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$
$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

(b) $8x^2 + 2x - 3 = (4x+3)(2x-1)$

Remaining factors: $(4x+3)$, $(2x-1)$

(c) $f(x) = (x+2)(x-4)(4x+3)(2x-1)$

(d) Real zeros: $-2, 4, -\frac{3}{4}, \frac{1}{2}$



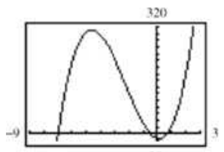
55. (a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

(b) $6x^2 + 38x - 28 = (3x-2)(2x+14)$

Remaining factors: $(3x-2)$, $(x+7)$

(c) $f(x) = (2x+1)(3x-2)(x+7)$

(d) Real zeros: $-\frac{1}{2}, \frac{2}{3}, -7$



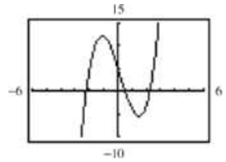
56. (a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

(b) $2x^2 - 10 = 2(x-\sqrt{5})(x+\sqrt{5})$

Remaining factors: $(x-\sqrt{5})$, $(x+\sqrt{5})$

(c) $f(x) = (2x-1)(x+\sqrt{5})(x-\sqrt{5})$

(d) Real zeros: $\frac{1}{2}, \pm\sqrt{5}$



57. $f(x) = x^3 + 3x^2 - x - 3$

$p =$ factor of -3
$q =$ factor of 1

Possible rational zeros: $\pm 1, \pm 3$

$f(x) = x^2(x+3) - (x+3) = (x+3)(x^2-1)$

Rational zeros: $\pm 1, -3$

58. $f(x) = x^3 - 4x^2 - 4x + 16$

$p =$ factor of 16
$q =$ factor of 1

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$f(x) = x^2(x-4) - 4(x-4) = (x-4)(x^2-4)$

Rational zeros: $4, \pm 2$

59. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

$p =$ factor of -45
$q =$ factor of 2

Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Using synthetic division, -1, 3, and 5 are zeros.

$f(x) = (x+1)(x-3)(x-5)(2x-3)$

Rational zeros: $-1, 3, 5, \frac{3}{2}$

60. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

$p =$ factor of -2
$q =$ factor of 4

Possible rational zeros: $\pm 2, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$

Using synthetic division, -1, 1, and 2 are zeros.

$f(x) = (x+1)(x-1)(x-2)(2x-1)(2x+1)$

Rational zeros: $\pm 1, \pm\frac{1}{2}, 2$

61. $f(x) = 2x^4 - x^3 + 6x^2 - x + 5$

4 variations in sign $\Rightarrow$ 4, 2, or 0 positive real zeros

$f(-x) = 2x^4 + x^3 + 6x^2 + x + 5$

0 variations in sign $\Rightarrow$ 0 negative real zeros

62. $f(x) = 3x^4 + 5x^3 - 6x^2 + 8x - 3$

3 sign changes $\Rightarrow$ 3 or 1 positive real zeros
$f(-x) = 3x^4 - 5x^3 - 6x^2 - 8x - 3$
1 sign change $\Rightarrow$ 1 negative real zero

63. $g(x) = 4x^3 - 5x + 8$

2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros
$g(-x) = -4x^3 + 5x + 8$
1 variation in sign $\Rightarrow$ 1 negative real zero

64. $g(x) = 2x^3 - 4x^2 - 5$

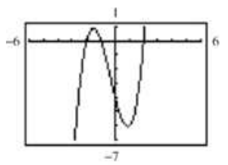
1 sign change $\Rightarrow$ 1 positive real zero
$g(-x) = -2x^3 - 4x^2 - 5$
No sign change $\Rightarrow$ no negative real zeros

65. $f(x) = x^3 + x^2 - 4x - 4$

(a) $f(x)$ has 1 variation in sign $\Rightarrow$ 1 positive real zero.
$f(-x) = -x^3 + x^2 + 4x - 4$ has 2 variations in sign $\Rightarrow$ 2 or 0 negative real zeros.

(b) Possible rational zeros: $\pm 1,\ \pm 2,\ \pm 4$

(c)



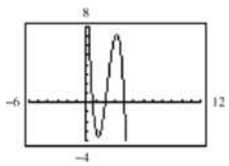
(d) Real zeros: $-2,\ -1,\ 2$

66. (a) $f(x) = -3x^3 + 20x^2 - 36x + 16$

3 sign changes $\Rightarrow$ 3 or 1 positive real zeros
$f(-x) = 3x^3 + 20x^2 + 36x + 16$
0 sign changes $\Rightarrow$ No negative real zeros

(b) Possible rational zeros:
$\pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{4}{3},\ \pm\frac{8}{3},\ \pm\frac{16}{3},\ \pm 1,\ \pm 2,\ \pm 4,\ \pm 8,\ \pm 16$

(c)



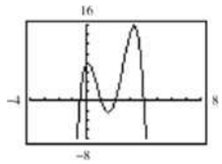
(d) Zeros: $\frac{2}{3}$, 2, 4

67. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) $f(x)$ has variations in sign $\Rightarrow$ 3 or 1 positive real zeros.
$f(-x) = -2x^4 - 13x^3 - 21x^2 - 2x + 8$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros: $\pm\frac{1}{2},\ \pm 1,\ \pm 2,\ \pm 4,\ \pm 8$

(c)



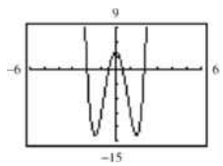
(d) Real zeros: $-\frac{1}{2},\ 1,\ 2,\ 4$

68. (a) $f(x) = 4x^4 - 17x^2 + 4$

2 sign changes $\Rightarrow$ 0 or 2 positive real zeros
$f(-x) = 4x^4 - 17x^2 + 4$
2 sign changes $\Rightarrow$ 0 or 2 negative real zeros

(b) Possible rational zeros: $\pm\frac{1}{4},\ \pm\frac{1}{2},\ \pm 1,\ \pm 2,\ \pm 4$

(c)



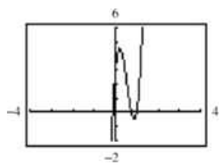
(d) Zeros: $\pm 2,\ \pm\frac{1}{2}$

69. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) $f(x)$ has 2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros.
$f(-x) = -32x^3 - 52x^2 - 17x + 3$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros:
$\pm\frac{1}{32}, \pm\frac{1}{16}, \pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{32}, \pm\frac{3}{16}, \pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{3}{2}, \pm 3$

(c)



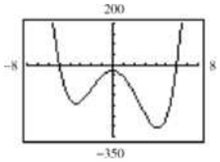
(d) Real zeros: $1,\ \frac{3}{4}, -\frac{1}{8}$

70. $f(x) = x^4 - x^3 - 29x^2 - x - 30$

(a) $f(x)$ has 1 variation in sign $\Rightarrow$ 1 positive real zero.
$f(-x) = x^4 + x^3 - 29x^2 + x - 30$ has 3 variations in signs $\Rightarrow$ 3 or 1 negative real zeros.

(b) Possible rational zeros:
$\pm 1,\ \pm 2,\ \pm 3,\ \pm 5,\ \pm 6,\ \pm 10,\ \pm 15,\ \pm 30$

(c)



(d) Real zeros: −5, 6

71. $f(x) = x^4 - 4x^3 + 15$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$$

-1 is a lower bound.

Real zeros: 1.937, 3.705

72. $f(x) = 2x^3 - 3x^2 - 12x + 8$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -12 & 8 \\ & & 8 & 20 & 32 \\ \hline & 2 & 5 & 8 & 40 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -12 & 8 \\ & & -6 & 27 & -45 \\ \hline & 2 & -9 & 15 & -37 \end{array}$$

-3 is a lower bound.

Real zeros: $-2.152, 0.611, 3.041$

73. $f(x) = x^4 - 4x^3 + 16x - 16$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 25 & 105 & 525 & 2705 \\ \hline & 5 & 21 & 105 & 541 & 2689 \end{array}$$

5 is an upper bound.

$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

-3 is a lower bound.

Real zeros: $-2, 2$

74. $f(x) = 2x^4 - 8x + 3$

$$\begin{array}{r|rrrrr} 3 & 2 & 0 & 0 & -8 & 3 \\ & & 6 & 18 & 54 & 138 \\ \hline & 2 & 6 & 18 & 46 & 141 \end{array}$$

3 is an upper bound.

$$\begin{array}{r|rrrrr} -4 & 2 & 0 & 0 & -8 & 3 \\ & & -8 & 32 & -128 & 544 \\ \hline & 2 & -8 & 32 & -136 & 547 \end{array}$$

-4 is lower bound.

Real zeros: 0.380, 1.435

75.
$$\begin{aligned} P(x) &= x^4 - \frac{25}{4}x^2 + 9 \\ &= \frac{1}{4}(4x^4 - 25x^2 + 36) \\ &= \frac{1}{4}(4x^2 - 9)(x^2 - 4) \\ &= \frac{1}{4}(2x+3)(2x-3)(x+2)(x-2) \end{aligned}$$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

76. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= \frac{1}{2}(x-4)(2x^2 + 5x - 3) \\ &= \frac{1}{2}(x-4)(2x-1)(x+3) \end{aligned}$$

Rational zeros: $-3, \frac{1}{2}, 4$

77.
$$\begin{aligned} f(x) &= x^3 - \frac{1}{4}x^2 - x + \frac{1}{4} \\ &= \frac{1}{4}(4x^3 - x^2 - 4x + 1) \\ &= \frac{1}{4}[x^2(4x-1) - (4x-1)] \\ &= \frac{1}{4}(4x-1)(x^2-1) \\ &= \frac{1}{4}(4x-1)(x+1)(x-1) \end{aligned}$$

The rational zeros are $\frac{1}{4}$ and ± 1.

78. $f(z)=\frac{1}{6}(6z^3+11z^2-3z-2)$

Possible rational zeros: $\pm 1,\ \pm 2,\ \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3},\ \pm\frac{1}{6}$

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

$$\begin{aligned} f(x) &= \frac{1}{6}(z+2)(6z^2-z-1) \\ &= \frac{1}{6}(z+2)(3z+1)(2z-1) \end{aligned}$$

Rational zeros: $-2,\ -\frac{1}{3},\ \frac{1}{2}$

79. $f(x)=x^3-1$
$\quad =(x-1)(x^2+x+1)$

Rational zeros: $1\ (x=1)$

Irrational zeros: 0

Matches (d).

80. $f(x)=x^3-2$
$\quad =(x-\sqrt[3]{2})\left(x^2+\sqrt[3]{2}\,x+\sqrt[3]{4}\right)$

Rational zeros: 0

Irrational zeros: $1\ \left(x=\sqrt[3]{2}\right)$

Matches (a).

81. $f(x)=x^3-x=x(x+1)(x-1)$

Rational zeros: $3\ (x=0, \pm 1)$

Irrational zeros: 0

Matches (b).

82. $f(x)=x^3-2x$
$\quad = x(x^2-2)$
$\quad = x\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$

Rational zeros: $1\ (x=0)$

Irrational zeros: $2\ \left(x=\pm\sqrt{2}\right)$

Matches (c).

83. $y=2x^4-9x^3+5x^2+3x-1$

Using the graph and synthetic division, $-\frac{1}{2}$ is a zero.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -9 & 5 & 3 & -1 \\ & & -1 & 5 & -5 & 1 \\ \hline & 2 & -10 & 10 & -2 & 0 \end{array}$$

$y=\left(x+\frac{1}{2}\right)(2x^3-10x^2+10x-2)$

$x=1$ is a zero of the cubic, so
$y=(2x+1)(x-1)(x^2-4x+1)$.

For the quadratic term, use the Quadratic Formula.

$$x=\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$$

The real zeros are $-\frac{1}{2},\ 1,\ 2\pm\sqrt{3}$.

84. $y=x^4-5x^3-7x^2+13x-2$

Using the graph and synthetic division, 1 and –2 are zeros.

$y=(x-1)(x+2)(x^2-6x+1)$

For the quadratic term, use the Quadratic Formula.

$$x=\frac{6\pm\sqrt{36-4}}{2}=3\pm 2\sqrt{2}$$

The real zeros are $1,\ -2,\ 3\pm 2\sqrt{2}$.

85. $y=-2x^4+17x^3-3x^2-25x-3$

Using the graph and synthetic division, -1 and $\frac{3}{2}$ are zeros.

$y=-(x+1)(2x-3)(x^2-8x-1)$

For the quadratic term, use the Quadratic Formula.

$$x=\frac{8\pm\sqrt{64+4}}{2}=4\pm\sqrt{17}$$

The real zeros are $-1,\ \frac{3}{2},\ 4\pm\sqrt{17}$.

86. $y=-x^4+5x^3-10x-4$

Using the graph and synthetic division, 2 and –1 are zeros.

$y=-(x-2)(x+1)(x^2-4x-2)$

For the quadratic term, use the Quadratic Formula.

$$x=\frac{4\pm\sqrt{16+8}}{2}=2\pm\sqrt{6}$$

The real zeros are $2,\ -1,\ 2\pm\sqrt{6}$.

87. $3x^4 - 14x^2 - 4x = 0$

$x(3x^3 - 14x - 4) = 0$

$x = 0$ is a real zero.

Possible ratoinal zeros: $\pm 1,\ \pm 2,\ \pm 4,\ \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{4}{3}$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -14 & -4 \\ & & -6 & 12 & 4 \\ \hline & 3 & -6 & -2 & 0 \end{array}$$

$x = -2$ is a real zero.

Use the Quadratic Formula. $3x^2 - 6x - 2 = 0$

$$x = \frac{3 \pm \sqrt{15}}{3}$$

Real zeros: $x = 0,\ -2,\ \dfrac{3 \pm \sqrt{15}}{3}$

88. $4x^4 - 11x^3 - 22x^2 + 8x = 0$

$x(4x^3 - 11x^2 - 22x + 8) = 0$

$x = 0$ is a real zero.

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{4}, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} 4 & 4 & -11 & -22 & 8 \\ & & 16 & 20 & -8 \\ \hline & 4 & 5 & -2 & 0 \end{array}$$

$x = 4$ is a real zero.

Using the Quadratic Formula.

$4x^2 + 5x - 2 = 0$ $\qquad x = \dfrac{-5 \pm \sqrt{57}}{8}$

Real zeros: $x = 0,\ 4,\ \dfrac{-5 \pm \sqrt{57}}{8}$

89. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1,\ \pm 2,\ \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z = -1$ and $z = 2$ are real zeros.

$z^4 - z^3 - 2z - 4 = (z+1)(z-2)(z^2+2) = 0$

The only real zeros are –1 and 2. You can verify this by graphing the function $f(z) = z^4 - z^3 - 2z - 4$.

90. $4x^3 + 7x^2 - 11x - 18 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18,$
$\pm\frac{1}{4},\ \pm\frac{1}{2},\ \pm\frac{3}{4},\ \pm\frac{3}{2},\ \pm\frac{9}{4},\ \pm\frac{9}{2}$

$$\begin{array}{r|rrrr} -2 & 4 & 7 & -11 & -18 \\ & & -8 & 2 & 18 \\ \hline & 4 & -1 & -9 & 0 \end{array}$$

$x = -2$ is a real zero.

Use the Quadratic Formula. $4x^2 - x - 9 = 0$

$$x = \frac{1 \pm \sqrt{145}}{8}$$

Real zeros: $x = -2,\ \dfrac{1 \pm \sqrt{145}}{8}$

91. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Possible rational zeros:

$\pm\frac{1}{2},\ \pm 1,\ \pm\frac{3}{2},\ \pm 2,\ \pm 3,\ \pm 6$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & 7 & -26 & 23 & -6 \\ & & 1 & 4 & -11 & 6 \\ \hline & 2 & 8 & -22 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 8 & -22 & 12 \\ & & 2 & 10 & -12 \\ \hline & 2 & 10 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrr} -6 & 2 & 10 & -12 \\ & & -12 & 12 \\ \hline & 2 & -2 & 0 \end{array}$$

$$\begin{array}{r|rr} 1 & 2 & -2 \\ & & 2 \\ \hline & 2 & 0 \end{array}$$

$x = \frac{1}{2},\ x = 1, x = -6,$ and $x = 1$ are real zeros.

$(y+6)(y-1)^2(2y-1) = 0$

Real zeros: $x = -6,\ 1,\ \dfrac{1}{2}$

92. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x(x^4 - x^3 - 3x^2 + 5x - 2) = 0$

$x = 0$ is a real zero.

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -3 & 5 & -2 \\ & & 1 & 0 & -3 & 2 \\ \hline & 1 & 0 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x = 1$ and $x = -2$ are real zeros.

$x(x-1)(x+2)(x^2-2x+1) = 0$
$x(x-1)(x+2)(x-1)(x-1) = 0$

Real zeros: $-2, 0, 1$

93. $4x^4 - 55x^2 - 45x + 36 = 0$

Possible rational zeros:
$\pm\frac{1}{2}, \pm\frac{3}{4}, \pm1, \pm\frac{3}{2}, \pm2, \pm\frac{9}{4}, \pm3, \pm4, \pm\frac{9}{2}, \pm6, \pm9, \pm18$

$$\begin{array}{r|rrrrr} 4 & 4 & 0 & -55 & -45 & 36 \\ & & 16 & 64 & 36 & -36 \\ \hline & 4 & 16 & 9 & -9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 9 & -9 \\ & & -12 & -12 & 9 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2} & 4 & 4 & -3 \\ & & 2 & 3 \\ \hline & 4 & 6 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{3}{2} & 4 & 6 \\ & & -6 \\ \hline & 4 & 0 \end{array}$$

$x = 4, x = -3, x = \frac{1}{2},$ and $x = -\frac{3}{2}$ are real zeros.
$(x-4)(x+3)(2x-1)(2x+3) = 0$

Real zeros: $x = 4, -3, \frac{1}{2}, -\frac{3}{2}$

94. $4x^4 - 43x^2 - 9x + 90 = 0$

Possible rational zeros:

$\pm1, \pm2, \pm3, \pm5, \pm6, \pm9, \pm10, \pm15, \pm18,$

$\pm30, \pm45, \pm90, \pm\frac{1}{2},$

$\pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm\frac{9}{2}, \pm\frac{9}{4}, \pm\frac{15}{2}, \pm\frac{15}{4}, \pm\frac{45}{2}, \pm\frac{45}{4}$

$$\begin{array}{r|rrrrr} -\frac{5}{2} & 4 & 0 & -43 & -9 & 90 \\ & & -10 & 25 & 45 & -90 \\ \hline & 4 & -10 & -18 & 36 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & -10 & -18 & 36 \\ & & -8 & 36 & -36 \\ \hline & 4 & -18 & 18 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & -18 & 18 \\ & & 6 & -18 \\ \hline & 4 & -12 & 0 \end{array}$$

$$\begin{array}{r|rr} 3 & 4 & -12 \\ & & 12 \\ \hline & 4 & 0 \end{array}$$

$x = -\frac{5}{2}, x = -2, x = \frac{3}{2},$ and $x = 3$ are real zeros.

$(2x+5)(x+2)(2x-3)(x-3) = 0$

Real zeros: $x = -\frac{5}{2}, -2, \frac{3}{2}, 3$

95. $8x^4 + 28x^3 + 9x^2 - 9x = 0$
$x(8x^3 + 28x^2 + 9x - 9) = 0$

$x = 0$ is a real zero.

Possible rational zeros:

$\pm1, \pm3, \pm9, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{9}{8}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

$$\begin{array}{r|rrrr} -3 & 8 & 28 & 9 & -9 \\ & & -24 & -12 & 9 \\ \hline & 8 & 4 & -3 & 0 \end{array}$$

$x = -3$ is a real zeros

Use the Quadratic Formula.
$8x^2 + 4x - 3 = 0 \qquad x = \dfrac{-\pm\sqrt{7}}{4}$

Real zeros: $x = 0, -3, \dfrac{-1\pm\sqrt{7}}{4}$

96. $x^5 + 5x^4 - 5x^3 - 15x^2 - 6x = 0$
$x(x^4 + 5x^3 - 5x^2 - 15x - 6) = 0$

$x = 0$ is a real zero.

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrrr} -1 & 1 & 5 & -5 & -15 & -6 \\ & & -1 & -4 & 9 & 6 \\ \hline & 1 & 4 & -9 & -6 & 0 \end{array}$$

$x = -1$ is a real zero.

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -9 & -6 \\ & & 2 & 12 & 6 \\ \hline & 1 & 6 & 3 & 0 \end{array}$$

$x = 2$ is a real zero.

Use the Quadratic Formula.
$x^2 + 6x + 3 = 0 \qquad x = -3\pm\sqrt{6}$
Real zeros: $x = 0, -1, 2, -3\pm\sqrt{6}$

97. $4x^5 + 12x^4 - 11x^3 - 42x^2 + 7x + 30 = 0$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30,$
$\pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm\frac{15}{2}, \pm\frac{15}{4}$

$$\begin{array}{r|rrrrrr} 1 & 4 & 12 & -11 & -42 & 7 & 30 \\ & & 4 & 16 & 5 & -37 & -30 \\ \hline & 4 & 16 & 5 & -37 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 16 & 5 & -37 & -30 \\ & & -4 & -12 & 7 & 30 \\ \hline & 4 & 12 & -7 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & 12 & -7 & -30 \\ & & -8 & -8 & 30 \\ \hline & 4 & 4 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & 4 & -15 \\ & & 6 & 15 \\ \hline & 4 & 10 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{5}{2} & 4 & 10 \\ & & -10 \\ \hline & 4 & 0 \end{array}$$

$x = 1$, $x = -1$, $x = -2$, $x = \dfrac{3}{2}$, and $x = -\dfrac{5}{2}$ are real zeros.

$(x-1)(x+1)(x+2)(2x-3)(2x+5) = 0$

Real zeros: $x = 1,\ -1,\ -2,\ \dfrac{3}{2},\ -\dfrac{5}{2}.$

98. $4x^5 + 8x^4 - 15x^3 - 23x^2 + 11x + 15 = 0$

Possible rational zeros:

$\pm 1,\ \pm 3,\ \pm 5,\ \pm 15,\ \pm\dfrac{1}{2},\ \pm\dfrac{1}{4},\ \pm\dfrac{3}{2},\ \pm\dfrac{3}{4},\ \pm\dfrac{5}{2},\ \pm\dfrac{5}{4},\ \pm\dfrac{15}{2},\ \pm\dfrac{15}{4}$

$$\begin{array}{r|rrrrrr} 1 & 4 & 8 & -15 & -23 & 11 & 15 \\ & & 4 & 12 & -3 & -26 & -15 \\ \hline & 4 & 12 & -3 & -26 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 12 & -3 & -26 & -15 \\ & & -4 & -8 & 11 & 15 \\ \hline & 4 & 8 & -11 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 4 & 8 & -11 & -15 \\ & & -4 & -4 & 15 \\ \hline & 4 & 4 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & 4 & -15 \\ & & 6 & 15 \\ \hline & 4 & 10 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{5}{2} & 4 & 10 \\ & & -10 \\ \hline & 4 & 0 \end{array}$$

$x = 1$, $x = -1$, $x = -1$, $x = \dfrac{3}{2}$, and $x = \dfrac{5}{2}$ are real zeros.

Real zeros: $x = 1,\ -1,\ -1,\ \dfrac{3}{2},\ -\dfrac{5}{2}.$

99. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) Zeros: -2, 3.732, 0.268

(b)
$$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$
$t = -2$ is a zero.

(c) $h(t) = (t+2)(t^2 - 4t + 1)$
$= (t+2)\left[t - (\sqrt{3}+2)\right]\left[t + (\sqrt{3}-2)\right]$

100. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) Zeros 6, 5.236, 0.764

(b)
$$\begin{array}{r|rrrr} 6 & 1 & -12 & 40 & -24 \\ & & 6 & -36 & 24 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$
$s = 6$ is a zeros.

(c) $f(s) = (s-6)(s^2 - 6s + 4)$
$= (s-6)(s-3-\sqrt{5})(s-3-\sqrt{5})$

101. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $x = 0,\ 3,\ 4,\ \pm 1.414$

(b)
$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$x = 3$ is a zero.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$
$x = 4$ is a zero.

(c) $h(x) = x(x-3)(x-4)(x^2-2)$
$= x(x-3)(x-4)(x-\sqrt{2})(x+\sqrt{2})$

102. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3.0,\ 1.5,\ 0.333$

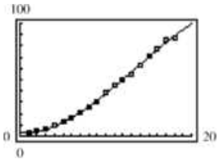
(b)
$$\begin{array}{r|rrrrr} 3 & 6 & -11 & -51 & 99 & -27 \\ & & 18 & 21 & -90 & 27 \\ \hline & 6 & 7 & -30 & 9 & 0 \end{array}$$
$x = 3$ is a zero.

$$\begin{array}{r|rrrr} -3 & 6 & 7 & -30 & 9 \\ & & -18 & 33 & -9 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

$x = -3$ is a zero.

(c) $g(x) = (x-3)(x+3)(6x^2 - 11x + 3)$
$= (x-3)(x+3)(3x-1)(2x-3)$

103. (a)



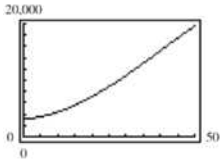
(b) The model fits the data well.

(c) $S = -0.0135t^3 + 0.545t^2 - 0.71t + 3.6$

$$\begin{array}{r|rrrr} 25 & -0.0135 & 0.545 & -0.71 & 3.6 \\ & & -0.3375 & 5.1875 & 111.9375 \\ \hline & -0.0135 & 0.2075 & 4.4775 & 115.5375 \end{array}$$

In 2015 (t = 25), the model predicts approximately 116 subscriptions per 100 people, obviously not a reasonable prediction because you cannot have more subscriptions than people.

104. (a)



(b) In 1960 $(t = 0)$, there were approximately 3,167,000 employees.

$$\begin{array}{r|rrrr} 40 & -0.084 & 10.32 & -23.5 & 3167 \\ & & -3.36 & 278.4 & 10,196 \\ \hline & -0.084 & 6.96 & 254.9 & 13,363 \end{array}$$

In 2000 (t = 40), the model predicts approximately 13,363,000 employees.

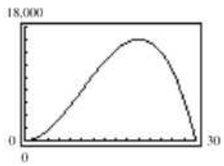
(c) Answers will vary.

105. (a) Combined length and width:

$$4x + y = 120 \Rightarrow y = 120 - 4x$$

$$\begin{aligned} \text{Volume} = l \cdot w \cdot h &= x^2 y \\ &= x^2(120 - 4x) \\ &= 4x^2(30 - x) \end{aligned}$$

(b)



Dimension with maximum volume: $20 \times 20 \times 40$

(c)

$$\begin{aligned} 13,500 &= 4x^2(30 - x) \\ 4x^3 - 120x^2 + 13,500 &= 0 \\ x^3 - 30x^2 + 3375 &= 0 \end{aligned}$$

$$\begin{array}{r|rrrr} 15 & 1 & -30 & 0 & 3375 \\ & & 15 & -225 & -3375 \\ \hline & 1 & -15 & -225 & 0 \end{array}$$

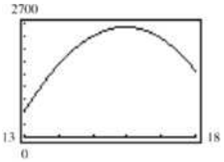
$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15$ or $\dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

106. $y = -5.05x^3 + 3857x - 38,411.25, \; 13 \le x \le 18$

(a)



(b) The second air-fuel ratio of 16.89 can be obtained by finding the second point where the curves y and $y_1 = 2400$ intersect.

(c) Solve $-5.05x^3 + 3857x - 38,411.25 = 2400$ or $-5.05x^3 + 3857x - 40,811.25 = 0$.

By synthetic division:

$$\begin{array}{r|rrrr} 15 & -5.05 & 0 & 3857 & -40,811.25 \\ & & -75.75 & -1136.25 & 40,811.25 \\ \hline & -5.05 & -75.75 & 2720.75 & 0 \end{array}$$

The positive zero of the quadratic
$-5.05x^2 - 75.75x + 2720.75$
can be found using the Quadratic Formula.

$$x = \frac{75.75 - \sqrt{(-75.75)^2 - 4(-5.05)(2720.75)}}{2(-5.05)} \approx 16.89$$

107. False, $-\dfrac{4}{7}$ is a zero of f.

108. False, $f\left(\dfrac{1}{7}\right) \approx -7.896 \ne 0$.

109. The zeros are 1, 1, and –2. The graph falls to the right.

$y = a(x - 1)^2(x + 2), \; a < 0$
Since $f(0) = -4, \; a = -2$.
$y = -2(x - 1)^2(x + 2)$

110. The zeros are 1, –1, and –2. The graph rises to the right.

$y = a(x - 1)(x + 1)(x + 2), \; a > 0$
Since $f(0) = -4, \; a = 2$.
$y = 2(x - 1)(x + 1)(x + 2)$

111. $f(x) = -(x + 1)(x - 1)(x + 2)(x - 2)$

112. Use synthetic division.

$$\begin{array}{r|rrrr} 3 & 1 & -k & 2k & -12 \\ & & 3 & 9 - 3k & 27 - 3k \\ \hline & 1 & 3 - k & 9 - k & 15 - 3k \end{array}$$

Since the remainder $15 - 3k$ should be 0, $k = 5$.

113. (a) $\dfrac{x^2-1}{x-1}=x+1,\ x\neq 1$

(b) $\dfrac{x^3-1}{x-1}=x^2+x+1,\ x\neq 1$

(c) $\dfrac{x^4-1}{x-1}=x^3+x^2+x+1,\ x\neq 1$

In general,

$$\frac{x^n-1}{x-1}=x^{n-1}+x^{n-2}+\cdots+x+1,\ x\neq 1.$$

114. (a) f has 1 negative real zero. $f(-x)$ has only 1 sign change.

(b) Because f has 4 sign changes, f has either 4 or 2 positive real zeros. The graph either turns upward and rises to the right or it turns upward and crosses the x-axis, then turns downward and crosses the x-axis and then turns back upward and rises to the right.

(c) No. No factor of 3 divided by a factor of 2 is equal to $-\dfrac{1}{3}$.

(d) Use synthetic division. If $r=0$, then $x-\dfrac{3}{2}$ is a factor of f. Otherwise, if each number in the last row is either positive or 0, then $x=\dfrac{3}{2}$ is an upper bound.

115. $9x^2-25 = 0$

$(3x+5)(3x-5)=0$

$x=-\dfrac{5}{3},\ \dfrac{5}{3}$

116. $16x^2-21=0$

$x^2=\dfrac{21}{16}$

$x=\pm\dfrac{\sqrt{21}}{4}$

117. $2x^2+6x+3=0$

$$x=\frac{-6\pm\sqrt{6^2-4(2)(3)}}{2(2)}$$

$$=\frac{-6\pm\sqrt{12}}{4}$$

$$=\frac{-3\pm\sqrt{3}}{2}$$

$$x=-\frac{3}{2}+\frac{\sqrt{3}}{2},-\frac{3}{2}-\frac{\sqrt{3}}{2}$$

118. $8x^2-22x+15=0$

$(4x-5)(2x-3)=0$

$x=\dfrac{5}{4},\ \dfrac{3}{2}$

Section 2.4

1. (a) ii
(b) iii
(c) i

2. $\sqrt{-1},\ -1$

3. $(7+6i)+(8+5i)=(7+8)+(6+5)i$
$=15+11i$

The real part is 15 and the imaginary part is $11i$.

4. When multiplying complex numbers, the FOIL Method can be used.

$(a+bi)(c+di)=ac+adi+bci+bdi^2$
$=(ac-bd)+(ad+bc)i$

5. The additive inverse of $2-4i$ is $-2+4i$ so that $(2-4i)+(-2+4i)=0$.

6. The complex conjugate of $2-4i$ is $2+4i$ so that $(2-4i)(2+4i)=4-16i^2=4+16=20$.

7. $a+bi=-9+4i$
$a=-9$
$b=4$

8. $a+bi=12+5i$
$a=12$
$b=5$

9. $3a+(b+3)i=9+8i$
$3a=9 \qquad b+3=8$
$a=3 \qquad b=5$

10. $(a+6)+26i=6-i$
$a+6=6 \qquad 2b=-1$
$-a=0 \qquad b=-\dfrac{1}{2}$

11. $5+\sqrt{-16}=5+\sqrt{16(-1)}$
$=5+4i$

12. $2-\sqrt{-9}=2-\sqrt{9(-1)}$
$=2-3i$

13. $-6=-6+0i$

14. $8=8+0i$

15. $-5i+i^2=-5i-1=-1-5i$

16. $-3i^2+i=-3(-1)+i$
$= 3 + i$

17. $\left(\sqrt{-75}\right)^2 = -75$

18. $\left(\sqrt{-4}\right)^2 - 7 = -4 - 7$
$= -11$

19. $\sqrt{-0.09} = \sqrt{0.09}i = 0.3i$

20. $\sqrt{-0.0004} = 0.02i$

21. $(4+i)-(7-2i) = (4-7)+(1+2)i$
$= -3+3i$

22. $(11-2i)-(-3+6i) = (11+3)+(-2-6)i$
$= 14-8i$

23. $(-1+8i)+(8-5i) = (-1+8)+(8-5)i$
$= 7+3i$

24. $(7+6i)+(3+12i) = (7+3)+(6+12)i$
$= 10+18i$

25. $13i-(14-7i) = 13i-14+7i = -14+20i$

26. $22+(-5+8i)-9i = (22+(-5))+(8-9)i$
$= 17-i$

27. $\left(\frac{3}{2}+\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{11}{3}i\right) = \left(\frac{3}{2}+\frac{5}{3}\right)+\left(\frac{5}{2}+\frac{11}{3}\right)i$
$= \frac{9+10}{6}+\frac{15+22}{6}i$
$= \frac{19}{6}+\frac{37}{6}i$

28. $\left(\frac{3}{4}+\frac{7}{5}i\right)-\left(\frac{5}{6}-\frac{1}{6}i\right) = \left(\frac{3}{4}-\frac{5}{6}\right)+\left(\frac{7}{5}+\frac{1}{6}\right)i$
$= -\frac{1}{12}+\frac{47}{30}i$

29. $(1.6+3.2i)+(-5.8+4.3i) = -4.2+7.5i$

30. $-(-3.7-12.8i)-(6.1-16.3i)$
$= (3.7+12.8i)+(-6.1+16.3i)$
$= (3.7-6.1)+(12.8+16.3)i$
$= 2.4+29.1i$

31. $4(3+5i) = 12+20i$

32. $-6(5-3i) = -30+18i$

33. $(1+i)(3-2i) = 3-2i+3i-2i^2$
$= 3+i+2$
$= 5+i$

34. $(6-2i)(2-3i) = 12-18i-4i+6i^2$
$= 12-22i-6$
$= 6-22i$

35. $4i(8+5i) = 32i+20i^2$
$= 32i+20(-1)$
$= -20+32i$

36. $-3i(6-i) = -18i-3 = -3-18i$

37. $\left(\sqrt{14}+\sqrt{10}\,i\right)\left(\sqrt{14}-\sqrt{10}\,i\right) = 14-10i^2 = 14+10 = 24$

38. $\left(\sqrt{3}+\sqrt{15}i\right)\left(\sqrt{3}-\sqrt{15}i\right) = 3-15i^2$
$= 3-15(-1)$
$= 3+15$
$= 18$

39. $(6+7i)^2 = 36+42i+42i+49i^2$
$= 36+84i-49$
$= -13+84i$

40. $(5-4i)^2 = 25-20i-20i+16i^2$
$= 25-40i-16$
$= 9-40i$

41. $(4+5i)^2-(4-5i)^2$
$= \left[(4+5i)+(4-5i)\right]\left[(4+5i)-(4-5i)\right] = 8(10i) = 80i$

42. $(1-2i)^2-(1+2i)^2 = 1-4i+4i^2-(1+4i+4i^2)$
$= 1-4i+4i^2-1-4i-4i^2$
$= -8i$

43. $4-3i$ is the complex conjugate of $4+3i$.

$(4+3i)(4-3i) = 16+9 = 25$

44. The complex conjugate of $7-5i$ is $7+5i$.

$(7-5i)(7+5i) = 49+25 = 74$

45. $-6+\sqrt{5}i$ is the complex conjugate of $-6-\sqrt{5}i$.

$\left(-6-\sqrt{5}i\right)\left(-6+\sqrt{5}i\right) = 36+5 = 41$

46. The complex conjugate of $-3+\sqrt{2}i$ is $-3-\sqrt{2}i$.

$\left(-3+\sqrt{2}i\right)\left(-3-\sqrt{2}i\right) = 9+2 = 11$

47. $-\sqrt{20}i$ is the complex conjugate of $\sqrt{-20} = \sqrt{20}i$.

$\left(\sqrt{20}i\right)\left(-\sqrt{20}i\right) = 20$

48. The complex conjugate of $\sqrt{-13} = \sqrt{13}i$ is $-\sqrt{13}i$.

$\sqrt{-13}\left(-\sqrt{13}i\right) = \left(\sqrt{13}i\right)\left(-\sqrt{13}i\right) = 13$

49. $3+\sqrt{2}i$ is the complex conjugate of
$3-\sqrt{-2} = 3-\sqrt{2}\,i$

$(3-\sqrt{2}i)(3+\sqrt{2}i) = 9+2 = 11$

50. The complex conjugate of $1+\sqrt{-8}=1+2\sqrt{2}i$ is $1-2\sqrt{2}i$.

$$\left(1+2\sqrt{2}i\right)\left(1-2\sqrt{2}i\right)=1+8=9$$

51. $\dfrac{6}{i}=\dfrac{6}{i}\cdot\dfrac{-i}{-i}=\dfrac{-6i}{-i^2}=\dfrac{-6i}{1}=-6i$

52. $\dfrac{-5}{2i}\cdot\dfrac{i}{i}=\dfrac{-5i}{-2}=\dfrac{5}{2}i$

53. $\dfrac{2}{4-5i}=\dfrac{2}{4-5i}\cdot\dfrac{4+5i}{4+5i}=\dfrac{8+10i}{16+25}=\dfrac{8}{41}+\dfrac{10}{41}i$

54. $\dfrac{3}{1-i}\cdot\dfrac{1+i}{1+i}=\dfrac{3+3i}{1-i^2}=\dfrac{3+3i}{2}=\dfrac{3}{2}+\dfrac{3}{2}i$

55.
$$\begin{aligned}\frac{2+i}{2-i}&=\frac{2+i}{2-i}\cdot\frac{2+i}{2+i}\\&=\frac{4+4i+i^2}{4+1}\\&=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i\end{aligned}$$

56.
$$\begin{aligned}\frac{8-7i}{1-2i}\cdot\frac{1+2i}{1+2i}&=\frac{8+16i-7i-14i^2}{1-4i^2}\\&=\frac{22+9i}{5}=\frac{22}{5}+\frac{9}{5}i\end{aligned}$$

57.
$$\begin{aligned}\frac{i}{(4-5i)^2}&=\frac{i}{16-25-40i}\\&=\frac{i}{-9-40i}\cdot\frac{-9+40i}{-9+40i}\\&=\frac{-40-9i}{81+40^2}\\&=-\frac{40}{1681}-\frac{9}{1681}i\end{aligned}$$

58.
$$\begin{aligned}\frac{5i}{(2+3i)^2}&=\frac{5i}{-5+12i}\cdot\frac{-5-12i}{-5-12i}\\&=\frac{-25i+60}{25+144}\\&=\frac{60}{169}-\frac{25}{169}i\end{aligned}$$

59.
$$\begin{aligned}\frac{2}{1+i}-\frac{3}{1-i}&=\frac{2(1-i)-3(1+i)}{(1+i)(1-i)}\\&=\frac{2-2i-3-3i}{1+1}\\&=\frac{-1-5i}{2}\\&=-\frac{1}{2}-\frac{5}{2}i\end{aligned}$$

60.
$$\begin{aligned}\frac{2i}{2+i}+\frac{5}{2-i}&=\frac{2i(2-i)}{(2+i)(2-i)}+\frac{5(2+i)}{(2+i)(2-i)}\\&=\frac{4i-2i^2+10+5i}{4-i^2}\\&=\frac{12+9i}{5}\\&=\frac{12}{5}+\frac{9}{5}i\end{aligned}$$

61.
$$\begin{aligned}\frac{i}{3-2i}+\frac{2i}{3+8i}&=\frac{3i+8i^2+6i-4i^2}{(3-2i)(3+8i)}\\&=\frac{-4+9i}{9+18i+16}\\&=\frac{-4+9i}{25+18i}\cdot\frac{25-18i}{25-18i}\\&=\frac{-100+72i+225i+162}{25^2+18^2}\\&=\frac{62+297i}{949}\\&=\frac{62}{949}+\frac{297}{949}i\end{aligned}$$

62.
$$\begin{aligned}\frac{1+i}{i}-\frac{3}{4-i}&=\frac{1+i}{i}\cdot\frac{-i}{-i}-\frac{3}{4-i}\cdot\frac{4+i}{4+i}\\&=\frac{-i+1}{1}-\frac{12+3i}{16+1}\\&=\frac{5}{17}-\frac{20}{17}i\end{aligned}$$

63.
$$\begin{aligned}\sqrt{-18}-\sqrt{-54}&=3\sqrt{2}i-3\sqrt{6}i\\&=3\left(\sqrt{2}-\sqrt{6}\right)i\end{aligned}$$

64.
$$\begin{aligned}\sqrt{-50}+\sqrt{-275}&=5\sqrt{2}i+5\sqrt{11}i\\&=5\left(\sqrt{2}+\sqrt{11}\right)i\end{aligned}$$

65.
$$\begin{aligned}\left(-3+\sqrt{-24}\right)+\left(7-\sqrt{-44}\right)&=\left(-3+2\sqrt{6}i\right)+\left(7-2\sqrt{11}i\right)\\&=4+\left(2\sqrt{6}-2\sqrt{11}\right)i\\&=4+2\left(\sqrt{6}-\sqrt{11}\right)i\end{aligned}$$

66.
$$\begin{aligned}\left(-12-\sqrt{-72}\right)+\left(9+\sqrt{-108}\right)&=\left(-12-6\sqrt{2}i\right)+\left(9+6\sqrt{3}i\right)\\&=-3+\left(-6\sqrt{2}+6\sqrt{3}\right)i\\&=-3+6\left(\sqrt{3}-\sqrt{2}\right)i\end{aligned}$$

67.
$$\begin{aligned}\sqrt{-6}\cdot\sqrt{-2}&=\left(\sqrt{6}i\right)\left(\sqrt{2}i\right)\\&=\sqrt{12}\,i^2=\left(2\sqrt{3}\right)(-1)=-2\sqrt{3}\end{aligned}$$

68.
$$\begin{aligned}\sqrt{-5}\cdot\sqrt{-10}&=\left(\sqrt{5}i\right)\left(\sqrt{10}i\right)\\&=\sqrt{50}i^2=5\sqrt{2}(-1)=-5\sqrt{2}\end{aligned}$$

69. $\left(\sqrt{-10}\right)^2=\left(\sqrt{10}i\right)^2=10i^2=-10$

70. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

71. $\left(2-\sqrt{-6}\right)^2 = \left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)$

$$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$$
$$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$$
$$= 4 - 6 - 4\sqrt{6}i$$
$$= -2 - 4\sqrt{6}i$$

72. $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right) = \left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)$

$$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$$
$$= 21 + \sqrt{50} + 7\sqrt{5}i - 3\sqrt{10}i$$
$$= \left(21 + 5\sqrt{2}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$$

73. $x^2 + 25 = 0$

$$x^2 = -25$$
$$x = \pm 5i$$

74. $x^2 + 32 = 0$

$$x^2 = -32$$
$$x = \pm\sqrt{32}i = \pm 4\sqrt{2}i$$

75. $x^2 - 2x + 2 = 0;\ a = 1,\ b = -2,\ c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{-4}}{2}$$
$$= \frac{2 \pm 2i}{2}$$
$$= 1 \pm i$$

76. $x^2 + 6x + 10 = 0;\ a = 1,\ b = 6,\ c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$$
$$= \frac{-6 \pm \sqrt{-4}}{2}$$
$$= \frac{-6 + 2i}{2}$$
$$= -3 \pm i$$

77. $4x^2 + 16x + 17 = 0;\ a = 4,\ b = 16,\ c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)}$$
$$= \frac{-16 \pm \sqrt{-16}}{8}$$
$$= \frac{-16 \pm 4i}{8}$$
$$= -2 \pm \frac{1}{2}i$$

78. $9x^2 - 6x + 37 = 0;\ a = 9,\ b = -6,\ c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}$$
$$= \frac{6 \pm \sqrt{-1296}}{18}$$
$$= \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

79. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$
$$= \frac{4 \pm \sqrt{-176}}{32}$$
$$= \frac{4 \pm 4\sqrt{11}\,i}{32}$$
$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

80. $4x^2 + 16x + 15 = 0;\ a = 4,\ b = 16,\ c = 15$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)}$$
$$= \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$
$$x = -\frac{12}{8} = -\frac{3}{2} \text{ or } x = -\frac{20}{8} = -\frac{5}{2}$$

81. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{-72}}{6}$$
$$= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

82. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14,\ b = -12,\ c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}$$
$$= \frac{12 \pm \sqrt{-136}}{28}$$
$$= \frac{12 \pm 2\sqrt{34}i}{28}$$
$$= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

83. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0;\ a = 7,\ b = -10,\ c = -50$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)}$$

$$= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}$$

$$= \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

84. $4.5x^2 - 3x + 12 = 0;\ a = 4.5,\ b = -3,\ c = 12$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)}$$

$$= \frac{3 \pm \sqrt{-207}}{9} = \frac{3 \pm 3\sqrt{23}i}{9} = \frac{1}{3} \pm \frac{\sqrt{23}}{3}i$$

85. $-6i^3 + i^2 = -6i^2 i + i^2 = -6(-1)i + (-1) = 6i - 1 = -1 + 6i$

86. $4i^2 - 2i^3 = -4 + 2i$

87. $\left(\sqrt{-75}\right)^3 = \left(5\sqrt{3}i\right)^3 = 5^3\left(\sqrt{3}\right)^3 i^3 = 125\left(3\sqrt{3}\right)(-i)$

$= -375\sqrt{3}i$

88. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^4 i^2 = -8$

89. $\dfrac{1}{i^3} = \dfrac{1}{i^3} \cdot \dfrac{i}{i} = \dfrac{i}{i^4} = \dfrac{i}{1} = i$

90. $\dfrac{1}{(2i)^3} = \dfrac{1}{8i^3} = \dfrac{1}{-8i} \cdot \dfrac{8i}{8i} = \dfrac{8i}{-64i^2} = \dfrac{1}{8}i$

91. (a) $(2)^3 = 8$

(b) $\left(-1 + \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$

$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$

$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$

$= 8$

(c) $\left(-1 - \sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$

$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$

$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$

$= 8$

The three numbers are cube roots of 8.

92. (a) $2^4 = 16$

(b) $(-2)^4 = 16$

(c) $(2i)^4 = 2^4 i^4 = 16(1) = 16$

(d) $(-2i)^4 = (-2)^4 i^4 = 16(1) = 16$

93. (a) $i^{20} = (i^4)^5 = (1)^5 = 1$

(b) $i^{45} = (i^4)^{11} i = (1)^{11} i = i$

(c) $i^{67} = (i^4)^{16} i^3 = (1)^{16}(-i) = -i$

(d) $i^{114} = (i^4)^{28} i^2 = (1)^{28}(-1) = -1$

94. (a) $z_1 = 5 + 2i$

$z_2 = 3 - 4i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{5 + 2i} + \frac{1}{3 - 4i}$$

$$= \frac{(3 - 4i) + (5 + 2i)}{(5 + 2i)(3 - 4i)}$$

$$= \frac{8 - 2i}{23 - 14i}$$

$$z = \frac{23 - 14i}{8 - 2i}\left(\frac{8 + 2i}{8 + 2i}\right)$$

$$= \frac{212 - 66i}{68} \approx 3.118 - 0.971i$$

(b) $z_1 = 16i + 9$

$z_2 = 20 - 10i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9 + 16i} + \frac{1}{20 - 10i}$$

$$= \frac{(20 - 10i) + (9 + 16i)}{(9 + 16i)(20 - 10i)}$$

$$= \frac{29 + 6i}{340 + 230i}$$

$$z = \frac{340 + 230i}{29 + 6i}\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11240 + 4630i}{877} \approx 12.816 + 5.279i$$

95. False. A real number $a + 0i = a$ is equal to its conjugate.

96. False. $i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = 1 - 1 + 1 - i + i = 1$

97. False. For example, $(1 + 2i) + (1 - 2i) = 2$, which is not an imaginary number.

98. False. For example, $(i)(i) = -1$, which is not an imaginary number.

99. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\overline{z_1 z_2} = \overline{(a_1 + b_1 i)(a_2 + b_2 i)}$$

$$= \overline{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + b_1 a_2)i}$$

$$= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + b_1 a_2)i$$

$$= (a_1 - b_1 i)\ (a_2 - b_2 i)$$

$$= \overline{a_1 + b_1 i}\ \overline{a_2 + b_2 i}$$

$$= \overline{z_1}\,\overline{z_2}.$$

100. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$. Then

$$\overline{z_1 + z_2} = \overline{(a_1 + b_1 i) + (a_2 + b_2 i)}$$

$$= \overline{(a_1 + a_2)} + \overline{(b_1 + b_2)i}$$

$$= (a_1 + a_2) - (b_1 + b_2)i$$

$$= (a_1 - b_1 i) + (a_2 - b_2 i)$$

$$= \overline{(a_1 + b_1 i)} + \overline{(a_2 + b_2 i)}$$

$$= \overline{z_1} + \overline{z_2}.$$

101. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}\,i = 6i^2 = -6$

102. $f(x) = 2(x-3)^2 - 4, g(x) = -2(x-3)^2 - 4$

(a) The graph of *f* is a parabola with vertex at the point $(3, -4)$. The *a* value is positive, so the graph opens upward.

The graph of *g* is also a parabola with vertex at the point $(3, -4)$. The *a* value is negative, so the graph opens downward.

f has an *x*-intercept and *g* does not because when $g(x) = 0$, *x* is a complex number.

(b)
$$\begin{aligned} f(x) &= 2(x-3)^2 - 4 \\ 0 &= 2(x-3)^2 - 4 \\ 4 &= 2(x-3)^2 \\ 2 &= (x-3)^2 \\ \pm\sqrt{2} &= x - 3 \\ 3 \pm \sqrt{2} &= x \end{aligned}$$

$$\begin{aligned} g(x) &= -2(x-3)^2 - 4 \\ 0 &= -2(x-3)^2 - 4 \\ 4 &= -2(x-3)^2 \\ -2 &= (x-3)^2 \\ \pm\sqrt{-2} &= x - 3 \\ 3 \pm \sqrt{2}i &= x \end{aligned}$$

(c) If all the zeros contain *i*, then the graph has no *x*-intercepts.

(d) If *a* and *k* have the same sign (both positive or both negative), then the graph of *f* has no *x*-intercepts and the zeros are complex. Otherwise, the graph of *f* has *x*-intercepts and the zeros are real.

103. $(4x-5)(4x+5) = 16x^2 - 20x + 20x - 25 = 16x^2 - 25$

104. $(x+2)^3 = x^3 + 3x^2 2 + 3x(2)^2 + 2^3$
$= x^3 + 6x^2 + 12x + 8$

105. $\left(3x - \frac{1}{2}\right)(x+4) = 3x^2 - \frac{1}{2}x + 12x - 2 = 3x^2 + \frac{23}{2}x - 2$

106. $(2x-5)^2 = 4x^2 - 20x + 25$

Section 2.5

1. Fundamental Theorem, Algebra

2. irreducible, reals

3. The Linear Factorization Theorem states that a polynomial function *f* of degree *n*, $n > 0$, has exactly *n* linear factors

$$f(x) = a(x - c_1)(x - c_2)\ldots(x - c_n).$$

4. Since complex zeros occur in conjugate pairs, if a fourth-degree polynomial function has zeros –1, 3, and 2*i*, then –2*i* is also a zero.

5. $f(x) = x^3 + x$ has exactly 3 zeros. Matches (c).

6. $f(x) = -x + 7$ has exactly 1 zero. Matches (a).

7. $f(x) = x^5 + 9x^3$ has exactly 5 zeros. Matches (d).

8. $f(x) = x^2 - 14x + 49$ has exactly 2 zeros. Matches (b).

9. $f(x) = x^2 + 25$
$$\begin{aligned} x^2 + 25 &= 0 \\ x^2 &= -25 \\ x &= \pm\sqrt{-25} \\ x &= \pm 5i \end{aligned}$$

10. $f(x) = x^2 + 2$
$$\begin{aligned} x^2 + 2 &= 0 \\ x^2 &= -2 \\ x &= \pm\sqrt{-2} \\ x &= \pm\sqrt{2}i \end{aligned}$$

11. $f(x) = x^3 + 9x$
$$\begin{aligned} x^3 + 9x &= 0 \\ x(x^2 + 9) &= 0 \end{aligned}$$
$x = 0 \quad x^2 + 9 = 0$
$$\begin{aligned} x^2 &= -9 \\ x &= \pm\sqrt{-9} \\ x &= \pm 3i \end{aligned}$$

12. $f(x) = x^3 + 49x$
$$\begin{aligned} x^3 + 49x &= 0 \\ x(x^2 + 49) &= 0 \end{aligned}$$
$x = 0 \quad x^2 + 49 = 0$
$$\begin{aligned} x^2 &= -49 \\ x &= \pm\sqrt{-49} \\ x &= \pm 7i \end{aligned}$$

13. $f(x) = x^3 - 4x^2 + x - 4 = x^2(x-4) + 1(x-4) = (x-4)(x^2+1)$
Zeros: $4, \pm i$
The only real zero of *f(x)* is $x = 4$. This corresponds to the *x*-intercept of $(4, 0)$ on the graph.

14.
$$\begin{aligned} f(x) &= x^3 - 4x^2 - 4x + 16 \\ &= x^2(x-4) - 4(x-4) \\ &= (x^2 - 4)(x-4) \\ &= (x+2)(x-2)(x-4) \end{aligned}$$

The zeros are $x = 2, -2$, and 4. This corresponds to the *x*-intercepts of $(-2, 0)$, $(2, 0)$, and $(4, 0)$ on the graph.

15. $f(x) = x^4 + 4x^2 + 4 = (x^2 + 2)^2$

Zeros: $\pm\sqrt{2}i,\ \pm\sqrt{2}i$

$f(x)$ has no real zeros and the graph of $f(x)$ has no x-intercepts.

16. $f(x) = x^4 - 3x^2 - 4$
$= (x^2 - 4)(x^2 + 1)$
$= (x + 2)(x - 2)(x^2 + 1)$

Zeros: $\pm 2, \pm i$

The only real zeros are $x = -2,\ 2$. This corresponds to the x-intercepts of $(-2,\ 0)$ and $(2,\ 0)$ on the graph.

17. $h(x) = x^2 - 4x + 1$

h has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}.$$

$$h(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$$
$$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

18. $g(x) = x^2 + 10x + 23$
g has no rational zeros. By the Quadratic Formula, the zeros are $x = \dfrac{-10 \pm \sqrt{8}}{2} = -5 \pm \sqrt{2}.$

$$g(x) = \left[x - (-5 + \sqrt{2})\right]\left[x - (-5 - \sqrt{2})\right]$$
$$= (x + 5 + \sqrt{2})(x + 5 - \sqrt{2})$$

19. $f(x) = x^2 - 12x + 26$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(26)}}{2} = 6 \pm \sqrt{10}.$$

$$f(x) = \left[x - (6 + \sqrt{10})\right]\left[x - (6 - \sqrt{10})\right]$$
$$= (x - 6 - \sqrt{10})(x - 6 + \sqrt{10})$$

20. $f(x) = x^2 + 6x - 2$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)}}{2} = -3 \pm \sqrt{11}.$$

$$f(x) = \left[x - (-3 + \sqrt{11})\right]\left[x - (-3 - \sqrt{11})\right]$$
$$= (x + 3 - \sqrt{11})(x + 3 + \sqrt{11})$$

21. $f(x) = x^2 + 25$
Zeros: $\pm 5i$
$f(x) = (x + 5i)(x - 5i)$

22. $f(x) = x^2 + 36$
Zeros: $\pm 6i$
$f(x) = (x + 6i)(x - 6i)$

23. $f(x) = 16x^4 - 81$
$= (4x^2 - 9)(4x^2 + 9)$
$= (2x - 3)(2x + 3)(2x + 3i)(2x - 3i)$

Zeros: $\pm\dfrac{3}{2},\ \pm\dfrac{3}{2}i$

24. $f(y) = 81y^4 - 625$
$= (9y^2 + 25)(9y^2 - 25)$
$= (3y + 5i)(3y - 5i)(3y + 5)(3y - 5)$

Zeros: $\pm\dfrac{5}{3}, \pm\dfrac{5}{3}i$

25. $f(z) = z^2 - z + 56$

$$z = \frac{1 \pm \sqrt{1 - 4(56)}}{2}$$
$$= \frac{1 \pm \sqrt{-223}}{2}$$
$$= \frac{1}{2} \pm \frac{\sqrt{223}}{2}i$$

Zeros: $\dfrac{1}{2} \pm \dfrac{\sqrt{223}}{2}i$

$$f(z) = \left(z - \frac{1}{2} + \frac{\sqrt{223}i}{2}\right)\left(z - \frac{1}{2} - \frac{\sqrt{223}i}{2}\right)$$

26. $h(x) = x^2 - 4x - 3$

$$x = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{7}$$

Zeros: $2 \pm \sqrt{7}$

$h(x) = \left(x - 2 + \sqrt{7}\right)\left(x - 2 - \sqrt{7}\right)$

27. $f(x) = x^4 + 10x^2 + 9$
$= (x^2 + 1)(x^2 + 9)$
$= (x + i)(x - i)(x + 3i)(x - 3i)$

The zeros of $f(x)$ are $x = \pm i$ and $x = \pm 3i$.

28. $f(x) = x^4 + 29x^2 + 100$
$= (x^2 + 25)(x^2 + 4)$
Zeros: $x = \pm 2i, \pm 5i$

$f(x) = (x + 2i)(x - 2i)(x + 5i)(x - 5i)$

29. $f(x) = 3x^3 - 5x^2 + 48x - 80$

Using synthetic division, $\dfrac{5}{3}$ is a zero:

$$\begin{array}{r|rrrr} \frac{5}{3} & 3 & -5 & 48 & -80 \\ & & 5 & 0 & 80 \\ \hline & 3 & 0 & 48 & 0 \end{array}$$

$$f(x) = \left(x - \frac{5}{3}\right)\left(3x^2 + 48\right)$$
$$= (3x-5)(x^2+16)$$
$$= (3x-5)(x+4i)(x-4i)$$

The zeros are $\frac{5}{3},\ 4i,\ -4i.$

30. $f(x) = 3x^3 - 2x^2 + 75x - 50$

Using synthetic division, $\frac{2}{3}$ is a zero:

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 75 & -50 \\ & & 2 & 0 & 50 \\ \hline & 3 & 0 & 75 & 0 \end{array}$$

$$f(x) = \left(x - \frac{2}{3}\right)\left(3x^3 + 75\right)$$
$$= (3x-2)(x^2+25)$$
$$= (3x-2)(x+5i)(x-5i)$$

The zeros are $\frac{2}{3},\ 5i,\ -5i.$

31. $f(t) = t^3 - 3t^2 - 15t + 125$

Possible rational zeros: $\pm1,\ \pm5,\ \pm25,\ \pm125$

$$\begin{array}{r|rrrr} -5 & 1 & -3 & -15 & 125 \\ & & -5 & 40 & -125 \\ \hline & 1 & -8 & 25 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $t^2 - 8t + 25$ are

$$t = \frac{8 \pm \sqrt{64-100}}{2} = 4 \pm 3i.$$

The zeros of *f*(*t*) are $t = -5$ and $t = 4 \pm 3i.$

$$f(t) = \left[t - (-5)\right]\left[t - (4+3i)\right]\left[t - (4-3i)\right]$$
$$= (t+5)(t-4-3i)(t-4+3i)$$

32. $f(x) = x^3 + 11x^2 + 39x + 29$

$$\begin{array}{r|rrrr} -1 & 1 & 11 & 39 & 29 \\ & & -1 & -10 & -29 \\ \hline & 1 & 10 & 29 & 0 \end{array}$$

Zeros: $x = -1,\ \dfrac{-10 \pm \sqrt{16}i}{2} = -5 \pm 2i$

$$f(x) = (x+1)(x+5+2i)(x+5-2i)$$

33. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros: $\pm6, \pm\frac{6}{5}, \pm3, \pm\frac{3}{5}, \pm2, \pm\frac{2}{5}, \pm1, \pm\frac{1}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30$ are those of $x^2 - 2x + 6$:

$$x = \frac{2 \pm \sqrt{4-4(6)}}{2} = 1 \pm \sqrt{5i}$$

Zeros: $-\frac{1}{5},\ 1 \pm \sqrt{5i}$

$$f(x) = 5\left(x + \frac{1}{5}\right)\left[x - \left(1 + \sqrt{5i}\right)\right]\left[x - \left(1 - \sqrt{5i}\right)\right]$$
$$= (5x+1)\left(x - 1 - \sqrt{5i}\right)\left(x - 1 + \sqrt{5i}\right)$$

34. $f(s) = 3s^3 - 4s^2 + 8s + 8$
$$= (3s+2)(s^2 - 2s + 4)$$

Factoring the quadratic,

$$s = \frac{2 \pm \sqrt{4-16}}{2} = 1 \pm \sqrt{3}i.$$

Zeros: $-\frac{2}{3}, 1 \pm \sqrt{3}i$

$$f(s) = (3s+2)(s - 1 + \sqrt{3i})(s - 1 - \sqrt{3i})$$

35. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm1,\ \pm2,\ \pm4,\ \pm8,\ \pm16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline 2 & 1 & -2 & 4 & -8 & 0 \\ & & 2 & 0 & 8 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$$g(x) = (x-2)(x-2)\left(x^2+4\right)$$
$$= (x-2)^2(x+2i)(x-2i)$$

The zeros of *g* are $2, 2,$ and $\pm 2i$.

36. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline -3 & 1 & 3 & 1 & 3 & 0 \\ & & -3 & 0 & -3 & \\ \hline & 1 & 0 & 1 & 0 & \end{array}$$

Zeros: $x = -3,\ -3,\ \pm i$

$$h(x) = (x+3)^2(x+i)(x-i)$$

37. (a) $f(x) = x^2 - 14x + 46.$

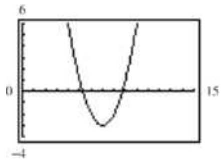
By the Quadric Formula,

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(46)}}{2} = 7 \pm \sqrt{3}.$$

The zeros are $7 + \sqrt{3}$ and $7 - \sqrt{3}.$

(b) $f(x) = \left[x - (7 + \sqrt{3})\right]\left[x - (7 - \sqrt{3})\right]$
$$= (x - 7 - \sqrt{3})(x - 7 + \sqrt{3})$$

(c) x-intercepts: $\left(7+\sqrt{3},\ 0\right)$ and $\left(7-\sqrt{3},\ 0\right)$



38. (a) $f(x) = x^2 - 12x + 34$

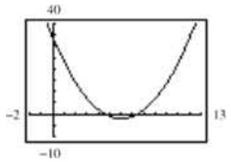
By the Quadric Formula,

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(34)}}{2} = 6 \pm \sqrt{2}.$$

The zeros are $6+\sqrt{2}$ and $6-\sqrt{2}$.

(b) $f(x) = \left(x - \left(6+\sqrt{2}\right)\right)\left(x - \left(6-\sqrt{2}\right)\right)$
$= \left(x - 6 - \sqrt{2}\right)\left(x - 6 + \sqrt{2}\right)$

(c) x-intercepts: $\left(6+\sqrt{2}, 0\right)$ $\left(6-\sqrt{2}, 0\right)$

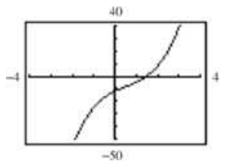


39. (a) $f(x) = 2x^3 - 3x^2 + 8x - 12$
$= (2x-3)\left(x^2+4\right)$

The zeros are $\dfrac{3}{2}$ and $\pm 2i$.

(b) $f(x) = (2x-3)(x+2i)(x-2i)$

(c) x-intercept: $\left(\dfrac{3}{2},\ 0\right)$

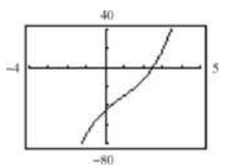


40. (a) $f(x) = 2x^3 - 5x^2 + 18x - 45$
$= (2x-5)(x^2+9)$

The zeros are $\dfrac{5}{2}$ and $\pm 3i$.

(b) $f(x) = (2x-5)(x+3i)(x-3i)$

(c) x-intercept: $\left(\dfrac{5}{2},\ 0\right)$



41. (a) $f(x) = x^3 - 11x + 150$
$= (x+6)\left(x^2 - 6x + 25\right)$

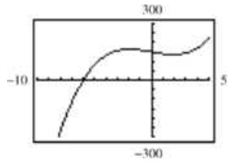
Use the Quadratic Formula to find the zeros of $x^2 - 6x + 25$.

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(25)}}{2} = 3 \pm 4i.$$

The zeros are $-6, 3+4i,$ and $3-4i$.

(b)
$f(x) = (x+6)(x-3+4i)(x-3-4i)$

(c) x-intercept: $(-6,\ 0)$



42. (a) $f(x) = x^3 + 10x^2 + 33x + 34$
$= (x+2)\left(x^2 + 8x + 17\right)$

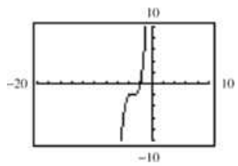
Use the Quadratic Formula to find the zeros of $x^2 + 8x + 17$.

$$x = \frac{-8 \pm \sqrt{8^2 - 4(17)}}{2}$$
$$= \frac{-8 \pm \sqrt{-4}}{2} = -4 + i$$

The zeros are $-2,\ -4+i,$ and $-4-i$.

(b) $f(x) = (x+2)(x+4+i)(x+4-i)$

(c) x-intercept: $(-2,\ 0)$

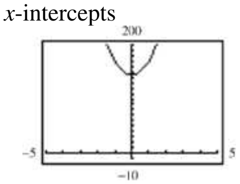


43. (a) $f(x) = x^4 + 25x^2 + 144$
$= \left(x^2+9\right)\left(x^2+16\right)$

The zeros are $\pm 3i,\ \pm 4i$.

(b) $f(x) = \left(x^2+9\right)\left(x^2+16\right)$
$= (x+3i)(x-3i)(x+4i)(x-4i)$

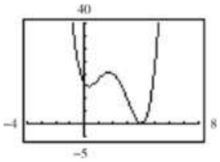
(c) No x-intercepts



44. (a) $f(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$
$$= \left(x^2+1\right)\left(x^2-8x+16\right)$$
$$= \left(x^2+1\right)\left(x-4\right)^2$$
The zeros are i, $-i$, 4, and 4.

(b) $f(x) = \left(x^2+1\right)\left(x-4\right)^2$

(c) x-intercept: (4, 0)



45. $f(x) = (x-2)(x-i)(x+i)$
$$= (x-2)\left(x^2+1\right)$$
Note that $f(x) = a(x^3 - 2x^2 + x - 2)$, where a is any nonzero real number, has zeros 2, $\pm i$.

46. $f(x) = (x-3)(x-4i)(x+4i)$
$$= (x-3)(x^2+16)$$
$$= x^3 - 3x^2 + 16x - 48$$
Note that $f(x) = a(x^3 - 3x^2 + 16x - 48)$, where a is any nonzero real number, has zeros 3, $\pm 4i$.

47. $f(x) = (x-2)^2(x-4-i)(x-4+i)$
$$= (x-2)^2(x-8x+16+1)$$
$$= \left(x^2-4x+4\right)\left(x^2-8x+17\right)$$
$$= x^4 - 12x^3 + 53x^2 - 100x + 68$$
Note that $f(x) = a(x^4 - 12x^3 + 53x^2 - 100x + 68)$, where a is any nonzero real number, has zeros 2, 2, $4 \pm i$.

48. Because $2+5i$ is a zero, so is $2-5i$.
$$f(x) = (x+1)^2(x-2-5i)(x-2+5i)$$
$$= (x+1)^2\left(x^2-4x+4+25\right)$$
$$= \left(x^2+2x+1\right)\left(x^2-4x+29\right)$$
$$= x^4 - 2x^3 + 22x^2 + 54x + 29$$
Note that $f(x) = a(x^4 - 2x^3 + 22x^2 + 54x + 29)$, where a is any nonzero real number, has zeros $-1, -1, 2 \pm 5i$.

49. Because $1+\sqrt{2}i$ is a zero, so is $1-\sqrt{2}i$.
$$f(x) = (x-0)(x+5)\left(x-1-\sqrt{2}i\right)\left(x-1+\sqrt{2}i\right)$$
$$= \left(x^2+5x\right)\left(x^2-2x+1+2\right)$$
$$= \left(x^2+5x\right)\left(x^2-2x+3\right)$$
$$= x^4 + 3x^3 - 7x^2 - 15x$$
Note that $f(x) = a(x^4 + 3x^3 - 7x^2 + 15x)$, where a is any nonzero real number, has zeros 0, -5, $1 \pm \sqrt{2}i$.

50. Because $1+\sqrt{2}i$ is a zero, so is $1 \pm \sqrt{2}i$.
$$f(x) = (x-0)(x-4)\left(x-1-\sqrt{2}i\right)\left(x-1+\sqrt{2}i\right)$$
$$= \left(x^2-4x\right)\left(x^2-2x+1+2\right)$$
$$= x^4 - 6x^3 + 11x^2 - 12x$$
Note that $f(x) = a(x^4 - 6x^3 + 11x^2 - 12x)$, where a is any nonzero real number, has zeros $0, 4, 1 \pm \sqrt{2}i$.

51. (a) $f(x) = a(x-1)(x+2)(x-2i)(x+2i)$
$$= a(x-1)(x+2)\left(x^2+4\right)$$
$$f(1) = 10 = a(-2)(1)(5) \Rightarrow a = -1$$
$$f(x) = -(x-1)(x+2)(x-2i)(x+2i)$$

(b) $f(x) = -(x-1)(x+2)\left(x^2+4\right)$
$$= -\left(x^2+x-2\right)\left(x^2+4\right)$$
$$= -x^4 - x^3 - 2x^2 - 4x + 8$$

52. (a) $f(x) = a(x+1)(-2)(x-i)(x+i)$
$$= a(x+1)(x-2)\left(x^2+1\right)$$
$$f(1) = 8 = a(2)(-1)(2) \Rightarrow a = -2$$
$$f(x) = -2(x+1)(x-2)(x-i)(x+i)$$

(b) $f(x) = -2\left(x^2-x-2\right)\left(x^2+1\right)$
$$= -2x^4 + 2x^3 + 2x^2 + 2x + 4$$

53. (a) $f(x) = a(x+1)\left(x-2-\sqrt{5}i\right)\left(x-2+\sqrt{5}i\right)$
$$= a(x+1)\left(x^2-4x+4+5\right)$$
$$= a(x+1)\left(x^2-4x+9\right)$$
$$f(-2) = 42 = a(-1)(4+8+9) \Rightarrow a = -2$$
$$f(x) = -2(x+1)\left(x-2-\sqrt{5}i\right)\left(x-2+\sqrt{5}i\right)$$

(b) $f(x) = -2(x+1)\left(x^2-4x+9\right)$
$$= -2x^3 + 6x^2 - 10x - 18$$

54. (a) $f(x) = a(x+2)\left(x-2-2\sqrt{2}i\right)\left(x-2+2\sqrt{2}i\right)$
$$= a(x+2)\left(x^2-4x+4+8\right)$$
$$= a(x+2)\left(x^2-4x+12\right)$$
$$f(-1) = -34 = a(1)(17) \Rightarrow a = -2$$
$$f(x) = -2(x+2)\left(x-2-2\sqrt{2}i\right)\left(x-2+2\sqrt{2}i\right)$$

(b) $f(x) = -2(x+2)\left(x^2-4x+12\right)$
$$= -2x^3 + 4x^2 - 8x - 48$$

55. $f(x) = x^4 - 6x^2 - 7$

(a) $f(x) = \left(x^2-7\right)\left(x^2+1\right)$

(b) $f(x) = \left(x-\sqrt{7}\right)\left(x+\sqrt{7}\right)\left(x^2+1\right)$

(c) $f(x) = \left(x-\sqrt{7}\right)\left(x+\sqrt{7}\right)(x+i)(x-i)$

56. $f(x)=x^4+6x^2-27$

(a) $f(x)=\left(x^2+9\right)\left(x^2-3\right)$

(b) $f(x)=\left(x^2+9\right)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$

(c) $f(x)=\left(x+3i\right)\left(x-3i\right)\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$

57. $f(x)=x^4-2x^3-3x^2+12x-18$

(a) $f(x)=\left(x^2-6\right)\left(x^2-2x+3\right)$

(b) $f(x)=\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)\left(x^2-2x+3\right)$

(c) $f(x)=\left(x+\sqrt{6}\right)\left(x-\sqrt{6}\right)\left(x-1-\sqrt{2}i\right)\left(x-1+\sqrt{2}i\right)$

58. $f(x)=x^4-3x^3-x^2-12x-20$

(a) $f(x)=\left(x^2+4\right)\left(x^2-3x-5\right)$

(b) $f(x)=\left(x^2+4\right)\left(x-\dfrac{3+\sqrt{29}}{2}\right)\left(x-\dfrac{3-\sqrt{29}}{2}\right)$

(c) $f(x)=\left(x+2i\right)\left(x-2i\right)\left(x-\dfrac{3+\sqrt{29}}{2}\right)\left(x-\dfrac{3-\sqrt{29}}{2}\right)$

59. $f(x)=2x^3+3x^2+50x+75$

Since is $5i$ a zero, so is $-5i$.

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75 \\ & & 10i & -50+15i & -75 \\ \hline & 2 & 3+10i & 15i & 0 \end{array}$$

$$\begin{array}{r|rrr} -5i & 2 & 3+10i & 15i \\ & & -10i & -15i \\ \hline & 2 & 3 & 0 \end{array}$$

The zero of $2x+3$ is $x=-\dfrac{3}{2}$. The zeros of f are $x=-\dfrac{3}{2}$ and $x=\pm 5i$.

Alternate solution

Since $x=\pm 5i$ are zeros of $f(x)$, $(x+5i)(x-5i)=x^2+25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r} 2x+3 \\ x^2+0x+25\ \overline{)\,2x^3+3x^2+50x+75} \\ \underline{2x^3+0x^2+50x} \\ 3x^2+0x+75 \\ \underline{3x^2+0x+75} \\ 0 \end{array}$$

Thus, $f(x)=\left(x^2+25\right)\left(2x+3\right)$ and the zeros of f are $x=\pm 5i$ and $x=-\dfrac{3}{2}$.

60. $f(x)=x^3+x^2+9x+9$

Since is $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|rrrr} 3i & 1 & 1 & 9 & 9 \\ & & 3i & -9+3i & -9 \\ \hline & 1 & 1+3i & 3i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3i & 1 & 1+3i & 3i \\ & & -3i & -3i \\ \hline & 1 & 1 & 0 \end{array}$$

The zeros of f are $3i, -3i,$ and -1 .

61. $g(x)=x^3-7x^2-x+87$. Since $5+2i$ is a zero, so is $5-2i$.

$$\begin{array}{r|rrrr} 5+2i & 1 & -7 & -1 & 87 \\ & & 5+2i & -14+6i & -87 \\ \hline & 1 & -2+2i & -15+6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5-2i & 1 & -2+2i & -15+6i \\ & & 5-2i & 15-6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x+3$ is $x=-3$.
The zeros of f are $-3, 5\pm 2i$. .

62. $g(x)=4x^3+23x^2+34x-10$

Since $-3+i$ is a zero, so is $-3-i$

$$\begin{array}{r|rrrr} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zero of $4x-1$ is $x=\dfrac{1}{4}$. The zeros of $g(x)$ are $x=-3\pm i$ and $x=\dfrac{1}{4}$.

Alternate solution

Since $-3\pm i$ are zeros of $g(x)$,

$[x-(-3+i)][x-(-3-i)]=[(x-3)-i][(x+3)+i]$

$=(x+3)^2-i^2=x^2+6x+10$

is a factor of $g(x)$. By long division we have:

$$\begin{array}{r} 4x-1 \\ x^2+6x+10\ \overline{)\,4x^3+23x^2+34x-10} \\ \underline{4x^3+24x^2+40x} \\ -x^2-6x-10 \\ \underline{-x^2-6x-10} \\ 0 \end{array}$$

Thus, $g(x)=(x^2+6x+10)(4x-1)$ and the zeros of g are $x=-3\pm i$ and $x=\dfrac{1}{4}$.

63. $h(x)=3x^3-4x^2+8x+8$ Since $1-\sqrt{3}i$ is a zero, so is $1+\sqrt{3}i$.

$$\begin{array}{r|cccc} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|ccc} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x+2$ is $x=-\dfrac{2}{3}$. The zeros of h are $x=-\dfrac{2}{3}, 1\pm\sqrt{3}i$.

64. $f(x)=x^3+4x^2+14x+20$

Since $-1-3i$ is a zero, so is $-1+3i$.

$$\begin{array}{r|cccc} -1-3i & 1 & 4 & 14 & 20 \\ & & -1-3i & -12-6i & -20 \\ \hline & 1 & 3-3i & 2-6i & 0 \end{array}$$

$$\begin{array}{r|ccc} -1+3i & 1 & 3-3i & 2-6i \\ & & -1+3i & -2+6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x+2$ is $x=-2$. The zeros of f are $x=-2, -1\pm 3i$.

65. $h(x)=8x^3-14x^2+18x-9$. Since $\dfrac{1}{2}(1-\sqrt{5}i)$ is a zero, so is $\dfrac{1}{2}(1+\sqrt{5}i)$.

$$\begin{array}{r|cccc} \frac{1}{2}\left(1-\sqrt{5}i\right) & 8 & -14 & 18 & -9 \\ & & 4-4\sqrt{5}i & -15+3\sqrt{5}i & 9 \\ \hline & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i & 0 \end{array}$$

$$\begin{array}{r|ccc} \frac{1}{2}\left(1+\sqrt{5}i\right) & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i \\ & & 4+4\sqrt{5}i & -3-3\sqrt{5}i \\ \hline & 8 & -6 & 0 \end{array}$$

The zero of $8x-6$ is $x=\dfrac{3}{4}$. The zeros of h are $x=\dfrac{3}{4},\ \dfrac{1}{2}(1\pm\sqrt{5}i)$.

66. $f(x)=25x^3-55x^2-54x-18$

Since $\dfrac{1}{5}(-2+\sqrt{2}i)=\dfrac{-2+\sqrt{2}i}{5}$ is a zero, so is $\dfrac{-2-\sqrt{2}i}{5}$.

$$\begin{array}{r|cccc} \frac{-2+\sqrt{2}i}{5} & 25 & -55 & -54 & -18 \\ & & -10+5\sqrt{2}i & 24-15\sqrt{2}i & 18 \\ \hline \end{array}$$

$$\begin{array}{r|cccc} \frac{-2-\sqrt{2}i}{5} & 25 & -65+5\sqrt{2}i & -30-15\sqrt{2}i & 0 \\ & & -10-5\sqrt{2}i & 30+15\sqrt{2}i & \\ \hline & 25 & -75 & 0 & \end{array}$$

The zero of $25x-75$ is $x=3$. The zeros of f are $x=3,\ \dfrac{-2\pm\sqrt{2}i}{5}$.

67. $f(x)=x^4+3x^3-5x^2-21x+22$

(a) The *root* feature yields the real roots 1 and 2, and the complex roots $-3\pm 1.414i$.

(b) By synthetic division:

$$\begin{array}{r|ccccc} 1 & 1 & 3 & -5 & -21 & 22 \\ & & 1 & 4 & -1 & -22 \\ \hline & 1 & 4 & -1 & -22 & 0 \end{array}$$

$$\begin{array}{r|cccc} 2 & 1 & 4 & -1 & -22 \\ & & 2 & 12 & 22 \\ \hline & 1 & 6 & 11 & 0 \end{array}$$

The complex roots of $x^2+6x+11$ are

$x=\dfrac{-6\pm\sqrt{6^2-4(11)}}{2}=-3\pm\sqrt{2}i.$

68. $f(x)=x^3+4x^2+14x+20$

(a) The *root* feature yields the real root – 2 and the complex roots $-1\pm 3i$.

(b) By synthetic division:

$$\begin{array}{r|cccc} -2 & 1 & 4 & 14 & 20 \\ & & -2 & -4 & -20 \\ \hline & 1 & 2 & 10 & 0 \end{array}$$

The complex roots of $x^2+2x+10$ are

$x=\dfrac{-2\pm\sqrt{4-4(1)(10)}}{2}=-1\pm 3i.$

69. $h(x)=8x^3-14x^2+18x-9$

(a) The *root* feature yields the real root 0.75, and the complex roots $0.5\pm 1.118i$.

(b) By synthetic division:

$$\begin{array}{r|cccc} \frac{3}{4} & 8 & -14 & 18 & -9 \\ & & 6 & -6 & 9 \\ \hline & 8 & -8 & 12 & 0 \end{array}$$

The complex roots of $8x^2-8x+12$ are

$x=\dfrac{8\pm\sqrt{64-4(8)(12)}}{2(8)}=\dfrac{1}{2}\pm\dfrac{\sqrt{5}}{2}i.$

70. $f(x) = 25x^3 - 55x^2 - 54x - 18$

(a) The *root* feature yields the real root 3 and the complex roots $-0.4 \pm 0.2828i$.

(b) By synthetic division:

$$\begin{array}{r|rrrr} 3 & 25 & -55 & -54 & -18 \\ & & 75 & 60 & 18 \\ \hline & 25 & 20 & 6 & 0 \end{array}$$

The complex roots of $25x^2 + 20x + 6$ are

$$x = \frac{-20 \pm \sqrt{400 - 4(25)6}}{2(25)} = \frac{-2 \pm \sqrt{2}i}{5}.$$

71. To determine if the football reaches a height of 50 feet, set $h(t) = 50$ and solve for t.

$$-16t^2 + 48t = 50$$
$$-16t^2 + 48t - 50 = 0$$

Using the Quadratic formula:

$$t = \frac{-(48) \pm \sqrt{48^2 - 4(-16)(-50)}}{2(-16)}$$
$$t = \frac{-(48) \pm \sqrt{-896}}{-32}$$

Because the discriminant is negative, the solutions are not real, therefore the football does not reach a height of 50 feet.

72. First find the revenue quation, $R = xP$.

$$R = x(140 - 0.001x)$$
$$R = 140x - 0.001x^2$$

Then $P = R - C$

$$P = (140x - 0.001x^2) - (40x + 150{,}000)$$
$$P = -0.001x^2 + 100x - 150{,}000$$

Next, set $P = 3{,}000{,}000$ and solve for x.

$$-0.001x^2 + 100x - 150{,}000 = 3{,}000{,}000$$
$$-0.001x^2 + 100x - 3{,}150{,}000 = 0$$

Using the Quadratic Formula

$$x = \frac{-(100) \pm \sqrt{(100)^2 - 4(-0.001)(-3{,}150{,}000)}}{2(-0.001)}$$
$$x = \frac{-100 \pm \sqrt{-260}}{-0.002}$$

Because the discriminant is negative, the solutions are not real. Therefore, there is no price P that will yield a profit of \$3 million.

73. False, a third-degree polynomial must have at least one real zero.

74. False. Because complex zeros occur in conjugate pairs, $[x + (4 - 3i)]$ is also a factor of f.

75. Answers will vary.

76. $f(x) = x^4 - 4x^2 + k$

$$x^2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(k)}}{2(1)} = \frac{4 \pm 2\sqrt{4-k}}{2} = 2 \pm \sqrt{4-k}$$
$$x = \pm\sqrt{2 \pm \sqrt{4-k}}$$

(a) For there to be four distinct real roots, both $4 - k$ and $2 \pm \sqrt{4-k}$ must be positive. This occurs when $0 < k < 4$. So, some possible k-values are $k = 1,\ k = 2,\ k = 3,\ k\frac{1}{2},\ k = \sqrt{2}$, etc.

(b) For there to be two real roots, each of multiplicity 2, $4 - k$ must equal zero. So, $k = 4$.

(c) For there to be two real zeros and two complex zeros, $2 + \sqrt{4-k}$ must be positive and $2 - \sqrt{4-k}$ must be negative. This occurs when $k < 0$. So, some possible k-values are $k = -1,\ k = -2,\ k = -\frac{1}{2}$, etc.

(d) For there to be four complex zeros, $2 \pm \sqrt{4-k}$ must be nonreal. This occurs when $k > 4$. Some possible k-values are $k = 5,\ k = 6,\ k = 7.4$, etc.

77. $f(x) = x^2 - 7x - 8$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$$

A parabola opening upward with vertex $\left(\frac{7}{2},\ -\frac{81}{4}\right)$

78. $f(x) = -x^2 + x + 6$

$$= -\left(x - \frac{1}{2}\right)^2 + \frac{25}{4}$$

A parabola opening downward with vertex $\left(\frac{1}{2},\ \frac{25}{4}\right)$

79. $f(x) = 6x^2 + 5x - 6$

$$= 6\left(x + \frac{5}{12}\right)^2 - \frac{169}{24}$$

A parabola opening upward with vertex $\left(-\frac{5}{12},\ -\frac{169}{24}\right)$

80. $f(x) = 4x^2 + 2x - 12$

$$= 4\left(x + \frac{1}{4}\right)^2 - \frac{49}{4}$$

A parabola opening upward with vertex $\left(-\frac{1}{4},\ -\frac{49}{4}\right)$

Section 2.6

1. rational functions

2. vertical asymptote

3. To determine the vertical asymptote(s) of the graph of $y = \frac{9}{x-3}$, find the real zeros of the denominator of the equation. (Assuming no common factors in the numerator and denominator)

4. No, the x-axis, $y = 0$, is the horizontal asymptote of the graph of $y = \frac{2x}{3x^2 - 5}$ because the numerator's degree is less than the denominator's degree

5. $f(x) = \frac{1}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	–2
0.9	–10
0.99	–100
0.999	–1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	$0.\overline{01}$
1000	$0.\overline{001}$

x	$f(x)$
–5	$-0.1\overline{6}$
–10	$-0.\overline{09}$
–100	$-0.\overline{0099}$
–1000	$-0.\overline{00099}$

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

6. $f(x) = \frac{5x}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	–5
0.9	–45
0.99	–495
0.999	–4995

x	$f(x)$
1.5	15
1.1	55
1.01	505
1.001	5005

x	$f(x)$
5	6.25
10	$5.\overline{55}$
100	$5.\overline{05}$
1000	$5.\overline{005}$

x	$f(x)$
–5	$4.1\overline{6}$
–10	$4.\overline{54}$
–100	$4.\overline{950495}$
–1000	4.995

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

7. $f(x) = \frac{3x}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	3
0.9	27
0.99	297
0.999	2997

x	$f(x)$
1.5	9
1.1	33
1.01	303
1.001	3003

x	$f(x)$
5	3.75
10	$3.\overline{33}$
100	$3.\overline{03}$
1000	$3.\overline{003}$

x	$f(x)$
–5	–2.5
–10	–2.727
–100	–2.970
–1000	–2.997

(c) f approaches ∞ from both the left and the right of 1.

8. $f(x) = \frac{3}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	6
0.9	30
0.99	300
0.999	3000

x	$f(x)$
1.5	6
1.1	30
1.01	300
1.001	3000

x	$f(x)$
5	0.75
10	$0.\overline{33}$
100	$0.\overline{03}$
1000	$0.\overline{003}$

x	$f(x)$
–5	0.5
–10	$0.\overline{27}$
–100	0.0297
–1000	0.003

(c) f approaches ∞ from both the left and the right of 1.

9. $f(x) = \frac{3x^2}{x^2 - 1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	–1
0.9	–12.79
0.99	–147.8
0.999	–1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502.3

x	$f(x)$
5	3.125
10	$3.0\overline{3}$
100	$3.0\overline{003}$
1000	3

x	$f(x)$
–5	3.125
–10	$3.0\overline{3}$
–100	$3.0\overline{003}$
–1000	3

(c) f approaches $-\infty$ from the left of 1, and ∞ from the right of 1. f approaches ∞ from the left of –1, and $-\infty$ from the right of –1.

10. $f(x) = \dfrac{4x}{x^2 - 1}$

(a) Domain: all $x \neq \pm 1$

(b)

x	$f(x)$
0.5	$-2.\overline{66}$
0.9	–18.95
0.99	–199
0.999	–1999

x	$f(x)$
1.5	4.8
1.1	20.95
1.01	201
1.001	2001

x	$f(x)$
5	$-0.8\overline{33}$
10	$0.\overline{40}$
100	0.04
1000	0.004

x	$f(x)$
–5	$-0.8\overline{33}$
–10	$-0.\overline{40}$
–100	–0.04
–1000	–0.004

(c) f approaches $-0.8\overline{33}$ ∞ from the left of 1, and ∞ from the right of 1. f approaches $-\infty$ from the left of
–1, and ∞ from the right of –1.

11. $f(x) = \dfrac{2}{x+2}$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$
Matches graph (a).

12. $f(x) = \dfrac{1}{x-3}$

Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$
Matches graph (d).

13. $f(x) = \dfrac{4x+1}{x}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 4$
Matches graph (c).

14. $f(x) = \dfrac{1-x}{x}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = -1$
Matches graph (e).

15. $f(x) = \dfrac{x-2}{x-4}$

Vertical asymptote: $x = 4$
Horizontal asymptote: $y = 1$
Matches graph (b).

16. $f(x) = -\dfrac{x+2}{x+4}$

Vertical asymptote: $x = -4$
Horizontal asymptote: $y = -1$
Matches graph (f).

17. $f(x) = \dfrac{1}{x^2}$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$ or x-axis

18. $f(x) = \dfrac{3}{(x-2)^2} = \dfrac{3}{x^2 - 4x + 4}$

Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 0$ or x-axis

19. $f(x) = \dfrac{2x^2}{x^2 + x - 6} = \dfrac{2x^2}{(x+3)(x-2)}$

Vertical asymptotes: $x = -3,\ x = 2$
Horizontal asymptote: $y = 2$

20. $f(x) = \dfrac{x^2 - 4x}{x^2 - 4} = \dfrac{x(x-4)}{(x+2)(x-2)}$

Vertical asymptotes: $x = -2,\ x = 2$
Horizontal asymptote: $y = 1$

21. $f(x) = \dfrac{x(2+x)}{2x - x^2} = \dfrac{\cancel{x}(x+2)}{-\cancel{x}(x-2)} = -\dfrac{x+2}{x-2},\ x \neq 0$

Vertical asymptote: $x = 2$
Horizontal asymptote: $y = -1$
Hole at $x = 0$

22. $f(x) = \dfrac{x^2 + 2x + 1}{2x^2 - x - 3} = \dfrac{(x+1)\cancel{(x+1)}}{(2x-3)\cancel{(x+1)}} = \dfrac{x+1}{2x-3},\ x \neq -1$

Vertical asymptote: $x = 3/2$
Horizontal asymptote: $y = 1/2$
Hole at $x = -1$

23. $f(x) = \dfrac{x^2 - 25}{x^2 + 5x} = \dfrac{\cancel{(x+5)}(x-5)}{x\cancel{(x+5)}} = \dfrac{x-5}{x},\ x \neq -5$

Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 1$
Hole at $x = -5$

24. $f(x) = \dfrac{3 - 14x - 5x^2}{3 + 7x + 2x^2} = \dfrac{-\cancel{(x+3)}(5x-1)}{\cancel{(x+3)}(2x+1)} = \dfrac{-(5x-1)}{2x+1}$

Vertical asymptote: $x = -1/2$
Horizontal asymptote: $y = -5/2$
Hole at $x = -3$

25. $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$

(a) Domain: all real numbers
(b) Continuous
(c) Vertical asymptote: none
Horizontal asymptote: $y = 3$

26. $f(x) = \dfrac{3x^2 + 1}{x^2 + x + 9}$

(a) Domain: all real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]
(b) Continuous
(c) Vertical asymptote: none
Horizontal asymptote: $y = 3$

27. $f(x) = \dfrac{x^2 + 3x - 4}{-x^3 + 27} = \dfrac{(x+4)(x-1)}{-(x-3)(x^2+3x+9)}$

(a) Domain: all real numbers x except $x = 3$
(b) Not continuous at $x = 3$
(c) Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$ or x-axis

28. $f(x) = \dfrac{4x^3 - x^2 + 3}{3x^3 + 24} = \dfrac{4x^3 - x^2 + 3}{3(x+2)(x^2 - 2x + 4)}$

(a) Domain: all real numbers x except $x = -2$
(b) Not continuous at $x = -2$
(c) Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 4/3$

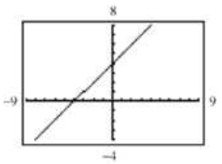
29. $f(x) = \dfrac{x^2 - 16}{x - 4} = \dfrac{(x+4)\cancel{(x-4)}}{\cancel{x-4}} = x + 4,\ x \neq 4$

$g(x) = x + 4$

(a) Domain of f: all real x except $x = 4$
Domain of g: all real x
(b) Vertical asymptote:
f has none. g has none.
Hole: f has a hole at $x = 4$; g has none.
(c)

x	1	2	3	4	5	6	7
$f(x)$	5	6	7	Undef.	9	10	11
$g(x)$	5	6	7	8	9	10	11

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

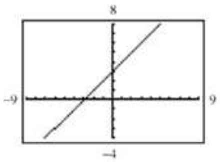
30. $f(x) = \dfrac{x^2 - 9}{x - 3} = \dfrac{(x+3)\cancel{(x-3)}}{\cancel{x-3}} = x + 3,\ x \neq 3$

$g(x) = x + 3$

(a) Domain of f: all real x except $x = 3$
Domain of g: all real x
(b) Vertical asymptote: f has none.
g has none.
Hole: f has a hole at $x = 3$; g has none.
(c)

x	0	1	2	3	4	5	6
$f(x)$	3	4	5	Undef.	7	8	9
$g(x)$	3	4	5	6	7	8	9

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

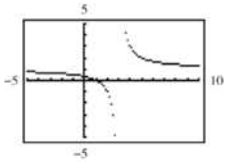
31. $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3} = \dfrac{\cancel{(x+1)}(x-1)}{(x-3)\cancel{(x+1)}} = \dfrac{x-1}{x-3},\ x \neq -1$

$g(x) = \dfrac{x-1}{x-3}$

(a) Domain of f: all real x except $x = 3$ and $x = -1$
Domain of g: all real x except $x = 3$
(b) Vertical asymptote: f has a vertical asymptote at $x = 3$.
g has a vertical asymptote at $x = 3$.
Hole: f has a hole at $x = -1$; g has none.
(c)

x	−2	−1	0	1	2	3	4
$f(x)$	$\frac{3}{5}$	Undef.	$\frac{1}{3}$	0	−1	Undef.	3
$g(x)$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	0	−1	Undef.	3

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

32. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x-1)} = \dfrac{x+2}{x-1},\ x \neq 2$

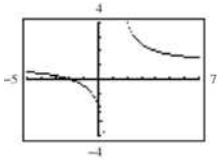
$g(x) = \dfrac{x+2}{x-1}$

(a) Domain of f: all real x except $x = 1$ and $x = 2$
Domain of g: all real x except $x = 1$
(b) Vertical asymptote: f has a vertical asymptote at $x = 1$.
g has a vertical asymptote at $x = 1$.
Hole: f has a hole at $x = 2$; g has none.

(c)

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	Undef.	Undef.	$\frac{5}{2}$
$g(x)$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	Undef.	4	$\frac{5}{2}$

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

33. $f(x) = 4 - \dfrac{1}{x}$

As $x \to \pm\infty$, $f(x) \to 4$.

As $x \to \infty$, $f(x) \to 4$ but is less than 4.

As $x \to -\infty$, $f(x) \to 4$ but is greater than 4.

34. $f(x) = 2 + \dfrac{1}{x-3}$

As $x \to \pm\infty$, $f(x) \to 2$.

As $x \to \infty$, $f(x) \to 2$ but is greater than 2.

As $x \to -\infty$, $f(x) \to 2$ but is less than 2.

35. $f(x) = \dfrac{2x-1}{x-3}$

As $x \to \pm\infty$, $f(x) \to 2$.

As $x \to \infty$, $f(x) \to 2$ but is greater than 2.

As $x \to -\infty$, $f(x) \to 2$ but is less than 2.

36. $f(x) = \dfrac{2x-1}{x^2+1}$

As $x \to \pm\infty$, $f(x) \to 0$. .

As $x \to \infty$, $f(x) \to 0$ but is greater than 0.

As $x \to -\infty$, $f(x) \to 0$ but is less than 0.

37. $g(x) = \dfrac{x^2-4}{x+3} = \dfrac{(x-2)(x+2)}{x+3}$

The zeros of g correspond to the zeros of the numerator and are $x = \pm 2$.

38. $g(x) = \dfrac{x^3-8}{x^2+4}$

The zero of g corresponds to the zero of the numerator and is $x = 2$.

39. $f(x) = 1 - \dfrac{2}{x-5} = \dfrac{x-7}{x-5}$

The zero of f corresponds to the zero of the numerator and is $x = 7$.

40. $h(x) = 5 + \dfrac{3}{x^2+1}$

There are no real zeros.

41. $g(x) = \dfrac{x^2-2x-3}{x^2+1} = \dfrac{(x-3)(x+1)}{x^2+1} = 0$

Zeros: $x = -1, 3$

42. $g(x) = \dfrac{x^2-5x+6}{x^2+4} = \dfrac{(x-3)(x-2)}{x^2+4} = 0$

Zeros: $x = 2, 3$

43. $f(x) = \dfrac{2x^2-5x+2}{2x^2-7x+3} = \dfrac{(2x-1)(x-2)}{(2x-1)(x-3)} = \dfrac{x-2}{x-3}, \; x \ne \dfrac{1}{2}$

Zero: $x = 2$ ($x = \dfrac{1}{2}$ is not in the domain.)

44. $f(x) = \dfrac{2x^2+3x-2}{x^2+x-2} = \dfrac{(x+2)(2x-1)}{(x+2)(x+1)} = \dfrac{2x-1}{x+1}, \; x = -2$

Zero: $x = \dfrac{1}{2}$ ($x = -2$ is not in the domain.)

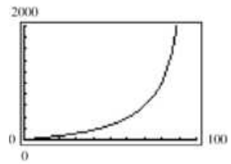
45. $C = \dfrac{255p}{100-p}, \; 0 \le p < 100$

(a) find $C(10) = \dfrac{255(10)}{100-10} = \$28.3$ million

$C(40) = \dfrac{255(40)}{100-40} = \$170$ million

$C(75) = \dfrac{255(75)}{100-75} = \$765$ million

(b)



(c) No. The function is undefined at $p = 100\,\%$.

46.

(a) Use data $\left(16, \dfrac{1}{3}\right), \left(32, \dfrac{1}{4.7}\right), \left(44, \dfrac{1}{9.8}\right), \left(50, \dfrac{1}{19.7}\right), \left(60, \dfrac{1}{39.4}\right)$.

$$\frac{1}{y} = -0.007x + 0.445$$

$$y = \frac{1}{0.445 - 0.007x}$$

(b)

x	16	32	44	50	60
y	3.0	4.5	7.3	10.5	40

(Answers will vary.)

(c) No, the function is negative for $x = 70$.

47.

(a)

M	200	400	600	800	1000	1200	1400	1600	1800	2000
t	0.472	0.596	0.710	0.817	0.916	1.009	1.096	1.178	1.255	1.328

The greater the mass, the more time required per oscillation. The model is a good fit to the actual data.

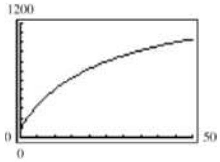
(b) You can find M corresponding to $t = 1.056$ by finding the point of intersection of

$$t = \frac{38M + 16{,}965}{10(M + 500)} \text{ and } t = 1.056.$$

If you do this, you obtain $M \approx 1306$ grams.

48. $N = \dfrac{20(5+3t)}{1+0.04t},\ 0 \le t$

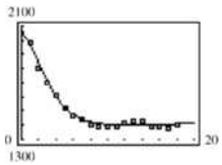
(a)



(b) $N(5) \approx 333$ deer
$N(10) = 500$ deer
$N(25) = 800$ deer

(c) The herd is limited by the horizontal asymptote:
$$N = \frac{60}{0.04} = 1500 \text{ deer}$$

49. (a) The model fits the data well.



(a) $N = \dfrac{77.095t^2 - 216.04t + 2050}{0.052t^2 - 0.8t + 1}$
$2009: N(19) \approx 1{,}412{,}000$
$2010: N(20) \approx 1{,}414{,}000$
$2011: N(11) \approx 1{,}416{,}000$
Answers will vary.

(b) Horizontal asymptote: $N = 1482.6$ (approximately) Answers will vary.

50. False. A rational function can have at most *n* vertical asymptotes, where *n* is the degree of the denominator.

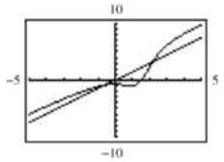
51. False. For example, $f(x) = \dfrac{1}{x^2+1}$ has no vertical asymptote.

52. If the domain of *f* (a rational function) is all real x except $x = c$, then either the graph of *f* has a vertical asymptote at $x = c$ (numerator and denominator do not have the like factor $x - c$) or the graph of f has a hole at $x = c$ (numerator and denominator do have like factor $x - c$) .

53. No. If $x = c$ is also a zero of the denominator of *f*, then *f* is undefined at $x = c$, and the graph of *f* may have a hole of vertical asymptote at $x = c$.

54. Yes. If the graph of *f*, a rational function, has a vertical asymptote at $x = 4$, then *f* may have a common factor of $x - 4$. For example,
$$f(x) = \frac{(x-4)(x+1)}{(x-4)(x-4)}.$$

55.



The graphs of $y_1 = \dfrac{3x^3 - 5x^2 + 4x - 5}{2x^2 - 6x + 7}$ and $y_2 = \dfrac{3x^3}{2x^2}$ are approximately the same graph as $x \to \infty$ and $x \to -\infty$.
Therefore as $x \to \pm\infty$, the graph of a rational function $y = \dfrac{a_n x^n + \ldots a_1 x + a_0}{b_m x^m + \ldots b_1 x + b_0}$ appears to be very close to the graph of $y = \dfrac{a_n x^n}{b_m x^m}$.

56.

(a) $f(x) = \dfrac{x-1}{x^3-8}$

(b) $f(x) = \dfrac{x-2}{(x+1)^2}$

(c) $f(x) = \dfrac{2(x+3)(x-3)}{(x+2)(x-1)} = \dfrac{2x^2-18}{x^2+x-2}$

(d) $f(x) = \dfrac{-2(x+2)(x-3)}{(x+1)(x-2)} = \dfrac{-2x^2+2x+12}{x^2-x-2}$

57.
$$\begin{aligned} y - 2 &= \frac{-1-2}{0-3}(x-3) = 1(x-3) \\ y &= x - 1 \\ y - x + 1 &= 0 \end{aligned}$$

58.
$$\begin{aligned} y - 1 &= \frac{1+5}{-6-4}(x+6) \\ -10y + 10 &= 6x + 36 \\ 3x + 5y + 13 &= 0 \end{aligned}$$

59.
$$\begin{aligned} y - 7 &= \frac{10-7}{3-2}(x-2) = 3(x-2) \\ y &= 3x + 1 \\ 3x - y + 1 &= 0 \end{aligned}$$

60.
$$\begin{aligned} y - 0 &= \frac{4-0}{-9-0}(x-0) \\ -9y &= 4x \\ 4x + 9y &= 0 \end{aligned}$$

61.
$$\begin{array}{r} x + 9 \\ x - 4 \overline{\smash{\big)}\, x^2 + 5x + 6} \\ \underline{x^2 - 4x} \\ 9x + 6 \\ \underline{9x - 36} \\ 42 \end{array}$$

$$\frac{x^2 + 5x + 6}{x - 4} = x + 9 + \frac{42}{x - 4}$$

62.

$$\begin{array}{r|rrr} 3 & 1 & -10 & 15 \\ & & 3 & -21 \\ \hline & 1 & -7 & -6 \end{array}$$

$$\frac{x^2-10x+15}{x-3} = x-7+\frac{-6}{x-3}$$

63.

$$\begin{array}{r} 2x^2-9 \\ x^2+5 \overline{\smash{)}2x^4+0x^3+x^2+0x-11} \\ \underline{2x^4+\quad 10x^2} \\ -9x^2-\quad 11 \\ \underline{-9x^2-\quad 45} \\ 34 \end{array}$$

$$\frac{2x^4+x^2-11}{x^2+5} = 2x^2-9+\frac{34}{x^2+5}$$

64.

$$\begin{array}{r} 2x^4-3x^3+6x^2-9x+\frac{27}{2} \\ 2x+3 \overline{\smash{)}4x^5+0x^4+3x^3+0x^2+0x-10} \\ \underline{4x^5+6x^4} \\ -6x^4+3x^3 \\ \underline{-6x^4-9x^3} \\ 12x^3 \\ \underline{12x^3+18x^2} \\ -18x^2 \\ \underline{-18x^2-27x} \\ 27x-10 \\ \underline{27x+\frac{81}{2}} \\ -\frac{101}{2} \end{array}$$

$$\frac{4x^5+3x^3-10}{2x+3} = 2x^4-3x^3+6x^2-9x+\frac{27}{2}-\frac{101}{4x+6}$$

Section 2.7

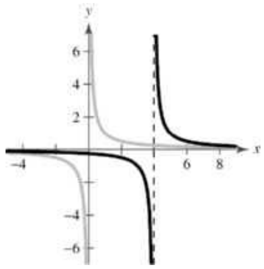
1. slant, asymptote

2. vertical

3. Yes. Because the numerator's degree is exactly 1 greater than that of the denominator, the graph of *f* has a slant asymptote.

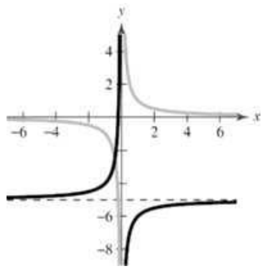
4. The slant asymptote's equation is that of the quotient, $y = x-1$. .

5.



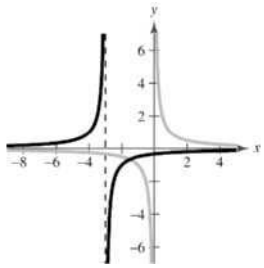
The graph of $g = \frac{1}{x-4}$ is a horizontal shift four units to the right of the graph of $f(x) = \frac{1}{x}$.

6.



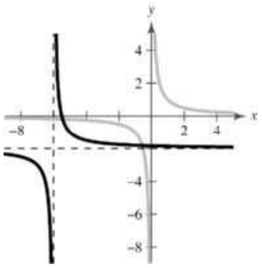
The graph of $g = \frac{-1}{x} - 5$ is a reflection in the *x*-axis and a vertical shift five units downward of the graph of $f(x) = \frac{1}{x}$.

7.



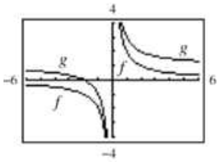
The graph of $g(x) = \frac{-1}{x+3}$ is a reflection in the *x*-axis and a horizontal shift three units to the left of the graph of $f(x) = \frac{1}{x}$.

8.



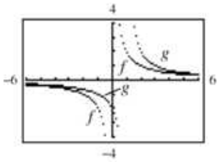
The graph of $g(x) = \dfrac{1}{x+6} - 2$ is a horizontal shift six units to the left and a vertical shift two units downward of the graph of $f(x) = \dfrac{1}{x}$.

9. $g(x) = \dfrac{2}{x} + 1$



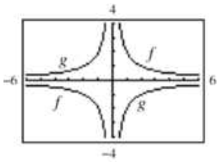
Vertical shift one unit upward

10. $g(x) = \dfrac{2}{x-1}$



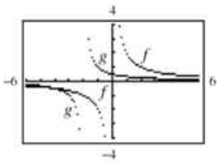
Horizontal shift one unit to the right

11. $g(x) = -\dfrac{2}{x}$



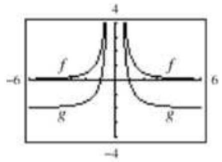
Reflection in the x-axis

12. $g(x) = \dfrac{1}{x+2}$



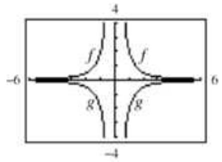
Horizontal shift two units to the left, and vertical shrink

13. $g(x) = \dfrac{2}{x^2} - 2$



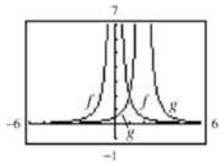
Vertical shift two units downward

14. $g(x) = -\dfrac{2}{x^2}$



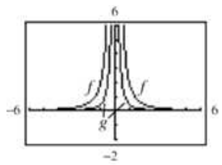
Reflection in the x–axis

15. $g(x) = \dfrac{2}{(x-2)^2}$



Horizontal shift two units to the right

16. $g(x) = \dfrac{1}{2x^2}$

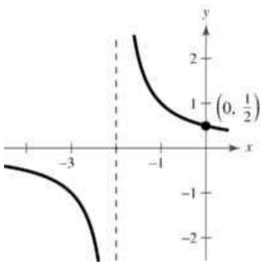


Vertical shrink

17. $f(x) = \dfrac{1}{x+2}$

y-intercept: $\left(0, \dfrac{1}{2}\right)$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$

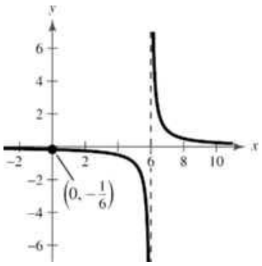


x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$

18. $f(x) = \dfrac{1}{x-6}$

y-intercept: $\left(0, -\dfrac{1}{6}\right)$

Vertical asymptote: $x = 6$
Horizontal asymptote: $y = 0$

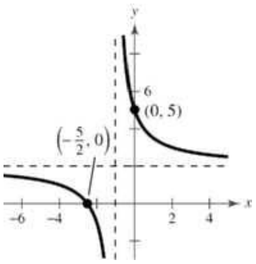


x	-1	0	2	4	8	10
y	$-\frac{1}{7}$	$-\frac{1}{6}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$

19. $C(x) = \dfrac{5+2x}{1+x} = \dfrac{2x+5}{x+1}$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

y-intercept: $(0, 5)$
Vertical asymptote: $x = -1$
Horizontal asymptote: $y = 2$

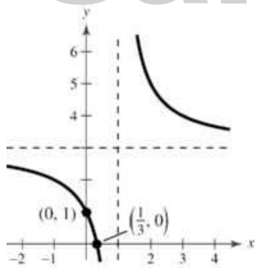


x	-4	-3	-2	0	1	2
$C(x)$	1	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3

20. $P(x) = \dfrac{1-3x}{1-x} = \dfrac{3x-1}{x-1}$

x-intercept: $\left(\dfrac{1}{3}, 0\right)$

y-intercept: $(0, 1)$
Vertical asymptote: $x = 1$
Horizontal asymptote: $y = 3$

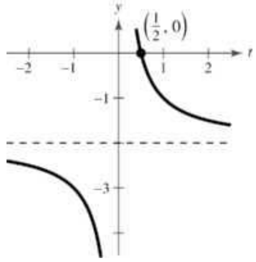


x	-1	0	2	3
y	2	1	5	4

21. $f(t) = \dfrac{1-2t}{t} = -\dfrac{2t-1}{t}$

t-intercept: $\left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $t = 0$
Horizontal asymptote: $y = -2$



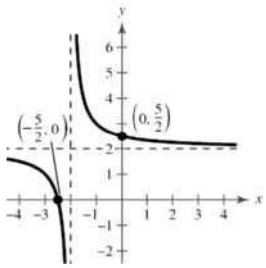
x	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$

22. $g(x) = \dfrac{1}{x+2} + 2 = \dfrac{2x+5}{x+2}$

y-intercept: $\left(0, \dfrac{5}{2}\right)$

x-intercept: $\left(-\dfrac{5}{2}, 0\right)$

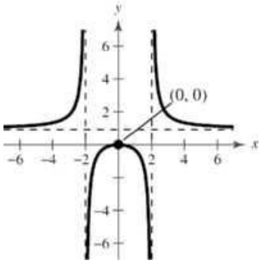
Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 2$



x	-4	$-\frac{5}{2}$	-1	0	2
y	$\frac{3}{2}$	0	3	$\frac{5}{2}$	$\frac{9}{4}$

23. $f(x) = \dfrac{x^2}{x^2 - 4}$

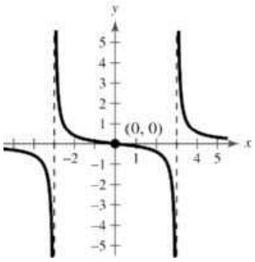
Intercept: $(0, 0)$
Vertical asymptotes: $x = 2,\ x = -2$
Horizontal asymptote: $y = 1$
y-axis symmetry



x	-4	-1	0	-1	4
y	$\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{4}{3}$

24. $g(x) = \dfrac{x}{x^2 - 9}$

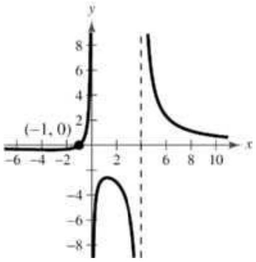
Intercepts: $(0, 0)$
Vertical asymptotes: $x = \pm 3$
Horizontal asymptote: $y = 0$
Origin symmetry



x	-5	-4	-2	0	2	4	5
y	$-\frac{5}{16}$	$-\frac{4}{7}$	$\frac{2}{5}$	0	$-\frac{2}{5}$	$\frac{4}{7}$	$\frac{5}{16}$

25. $g(x) = \dfrac{4(x+1)}{x(x-4)}$

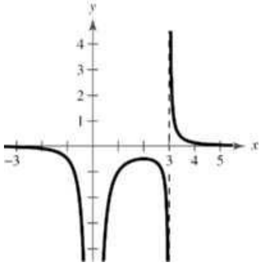
Intercept: $(-1, 0)$
Vertical asymptotes: $x = 0$ and $x = 4$
Horizontal asymptote: $y = 0$



x	-2	-1	1	2	3	5	6
y	$-\frac{1}{3}$	0	$-\frac{8}{3}$	-3	$-\frac{16}{3}$	$\frac{24}{5}$	$\frac{7}{3}$

26. $h(x) = \dfrac{2}{x^2(x-3)}$

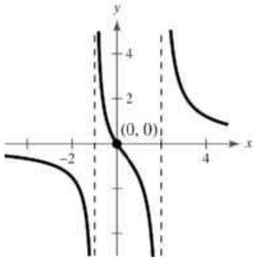
Vertical asymptotes: $x = 0,\ x = 3$
Horizontal asymptote: $y = 0$



x	-2	0	1	2	3	4
y	$-\frac{1}{10}$	Undef.	-1	$-\frac{1}{2}$	Undef.	$\frac{1}{8}$

27. $f(x) = \dfrac{3x}{x^2 - x - 2} = \dfrac{3x}{(x+1)(x-2)}$

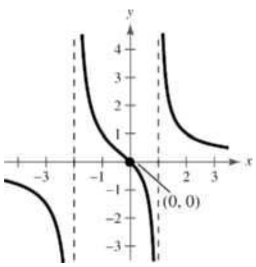
Intercept: $(0, 0)$
Vertical asymptotes: $x = -1,\ 2$
Horizontal asymptote: $y = 0$



x	-3	0	1	3	4
y	$-\frac{9}{10}$	0	$-\frac{3}{2}$	$\frac{9}{4}$	$\frac{6}{5}$

28. $f(x) = \dfrac{2x}{x^2 + x - 2} = \dfrac{2x}{(x+2)(x-1)}$

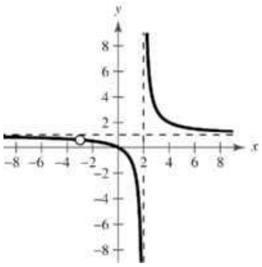
Intercept: $(0, 0)$
Vertical asymptotes: $x = -2,\ 1$
Horizontal asymptote: $y = 0$



x	-4	-3	-1	0	$\frac{1}{2}$	2	3
y	$-\frac{4}{5}$	$-\frac{3}{2}$	1	0	$-\frac{4}{5}$	1	$\frac{3}{5}$

29. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x-2)(x+3)} = \dfrac{x}{x-2},$

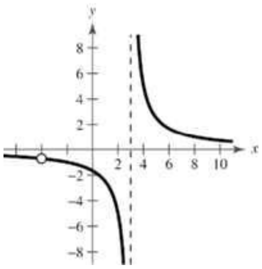
$x \neq -3$
Intercept: $(0, 0)$
Vertical asymptote: $x = 2$
(There is a hole at $x = -3$.)
Horizontal asymptote: $y = 1$



x	-2	-1	0	1	2	3
y	$\frac{1}{2}$	$\frac{1}{3}$	0	-1	Undef.	3

30. $g(x) = \dfrac{5(x+4)}{x^2 + x - 12} = \dfrac{5(x+4)}{(x+4)(x-3)} = \dfrac{5}{x-3},$

$x \neq -4$
Vertical asymptote: $x = 3$
Horizontal asymptote: $y = 0$

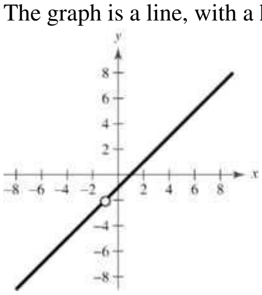


Hole at $x = -4$

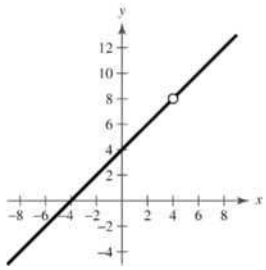
x	-4	0	1	3	4
y	Undef.	$-\frac{5}{3}$	$-\frac{5}{2}$	Undef.	5

31. $f(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x+1)(x-1)}{x+1} = x - 1,$

$x \neq -1$
The graph is a line, with a hole at $x = -1$.



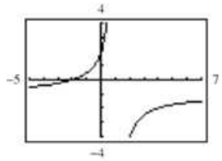
32. $f(x) = \dfrac{x^2 - 16}{x - 4} = x + 4,\ x \neq 4$



Hole at $x = 4$

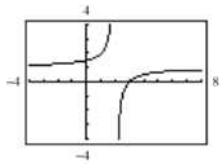
33. $f(x) = \dfrac{2 + x}{1 - x} = -\dfrac{x + 2}{x - 1}$

Vertical asymptote: $x = 1$
Horizontal asymptote: $y = -1$



Domain: $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

34. $f(x) = \dfrac{3 - x}{2 - x} = \dfrac{x - 3}{x - 2}$



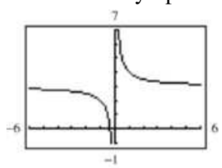
x-intercept: $(3, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$
Horizontal asymptote: $y = 1$
Domain: all $x \neq 2$

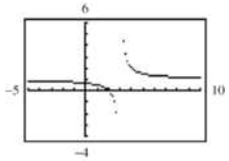
35. $f(t) = \dfrac{3t + 1}{t}$

Vertical asymptote: $t = 0$
Horizontal asymptote: $y = 3$



Domain: $t \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

36. $h(x) = \dfrac{x-2}{x-3}$



x-intercept: $(2, 0)$

y-intercept: $\left(0, \dfrac{2}{3}\right)$

Vertical asymptote: $x = 3$

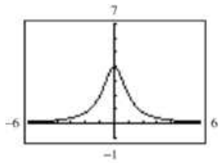
Horizontal asymptote: $y = 1$

Domain: all $x \neq 3$

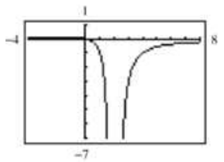
37. $h(t) = \dfrac{4}{t^2 + 1}$

Domain: all real numbers or $(-\infty, \infty)$

Horizontal asymptote: $y = 0$



38. $g(x) = -\dfrac{x}{(x-2)^2}$



Domain: all real numbers except 2 or $(-\infty, 2) \cup (2, \infty)$

Vertical asymptote: $x = 2$

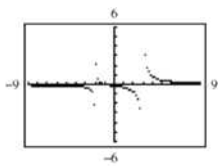
Horizontal asymptote: $y = 0$

39. $f(x) = \dfrac{x+1}{x^2 - x - 6} = \dfrac{x+1}{(x-3)(x+2)}$

Domain: all real numbers except $x = 3, \ -2$

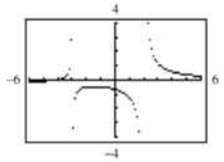
Vertical asymptotes: $x = 3, x = -2$

Horizontal asymptote: $y = 0$



40. $f(x) = \dfrac{x+4}{x^2 + x - 6}$

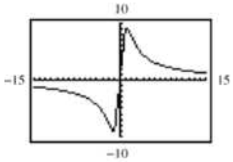
Domain: all real numbers except –3 and 2 or $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$



Vertical asymptotes: $x = -3, x = 2$

Horizontal asymptote: $y = 0$

41. $f(x) = \dfrac{20x}{x^2+1} - \dfrac{1}{x} = \dfrac{19x^2 - 1}{x(x^2+1)}$

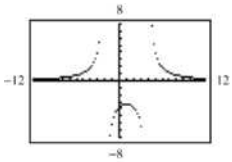


Domain: all real numbers except 0, or $(-\infty, 0) \cup (0, \infty)$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

42. $f(x) = 5\left(\dfrac{1}{x-4} - \dfrac{1}{x-2}\right) = \dfrac{30}{(x-4)(x+2)}$

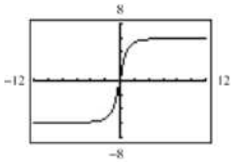


Domain: all real numbers except -2 and 4

Vertical asymptotes: $x = -2$, $x = 4$

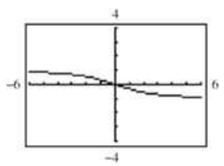
Horizontal asymptote: $y = 0$

43. $h(x) = \dfrac{6x}{\sqrt{x^2+1}}$



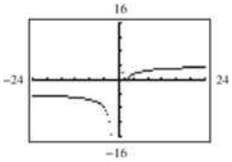
There are two horizontal asymptotes, $y = \pm 6$.

44. $f(x) = \dfrac{-x}{\sqrt{9 + x^2}}$



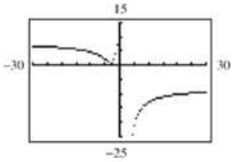
There are two horizontal asymptotes, $y = \pm 1$.

45. $g(x) = \dfrac{4|x-2|}{x+1}$



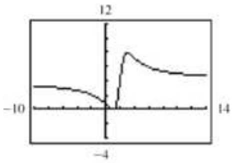
There are two horizontal asymptotes, $y = \pm 4$ and one vertical asymptote, $x = -1$.

46. $f(x) = \dfrac{-8|3+x|}{x-2} = \dfrac{8|3+x|}{2-x}$



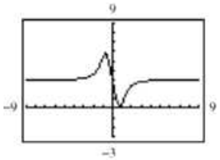
There are two horizontal asymptotes, $y = -8$ and $y = 8$, and one vertical asymptote, $x = 2$.

47. $f(x) = \dfrac{4(x-1)^2}{x^2 - 4x + 5}$



The graph crosses its horizontal asymptote, $y = 4$.

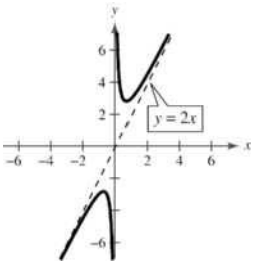
48. $g(x) = \dfrac{3x^4 - 5x + 3}{x^4 + 1}$



The graph crosses its horizontal asymptote, $y = 3$.

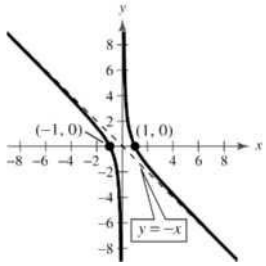
49. $f(x) = \dfrac{2x^2 + 1}{x} = 2x + \dfrac{1}{x}$

Vertical asymptote: $x = 0$
Slant asymptote: $y = 2x$
Origin symmetry



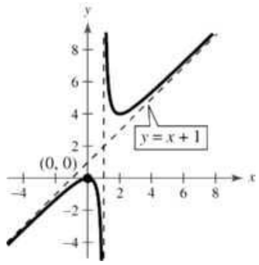
50. $g(x) = \dfrac{1 - x^2}{x}$

Intercepts: $(1, 0), (-1, 0)$
Vertical asymptote: $x = 0$
Slant asymptote: $y = -x$



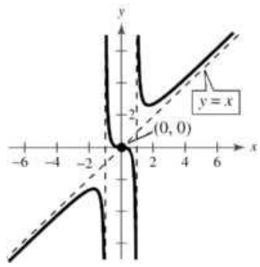
51. $h(x) = \dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$

Intercept: $(0, 0)$
Vertical asymptote: $x = 1$
Slant asymptote: $y = x + 1$



52. $f(x) = \dfrac{x^3}{x^2 - 1} = x + \dfrac{x}{x^2 - 1}$

Intercept: $(0, 0)$
Vertical asymptotes: $x = \pm 1$
Slant asymptote: $y = x$
Origin symmetry



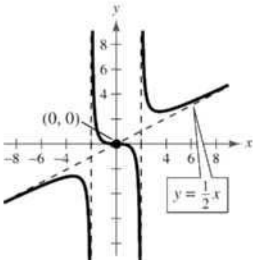
53. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

Intercept: $(0, 0)$

Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

Origin symmetry



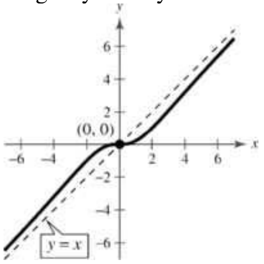
54. $f(x) = \dfrac{x^3}{x^2 + 4} = x - \dfrac{4x}{x^2 + 4}$

Intercept: $(0, 0)$

Vertical asymptotes: None

Slant asymptote: $y = x$

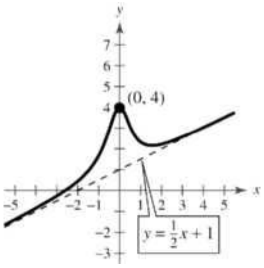
Origin symmetry



55. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1} = \dfrac{x}{2} + 1 + \dfrac{3 - \dfrac{x}{2}}{2x^2 + 1}$

Intercepts: $(-2.594, 0)$, $(0, 4)$

Slant asymptote: $y = \dfrac{x}{2} + 1$

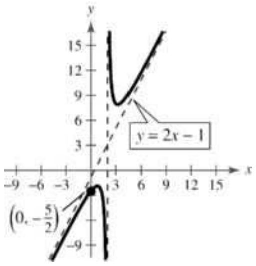


56. $f(x) \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

y-intercept: $\left(0, -\dfrac{5}{2}\right)$

Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x - 1$



57. $y = \dfrac{x + 1}{x - 3}$

x-intercept: $(-1, 0)$

$0 = \dfrac{x + 1}{x - 3}$

$0 = x + 1$

$-1 = x$

58. $y = \dfrac{2x}{x - 3}$

x-intercept: $(0, 0)$

$0 = \dfrac{2x}{x - 3}$

$0 = 2x$

$0 = x$

59. $y = \dfrac{1}{x} - x$

x-intercepts: $(\pm 1, 0)$

$0 = \dfrac{1}{x} - x$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$

60. $y = x - 3 + \dfrac{2}{x}$

x-intercepts: $(1, 0)$, $(2, 0)$

$0 = x - 3 + \dfrac{2}{x}$

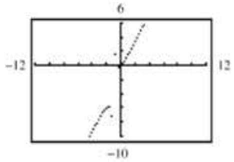
$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

$x = 1, 2$

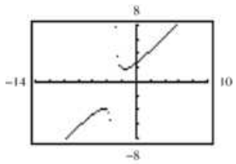
61. $y = \dfrac{2x^2 + x}{x+1} = 2x - 1 + \dfrac{1}{x+1}$

Domain: all real numbers except $x = -1$
Vertical asymptote: $x = -1$
Slant asymptote: $y = 2x - 1$



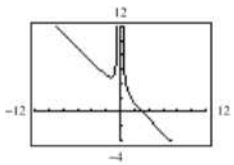
62. $y = \dfrac{x^2 + 5x + 8}{x+3} = x + 2 + \dfrac{2}{x+3}$

Domain: all $x \neq -3$
Vertical asymptote: $x = -3$
Slant asymptote: $y = x + 2$



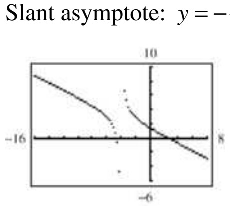
63. $y = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers except 0
or $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 0$
Slant asymptote: $y = -x + 3$



64. $y = \dfrac{12 - 2x - x^2}{2(4+x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4+x}$

Domain: all real numbers except –4 or
$(-\infty, -4) \cup (-4, \infty)$
x-intercepts: $(-4.61, 0)$, $(2.61, 0)$
y-intercept: $\left(0, \dfrac{3}{2}\right)$
Vertical asymptote: $x = -4$
Slant asymptote: $y = -\dfrac{1}{2}x + 1$



65. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x-4)(x-1)}{(x-2)(x+2)}$

Vertical asymptotes: $x = 2,\ x = -2$
Horizontal asymptote: $y = 1$
No slant asymptotes, no holes

66. $f(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9} = \dfrac{(x-4)(x+2)}{(x-3)(x+3)}$

Vertical asymptotes: $x = 3,\ x = -3$
Horizontal asymptote: $y = 1$
No slant asymptotes, no holes

67. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6} = \dfrac{(2x-1)(x-2)}{(2x+3)(x-2)} = \dfrac{2x-1}{2x+3},$

$x \neq 2$

Vertical asymptote: $x = -\dfrac{3}{2}$
Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 2,\ \left(2, \dfrac{3}{7}\right)$

68. $f(x) = \dfrac{3x^2 - 8x + 4}{2x^2 - 3x - 2} = \dfrac{(3x-2)(x-2)}{(2x+1)(x-2)} = \dfrac{3x-2}{2x+1},$

$x \neq 2$

Vertical asymptote: $x = -\dfrac{1}{2}$
Horizontal asymptote: $y = \dfrac{3}{2}$
No slant asymptotes
Hole at $x = 2,\ \left(2, \dfrac{4}{5}\right)$

69. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$= \dfrac{(x-1)(x+1)(2x-1)}{(x+1)(x+2)}$

$= \dfrac{(x-1)(2x-1)}{x+2},\ x \neq -1$

Long division gives

$f(x) = \dfrac{2x^2 - 3x + 1}{x+2} = 2x - 7 + \dfrac{15}{x+2}.$

Vertical asymptote: $x = -2$
No horizontal asymptote
Slant asymptote: $y = 2x - 7$
Hole at $x = -1,\ (-1, 6)$

70. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

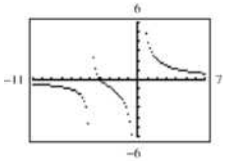
$= \dfrac{(x-2)(x+2)(2x+1)}{(x-2)(x-1)}$

$= \dfrac{(x+2)(2x+1)}{x-1},\ x \neq 2$

Long division gives $f(x) = 2x + 7 + \dfrac{9}{x-1}$.

Vertical asymptote: $x = 1$
No horizontal asymptote
Slant asymptote: $y = 2x + 7$
Hole at $x = 2,\ (2,\ 20)$

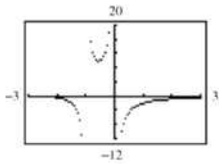
71. $y = \dfrac{1}{x+5} + \dfrac{4}{x}$



x-intercept: $(-4,\ 0)$

$$0 = \frac{1}{x+5} + \frac{4}{x}$$
$$-\frac{4}{x} = \frac{1}{x+5}$$
$$-4(x+5) = x$$
$$-4x - 20 = x$$
$$-5x = 20$$
$$x = -4$$

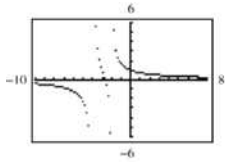
72. $y = \dfrac{2}{x+1} - \dfrac{3}{x}$



x-intercept: $(-3,\ 0)$

$$\frac{2}{x+1} - \frac{3}{x} = 0$$
$$\frac{2}{x+1} = \frac{3}{x}$$
$$2x = 3x + 3$$
$$-3 = x$$

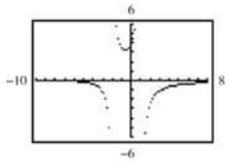
73. $y = \dfrac{1}{x+2} + \dfrac{2}{x+4}$



x-intercept: $\left(-\dfrac{8}{3},\ 0\right)$

$$\frac{1}{x+2} + \frac{2}{x+4} = 0$$
$$\frac{1}{x+2} = \frac{-2}{x+4}$$
$$x + 4 = -2x - 4$$
$$3x = -8$$
$$x = -\frac{8}{3}$$

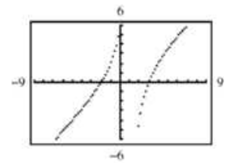
74. $y = \dfrac{2}{x+2} - \dfrac{3}{x-1}$



x-intercept: $(-8,\ 0)$

$$\frac{2}{x+2} = \frac{3}{x-1}$$
$$2x - 2 = 3x + 6$$
$$-8 = x$$

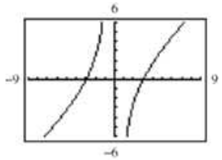
75. $y = x - \dfrac{6}{x-1}$



x-intercepts: $(-2, 0),\ (3, 0)$

$$0 = x - \frac{6}{x-1}$$
$$\frac{6}{x-1} = x$$
$$6 = x(x-1)$$
$$0 = x^2 - x - 6$$
$$0 = (x+2)(x-3)$$
$$x = -2,\ x = 3$$

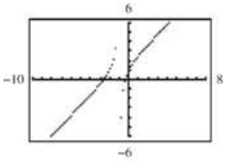
76. $y = x - \dfrac{9}{x}$



x-intercepts: $(-3, 0)$, $(3, 0)$

$$0 = x - \frac{9}{x}$$
$$\frac{9}{x} = x$$
$$9 = x^2$$
$$\pm 3 = x$$

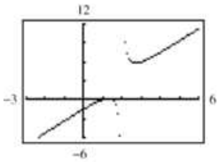
77. $y = x + 2 - \dfrac{1}{x+1}$



x-intercepts: $(-2.618, 0)$, $(-0.382, 0)$

$$x + 2 = \frac{1}{x+2}$$
$$x^2 + 3x + 2 = 1$$
$$x^2 + 3x + 1 = 0$$
$$x = \frac{-3 \pm \sqrt{9-4}}{2}$$
$$= -\frac{3}{2} \pm \frac{\sqrt{5}}{2}$$
$$\approx -2.618, -0.382$$

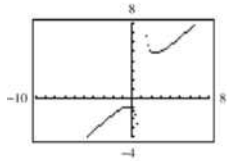
78. $y = 2x - 1 + \dfrac{1}{x-2}$



x-intercepts: $(1, 0)$, $\left(\dfrac{3}{2}, 0\right)$

$$2x - 1 + \frac{1}{x-2} = 0$$
$$\frac{1}{x-2} = 1 - 2x$$
$$1 = -2x^2 + 5x - 2$$
$$2x^2 - 5x + 3 = 0$$
$$(x-1)(2x-3) = 0$$
$$x = 1, \frac{3}{2}$$

79. $y = x + 1 + \dfrac{2}{x-1}$

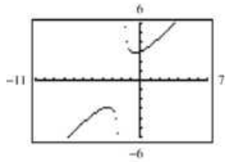


No x-intercepts

$$x + 1 + \frac{2}{x-1} = 0$$
$$\frac{2}{x-1} = -x - 1$$
$$2 = -x^2 + 1$$
$$x^2 + 1 = 0$$

No real zeros

80. $y = x + 2 + \dfrac{2}{x+2}$

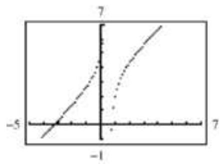


No x-intercepts

$$x + 2 + \frac{2}{x+2} = 0$$
$$\frac{2}{x+2} = -x - 2$$
$$2 = -x^2 - 4x - 4$$
$$x^2 + 4x + 6 = 0$$

Because $b^2 - 4ac = 16 - 24 < 0$, there are no real zeros.

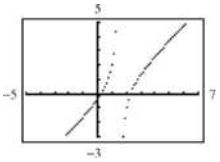
81. $y = x + 3 - \dfrac{2}{2x-1}$



x-intercepts: $(0.766, 0)$, $(-3.266, 0)$

$$x + 3 - \frac{2}{2x-1} = 0$$
$$x + 3 = \frac{2}{2x-1}$$
$$2x^2 + 5x - 3 = 2$$
$$2x^2 + 5x - 5 = 0$$
$$x = \frac{-5 \pm \sqrt{25 - 4(2)(-5)}}{4}$$
$$= \frac{-5 \pm \sqrt{65}}{4}$$
$$\approx 0.766, -3.266$$

82. $y = x - 1 - \dfrac{2}{2x-3}$



x-intercepts: $(0.219,\ 0)$, $(2.281,\ 0)$

$$x - 1 - \frac{2}{2x-3} = 0$$
$$x - 1 = \frac{2}{2x-3}$$
$$2x^2 - 5x + 3 = 2$$
$$2x^2 - 5x + 1 = 0$$
$$x = \frac{5 \pm \sqrt{25-8}}{4}$$
$$= \frac{5 \pm \sqrt{17}}{4} \approx 0.219,\ 2.281$$

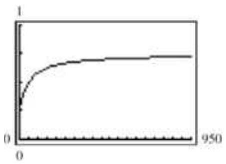
83.

(a) $0.25(50) + 0.75(x) = C(50 + x)$

$$\frac{12.5 + 0.75x}{50 + x} = C$$
$$\frac{50 + 3x}{200 + 4x} = C$$
$$C = \frac{3x + 50}{4(x + 50)}$$

(b) Domain: $x \ge 0$ and $x \le 1000 - 50 = 950$
Thus, $0 \le x \le 950$.

(c)



As the tank fills, the rate that the concentration is increasing slows down. It approaches the horizontal asymptote $C = \dfrac{3}{4} = 0.75$. When the tank is full $(x = 950)$, the concentration is $C = 0.725$.

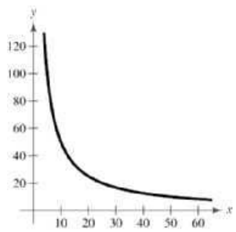
84.

(a) Area $= xy = 500$

$$y = \frac{500}{x}$$

(b) Domain: $x > 0$

(c)



For $x = 30$, $y = \dfrac{500}{30} = 16\dfrac{2}{3}$ meters.

85.

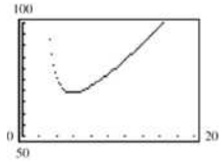
(a) $A = xy$ and

$$(x-2)(y-4) = 30$$
$$y - 4 = \frac{30}{x-2}$$
$$y = 4 + \frac{30}{x-2} = \frac{4x+22}{x-2}$$

Thus, $A = xy = x\left(\dfrac{4x+22}{x-2}\right) = \dfrac{2x(2x+11)}{x-2}$.

(b) Domain: Since the margins on the left and right are each 1 inch, $x > 2$, or $(2, \infty)$.

(c)



The area is minimum when $x \approx 5.87$ in. and $y \approx 11.75$ in.

86.

(a) The line passes through the points $(a,\ 0)$ and $(3,\ 2)$ and has a slope of

$$m = \frac{2-0}{3-a} = \frac{2}{3-a}.$$
$$y - 0 = \frac{2}{3-a}(x - a)$$
$$y = \frac{2(x-a)}{3-a} = \frac{-2(a-x)}{-1(a-3)} \text{ by the point-slope form}$$
$$= \frac{2(a-x)}{a-3},\ 0 \le x \le a$$

(b)

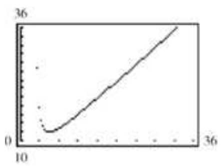
The area of a triangle is $A = \dfrac{1}{2}bh$.

$b = a$

$h = y$ when $x = 0$, so $h = \dfrac{2(a-0)}{a-3} = \dfrac{2a}{a-3}$.

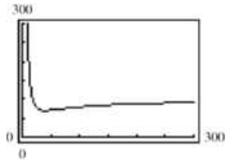
$$A = \frac{1}{2}a\left(\frac{2a}{a-3}\right) = \frac{a^2}{a-3}$$

(c) $A = \dfrac{a^2}{a-3} = a + 3 + \dfrac{9}{a-3}$
Vertical asymptote: $a = 3$



Slant asymptote: $A = a + 3$
A is a minimum when $a = 6$ and $A = 12$.

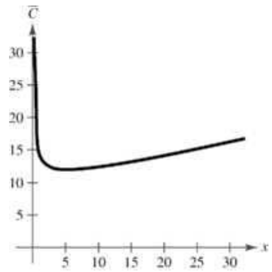
87. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right), \quad 1 \le x$



The minimum occurs when $x = 40.4 \approx 40$.

88. $\overline{C} = \dfrac{C}{x} = \dfrac{0.2x^2 + 10x + 5}{x}, \; x > 0$

x	0.5	1	2	3	4	5	6	7
$\overline{C}$	20.1	15.2	12.9	12.3				



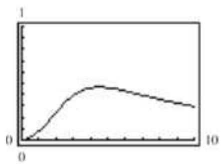
The minimum average cost occurs when $x = 5$.

89.

$C = \dfrac{3t^2 + t}{t^3 + 50}, \; 0 \le t$

(a) The horizontal asymptote is the t-axis, or $C = 0$. This indicates that the chemical eventually dissipates.

(b)



The maximum occurs when $t \approx 4.5$.

(c) Graph C together with $y = 0.345$. The graphs intersect at $t \approx 2.65$ and $t \approx 8.32$. $C < 0.345$ when $0 \le t \le 2.65$ hours and when $t > 8.32$ hours.

90.

(a) $\text{Rate} \times \text{Time} = \text{Distance}$ or $\dfrac{\text{Distance}}{\text{Rate}} = \text{Time}$

$$\frac{100}{x} + \frac{100}{y} = \frac{200}{50} = 4$$

$$\frac{25}{x} + \frac{25}{y} = 1$$

$$25y + 25x = xy$$

$$25x = xy - 25y$$

$$25x = y(x - 25)$$

$$y = \frac{25x}{x - 25}$$

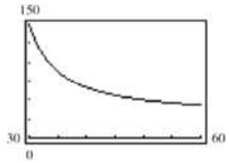
(b) Vertical asymptote: $x = 25$
Horizontal asymptote: $y = 25$

(c)

x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

The results in the table are unexpected. You would expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

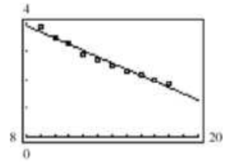
(d)



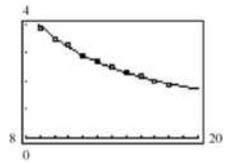
(e) No, it is not possible to average 20 miles per hour in one direction and still average 50 miles per hour on the round trip. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

91.

(a) $A = -0.2182t + 5.665$



(b) $A = \dfrac{1}{0.0302t - 0.020}$



(c)

Year	1999	2000	2001	2002
Original data, A	3.9	3.5	3.3	2.9
Model from (a), A	3.7	3.5	3.3	3.0
Model from (b), A	4.0	3.5	3.2	2.9

Year	2003	2004	2005	2006
Original data, A	2.7	2.5	2.3	2.2
Model from (a), A	2.8	2.6	2.4	2.2
Model from (b), A	2.7	2.5	2.3	2.2

Year	2007	2008
Original data, A	2.0	1.9
Model from (a), A	2.0	1.7
Model from (b), A	2.0	1.9

Answers will vary.

92.

(a) Domain: $t \geq 0$; the model is valid only after the elk have been released.

(b) At $t = 0$, $P = 10$.

(c) $P(25) \approx 22$ elk

$P(50) \approx 24$ elk

$P(100) \approx 25$ elk

(d) Yes, the horizontal asymptote $y = \dfrac{2.7}{0.1} \approx 27$ is the limit.

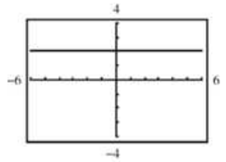
93. False. The graph of a rational function is continuous when the polynomial in the denominator has no real zeros.

94. False. $f(x) = \dfrac{x}{x^3 + 1}$ crosses its horizontal asymptote $y = 0$ at $x = 0$.

95. $h(x) = \dfrac{6 - 2x}{3 - x} = \dfrac{2(3 - x)}{3 - x} = 2,\ x \neq 3$

Since *h(x)* is not reduced and $(3 - x)$ is a factor of both the numerator and the denominator, $x = 3$ is not a horizontal asymptote.

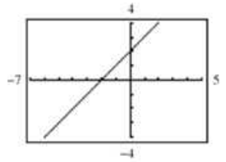
There is a hole in the graph at $x = 3$.



96. $g(x) = \dfrac{x^2 + x - 2}{x - 1}$

$= \dfrac{(x + 2)(x - 1)}{x - 1} = x + 2,\ \ x \neq 1$

Since *g(x)* is not reduced $(x - 1)$ is a factor of both the numerator and the denominator, $x = 1$ is not a horizontal asymptote.



There is a hole at $x = 1$.

97. *Horizontal asymptotes*:

If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote. If the degree of the numerator is less than the degree of the denominator, then there is a horizontal asymptote at $y = 0$.

If the degree of the numerator is equal to the degree of the denominator, then there is a horizontal asymptote at the line given by the ratio of the leading coefficients.

Vertical asymptotes:

Set the denominator equal to zero and solve.

Slant asymptotes:

If there is no horizontal asymptote and the degree of the numerator is exactly one greater than the degree of the denominator, then divide the numerator by the denominator. The slant asymptote is the result, not including the remainder.

98.

(a) $y = x + 1 + \dfrac{a}{x + 2}$ has a slant asymptote $y = x + 1$ and a vertical asymptote $x = -2$.

$0 = 2 + 1 + \dfrac{a}{2 + 2}$

$0 = 3 + \dfrac{a}{4}$

$\dfrac{a}{4} = -3$

$a = -12$

Hence, $y = x + 1 - \dfrac{12}{x + 2} = \dfrac{x^2 + 3x - 10}{x + 2}$.

(b) $y = x - 2 + \dfrac{a}{x + 4}$ has slant asymptote $y = x - 2$ and vertical asymptote at $x = -4$. We determine a so that y has a zero at $x = 3$:

$0 = 3 - 2 + \dfrac{a}{3 + 4} = 1 + \dfrac{a}{7} \Rightarrow a = -7$

Hence, $y = x - 2 + \dfrac{-7}{x + 4} = \dfrac{x^2 + 2x - 15}{x + 4}$.

99. $\left(\dfrac{x}{8}\right)^{-3} = \left(\dfrac{8}{x}\right)^3 = \dfrac{512}{x^3}$

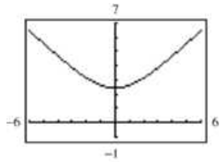
100. $\left(4x^2\right)^{-2} = \dfrac{1}{\left(4x^2\right)^2} = \dfrac{1}{16x^4}$

101. $\dfrac{3^{7/6}}{3^{1/6}} = 3^{6/6} = 3$

102. $\dfrac{x^{-2} \cdot x^{1/2}}{x^{-1} \cdot x^{5/2}} = \dfrac{x \cdot x^{1/2}}{x^2 \cdot x^{5/2}} = \dfrac{x^{3/2}}{x^{9/2}}$

$= \dfrac{1}{x^3}$

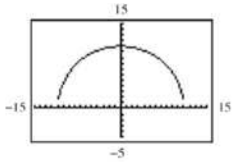
103.



Domain: all x

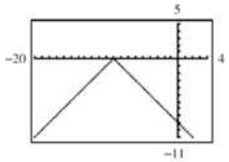
Range: $y \geq \sqrt{6}$

104.



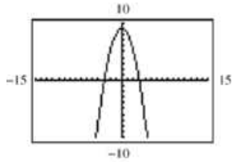
Domain: $-11 \le x \le 11$
Range: $0 \le y \le 11$

105.



Domain: all x
Range: $y \le 0$

106.



Domain: all x
Range: $y \le 9$

107. Answers will vary.

Section 2.8

1. quadratic

2. $S^2 = 0.9688$, the closer the value of S^2 is to 1, the better fit of the model.

3. Quadratic

4. Linear

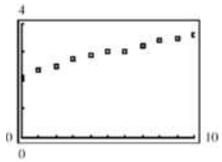
5. Linear

6. Neither

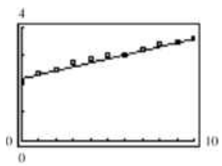
7. Neither

8. Quadratic

9. (a)



(b) Linear model is better.
(c) $y = 0.14x + 2.2$, linear
(d)

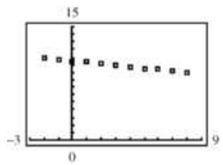


(e)

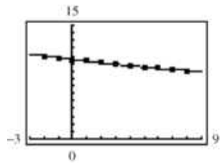
x	0	1	2	3	4
y	2.1	2.4	2.5	2.8	2.9
Model	2.2	2.4	2.5	2.6	2.8

x	5	6	7	8	9	10
y	3.0	3.0	3.2	3.4	3.5	3.6
Model	2.9	3.0	3.2	3.4	3.5	3.6

10. (a)



(b) Quadratic model is better.
(c) $= 0.006x^2 - 0.23x + 10.5$, quadratic
(d)

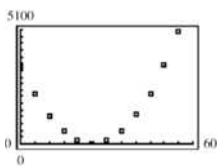


(e)

x	−2	−1	0	1	2
y	11	10.7	10.4	10.3	10.1
Model	11	10.7	10.4	10.3	10.1

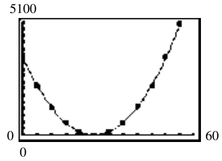
x	3	4	5	6	7	8
y	9.9	9.6	9.4	9.4	9.2	9.0
Model	9.9	9.7	9.5	9.3	9.2	9.0

11. (a)



(b) Quadratic model is better.
(c) $y = 5.55x^2 - 277.5x + 3478$

(d)

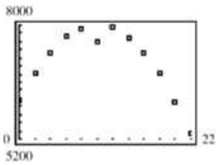


(e)

x	0	5	10	15	20	25
y	3480	2235	1250	565	150	12
Model	3478	2229	1258	564	148	9

x	30	35	40	45	50	55
y	145	575	1275	2225	3500	5010
Model	148	564	1258	2229	3478	5004

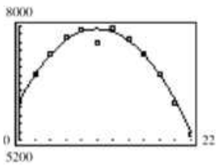
12. (a)



(b) Quadratic model is better.

(c) $y = -17.79x^2 + 354.8x + 6163$

(d)

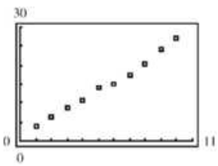


(e)

x	0	2	4	6	8	10
y	6140	6815	7335	7710	7915	7590
Model	6163	6801	7298	7651	7863	7932

x	12	14	16	18	20	22
y	7975	7700	7325	6820	6125	5325
Model	7859	7643	7286	6785	6143	5358

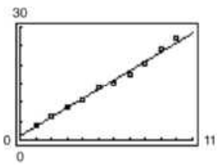
13. (a)



(b) Linear model is better.

(c) $y = 2.48x + 1.1$

(d)

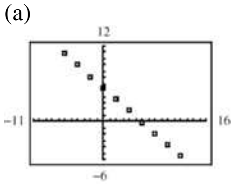


(e)

x	1	2	3	4	5
Actual, y	4.0	6.5	8.8	10.6	13.9
Model, y	3.6	6.1	8.5	11.0	13.5

x	6	7	8	9	10
Actual, y	15.0	17.5	20.1	24.0	27.1
Model, y	16.0	18.5	20.9	23.4	25.9

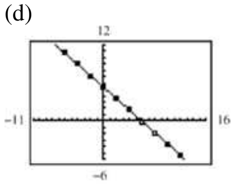
14. (a)



(b) Linear model is better.

(c) $y = -0.89x + 5.3$

(d)

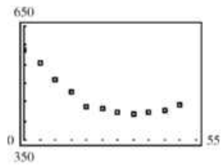


(e)

x	−6	−4	−2	0	2
Actual, y	10.7	9.0	7.0	5.4	3.5
Model, y	10.6	8.9	7.1	5.3	3.5

x	4	6	8	10	12
Actual, y	1.7	−0.1	−1.8	−3.6	−5.3
Model, y	1.7	0.0	−1.8	−3.6	−5.4

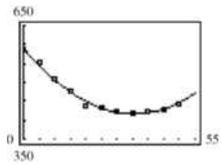
15. (a)



(b) Quadratic is better.

(c) $y = 0.14x^2 - 9.9x + 591$

(d)

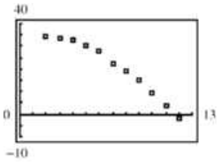


(e)

x	0	5	10	15	20	25
Actual, y	587	551	512	478	436	430
Model, y	591	545	506	474	449	431

x	30	35	40	45	50
Actual, y	424	420	423	429	444
Model, y	420	416	419	429	446

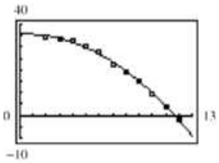
16. (a)



(b) Quadratic model is better.

(c) $y = -0.283x^2 + 0.25x + 35.6$

(d)

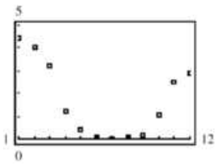


(e)

x	2	3	4	5	6	7
Actual, y	34.3	33.8	32.6	30.1	27.8	22.5
Model, y	35.0	33.8	32.1	29.8	26.9	23.5

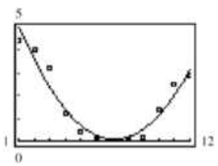
x	8	9	10	11	12
Actual, y	19.1	14,8	9.4	3.7	−1.6
Model, y	19.5	14.9	9.8	4.1	−2.2

17. (a)



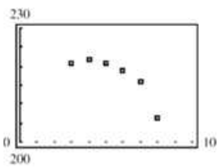
(b) $P = 0.1323t^2 - 1.893t + 6.85$

(c)



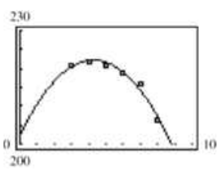
(d) The model's minimum is $H \approx 0.1$ at $t = 7.4$. This corresponds to July.

18. (a)



(b) $S = -1.093t^2 + 9.32t + 202.4$

(c)



(d) To determine when sales will be less than $180 billion, set $S = 180$. Using Quadratic Formula, solve for t.

$-1.093t^2 + 9.32t + 202.4 = 180$

$-1.093t^2 + 9.32t + 22.4 = 0$

$$t = \frac{-9.32 \pm \sqrt{(9.32)^2 - 4(-1.093)(22.4)}}{2(-1.093)}$$

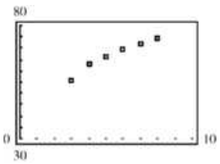
$$t = \frac{-9.32 \pm \sqrt{184.7952}}{-2.186}$$

$t \approx 10.48$ or 2010

Therefore after 2010, sales are expected to be less than $180 billion.

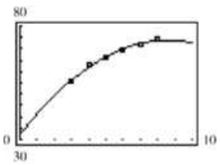
This is not a good model for predicting future years. In fact, by 2019 the model gives negative values for sales.

19. (a)



(b) $P = -0.5638t^2 + 9.690t + 32.17$

(c)



(d) To determine when the percent P of the U.S. population who used the Internet falls below 60%, set P = 60. Using the Quadratic Formula, solve for t.

$-0.5638t^2 + 9.690t + 32.17 = 60$

$-0.5638t^2 + 9.690t - 27.83 = 0$

$$t = \frac{-(9.690) \pm \sqrt{(9.690)^2 - 4(-0.5638)(-27.83)}}{2(-0.5638)}$$

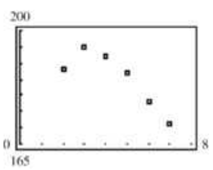
$$t = \frac{-9.690 \pm \sqrt{156.658316}}{-1.1276}$$

$t \approx 13.54$ or 2014

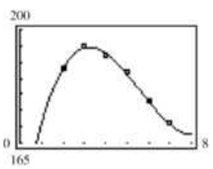
Therefore after 2014, the percent of the U.S. population who use the Internet will fall below 60%.

This is not a good model for predicting future years. In fact, by 2021, the model gives negative values for the percentage.

20. (a)



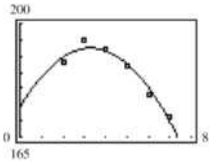
(b)



(c) $H = -1.68t^2 + 11.1t + 174$

$r^2 \approx 0.9567$

(d)



The model fits the data fairly well.

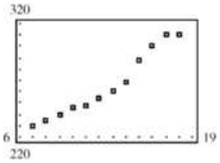
(e) The cubic model is a better fit because its coefficient of determination is closer to 1.

(f)

Year	2008	2009	2010
$H*$	164	159	155
Cubic model	168	172	186
Quadratic model	155	138	117

Answers will vary.

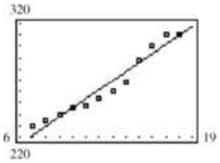
21. (a)



(b) $T = 7.97t + 166.1$

$r^2 \approx 0.9469$

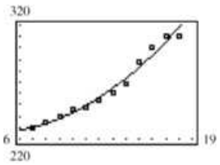
(c)



(d) $T = 0.459t^2 - 3.51t + 232.4$

$r^2 \approx 0.9763$

(e)



(f) The quadratic model is a better fit. Answers will vary.

(g) To determine when the number of televisions, T, in homes reaches 350 million, set T for the model equal to 350 and solve for t.

Linear model:

$$7.97t + 166.1 = 350$$
$$7.97t = 183.9$$
$$t \approx 23.1 \text{ or } 2014$$

Quadratic model:

$$0.459t^2 - 3.51t + 232.4 = 350$$
$$0.459t^2 - 3.51t - 117.6 = 0$$

Using the Quadratic Formula,

$$t = \frac{-(-3.51) \pm \sqrt{(-3.51)^2 - 4(0.459)(-117.6)}}{2(0.459)}$$
$$t = \frac{3.51 \pm \sqrt{228.2337}}{0.918}$$
$$t \approx 20.3 \text{ or } 2011$$

22. True. A quadratic model with a negative leading coefficient opens downward. So, its vertex is at its highest point.

23. True. A quadratic model with a positive leading coefficient opens upward. So, its vertex is at its lowest point.

24. False. A quadratic model could be a better fit for data that are positively correlated.

25. The model is above all data points.

26. Because the data appears more quadratic than linear, a quadratic model would be the better fit. Therefore the S^2-value of 0.9995 must be that of the quadratic model.

27. (a) $(f \circ g)(x) = f(x^2 + 3) = 2(x^2 + 3) - 1 = 2x^2 + 5$

(b) $(g \circ f)(x) = g(2x - 1) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4$

28. (a) $(f \circ g)(x) = f(2x^2 - 1) = 5(2x^2 - 1) + 8 = 10x^2 + 3$

(b) $(g \circ f)(x) = g(5x + 8) = 2(5x + 8)^2 - 1 = 50x^2 + 160x + 127$

29. (a) $(f \circ g)(x) = f(\sqrt[3]{x + 1}) = x + 1 - 1 = x$

(b) $(g \circ f)(x) = g(x^3 - 1) = \sqrt[3]{x^3 - 1 + 1} = x$

30. (a) $(f \circ g)(x) = f(x^3 - 5) = \sqrt[3]{x^3 - 5 + 5} = x$

(b) $(g \circ f)(x) = g(\sqrt[3]{x + 5}) = \left[\sqrt[3]{x + 5}\right]^3 - 5 = x$

31. f is one-to-one.

$$y = 2x + 5$$
$$x = 2y + 5$$
$$2y = x - 5$$
$$y = \frac{(x - 5)}{2} \Rightarrow f^{-1}(x) = \frac{x - 5}{2}$$

32. f is one-to-one.

$$y = \frac{x - 4}{5}$$
$$x = \frac{y - 4}{5}$$
$$5x + 4 = y \Rightarrow f^{-1}(x) = 5x + 4$$

33. f is one-to-one on $[0, \infty)$.

$$y = x^2 + 5,\ x \ge 0$$
$$x = y^2 + 5,\ y \ge 0$$
$$y^2 = x - 5$$
$$y = \sqrt{x - 5} \Rightarrow f^{-1}(x) = \sqrt{x - 5},\ x \ge 5$$

34. f is one-to-one.

$$y = 2x^2 - 3,\ x \ge 0$$
$$x = 2y^2 - 3,\ y \ge 0$$
$$y^2 = \frac{(x+3)}{2}$$
$$y = \sqrt{\frac{x+3}{2}} \Rightarrow f^{-1}(x) = \sqrt{\frac{x+3}{2}}$$
$$= \frac{\sqrt{2x+6}}{2},\ x \ge -3$$

35. For $1 - 3i$, the complex conjugate is $1 + 3i$.

$$(1-3i)(1+3i) = 1 - 9i^2$$
$$= 1 + 9$$
$$= 10$$

36. For $-2 + 4i$, the complex conjugate is $-2 - 4i$.

$$(-2+4i)(-2-4i) = 4 - 16i^2$$
$$= 4 + 16$$
$$= 20$$

37. For $-5i$, the complex conjugate is $5i$.

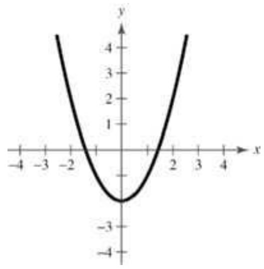
$$(-5i)(5i) = -25i^2$$
$$= 25$$

38. For $8i$, the complex conjugate is $-8i$.

$$(8i)(-8i) = -64i^2$$
$$= 64$$

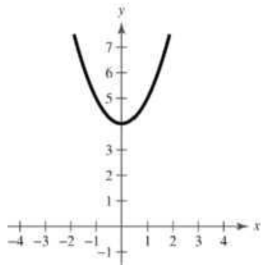
Chapter 2 Review Exercises

1.



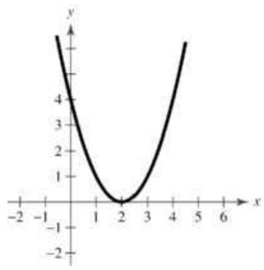
The graph of $y = x^2 - 2$ is a vertical shift two units downward of $y = x^2$.

2.



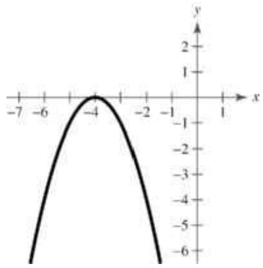
The graph of $y = x^2 + 4$ is a vertical shift four units upward of $y = x^2$.

3.



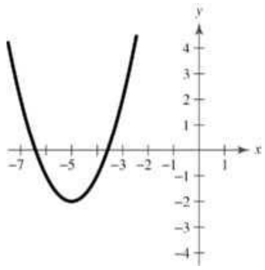
The graph of $y = (x-2)^2$ is a horizontal shift two units to the right of $y = x^2$.

4.



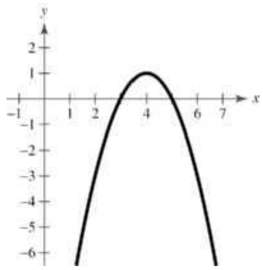
The graph of $y = -(x+4)^2$ is a horizontal shift four units to the left and a reflection in the x-axis of $y = x^2$.

5.



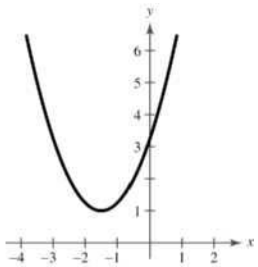
The graph of $y = (x+5)^2 - 2$ is a horizontal shift five units to the left and a vertical shift two units downward of $y = x^2$.

6.



The graph of $y = -(x-4)^2 + 1$ is a horizontal shift four units to the right, a reflection in the x-axis, and a vertical shift one unit upward of $y = x^2$.

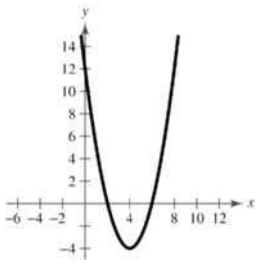
7. The graph of $f(x) = \left(x + 3/2\right)^2 + 1$ is a parabola opening upward with vertex $\left(-\frac{3}{2},\ 1\right)$, and no x-intercepts.



8. The graph of $f(x) = (x-4)^2 - 4$ is a parabola opening upward with vertex $(4,\ -4)$.

x-intercepts: $(x-4)^2 - 4 = 0$

$$(x-4)^2 = 4$$
$$x - 4 = \pm 2$$
$$x = 6,\ 2$$
$$(6,\ 0), (2,\ 0)$$



9. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

$$= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) - \frac{4}{3}$$

$$= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12}$$

The graph of f is a parabola opening upward with vertex $\left(-\frac{5}{2},\ -\frac{41}{12}\right)$.

x-intercepts: $\frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12} = 0.$

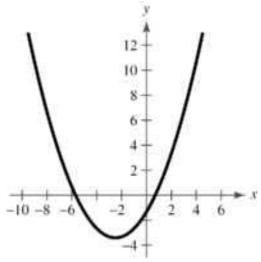
or

$$\frac{1}{3}\left(x^2 + 5x - 4\right) = 0$$
$$x^2 + 5x - 4 = 0$$

Use Quadratic formula.

$$x = \frac{-5 \pm \sqrt{41}}{2}$$

$$\left(\frac{-5 + \sqrt{41}}{2},\ 0\right), \left(\frac{-5 - \sqrt{41}}{2},\ 0\right)$$



10. $f(x) = 3x^2 - 12x + 11$

$$= 3(x^2 - 4x) + 11$$
$$= 3(x^2 - 4x + 4 - 4) + 11$$
$$= 3(x-2)^2 - 1$$

The graph of f is a parabola opening upward with vertex $(2, -1)$.

x-intercepts: $3(x-2)^2 - 1 = 0$

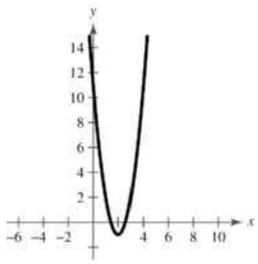
or

$$3x^2 - 12x + 11 = 0$$

Use Quadratic Formula.

$$x = \frac{6 \pm \sqrt{3}}{3}$$

$$\left(\frac{6 + \sqrt{3}}{3},\ 0\right), \left(\frac{6 - \sqrt{3}}{3}\right)$$



11. Vertex : $(1, -4) \Rightarrow f(x) = a(x-1)^2 - 4$

Point: $(2, -3) \Rightarrow -3 = a(2-1)^2 - 4$

$$1 = a$$

Thus, $f(x) = (x-1)^2 - 4.$

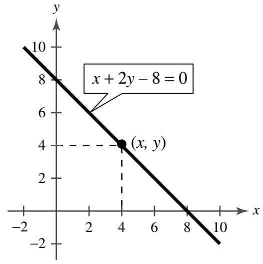
12. Vertex: $(2,\ 3) \Rightarrow y = a(x-2)^2 + 3$

Point: $(0,\ 2) \Rightarrow 2 = a(0-2)^2 + 3$

$$= 4a + 3 \Rightarrow a = -\frac{1}{4}$$

Thus, $f(x) = -\frac{1}{4}(x-2)^2 + 3.$

13.



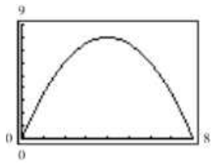
(a) $A = xy$

If $x + 2y - 8 = 0$, then $y = \frac{8 - x}{2}$.

$$A = x\left(\frac{8 - x}{2}\right)$$

$$A = 4x - \frac{1}{2}x^2, \ 0 < x < 8$$

(b)



Using the graph, when $x = 4$, the area is a maximum.

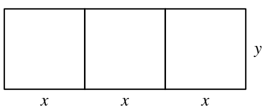
When $x = 4$, $y = \frac{8 - 4}{2} = 2$.

(c)
$$\begin{aligned} A &= -\frac{1}{2}x^2 + 4x \\ &= -\frac{1}{2}(x^2 - 8x) \\ &= -\frac{1}{2}(x^2 - 8x + 16 - 16) \\ &= -\frac{1}{2}(x - 4)^2 + 8 \end{aligned}$$

The graph of A is a parabola opening downward, with vertex $(4, 8)$. Therefore, when $x = 4$, the maximum area is 8 square units.

Yes, graphically and algebraically the same dimensions result.

14.



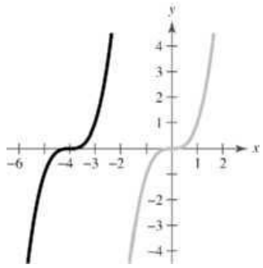
$6x + 4y = 1500$	Total amount of fencing
$A = 3xy$	Area enclosed

Because $y = \frac{1}{4}(1500 - 6x)$,

$$\begin{aligned} A &= 3x\left(\frac{1}{4}\right)(1500 - 6x) \\ &= -\frac{9}{2}x^2 + 1125x. \end{aligned}$$

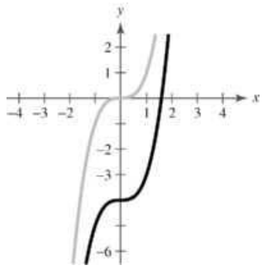
The vertex is at $x = \frac{-b}{2a} = \frac{-1125}{2\left(-\frac{9}{2}\right)} = 125$. Thus $x = 125$ feet, $y = \frac{1}{4}(1500 - 6(125)) = 187.5$, and the dimensions are 375 feet by 187.5 feet.

15.



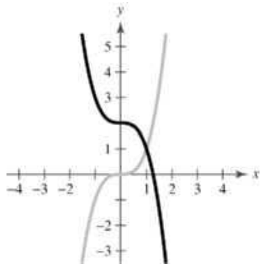
The graph of $f(x) = (x + 4)^3$ is a horizontal shift four units to the left of $y = x^3$.

16.



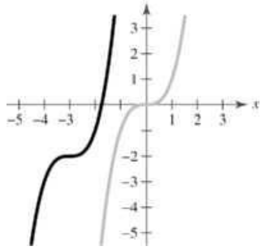
The graph of $f(x) = x^3 - 4$ is a vertical shift four units downward of $y = x^3$.

17.



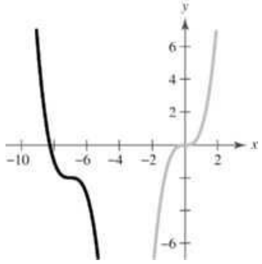
The graph if $f(x) = -x^3 + 2$ is a reflection in the x-axis and a vertical shift two units upward of $y = x^3$.

18.



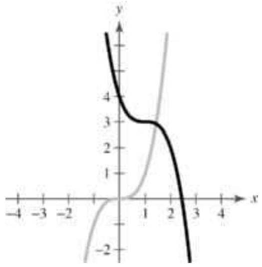
The graph of $f(x) = (x + 3)^3 - 2$ is a horizontal shift three units to the left and a vertical shift two units downward of $y = x^3$.

19.



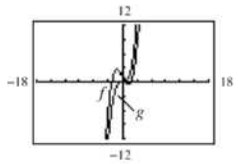
The graph of $f(x) = -(x+7)^3 - 2$ is a horizontal shift seven units to the left, a reflection in the x-axis, and a vertical shift two units downward of $y = x^3$.

20.



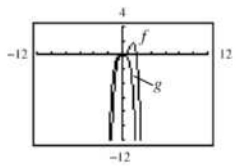
The graph of $f(x) = -(x-1)^3 + 3$ is a horizontal shift one units to the right, a reflection in the x-axis, and a vertical shift three units upward of $y = x^3$.

21. $f(x) = \dfrac{1}{2}x^3 - 2x + 1;\quad g(x) = \dfrac{1}{2}x^3$



The graphs have the same end behavior. Both functions are of the same degree and have positive leading coefficients.

22. $f(x) = -x^4 + 2x^3;\quad g(x) = -x^4$



The graphs have the same end behavior. Both functions are of the same degree and have negative leading coefficients.

23. $f(x) = -x^2 + 6x + 9$
The degree is even and the leading coefficient is negative. The graph falls to the left and right.

24. $f(x) = \dfrac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

25. $f(x) = \dfrac{3}{4}\left(x^4 + 3x^2 + 2\right)$

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

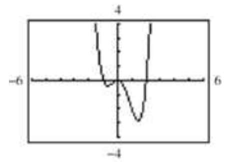
26. $h(x) = -x^5 - 7x^2 + 10x$
The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

27. (a) $x^4 - x^3 - 2x^2 = x^2\left(x^2 - x - 2\right)$

$= x^2(x-2)(x+1) = 0$

Zeros: $x = -1,\ 0,\ 0,\ 2$

(b)



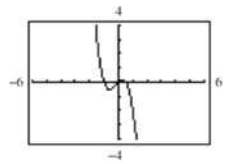
(c) Zeros: $x = -1,\ 0,\ 0,\ 2$; the same

28. (a) $-2x^3 - x^2 + x = -x\left(2x^2 + x - 1\right)$

$= -x(2x-1)(x+1) = 0$

Zeros: $x = -1,\ 0,\ \frac{1}{2}$

(b)

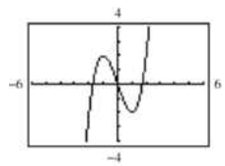


(c) Zeros: $x = -1,\ 0,\ 0.5$; the same

29. (a) $t^3 - 3t = t\left(t^2 - 3\right) = t\left(t + \sqrt{3}\right)\left(t - \sqrt{3}\right) = 0$

Zeros: $t = 0, \pm\sqrt{3}$

(b)



(c) Zeros: $t = 0, \pm 1.732$; the same

30. (a) For the quadratic, $x = \dfrac{-10 \pm \sqrt{100 - 112}}{2}$

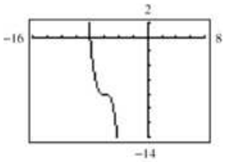
$= -5 \pm \sqrt{3}i.$

Zeros: $-8,\ -5 \pm \sqrt{3}i$

$-(x+6)^3 - 8 = x^3 + 18x^2 + 108x + 224$

$= (x+8)\left(x^2 + 10x + 28\right) = 0$

(b)

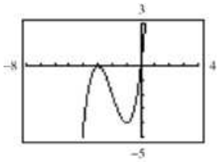


(c) Real zero: $x = -8$

31. (a) $x(x+3)^2 = 0$

Zeros: $x = 0, -3, -3$

(b)



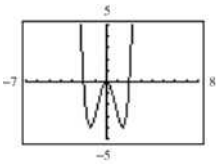
(c) Zeros: $x = -3, \ -3, \ 0$; the same

32. (a)

$$t^4 - 4t^2 = 0$$
$$t^2\left(t^2 - 4\right) = 0$$
$$t^2(t+2)(t-2) = 0$$

Zeros: $t = 0, \ 0, \ \pm 2$

(b)



(c) Zeros: $t = 0, \ 0, \ \pm 2$; the same

33. $f(x) = (x+2)(x-1)^2(x-5)$
$= x^4 - 5x^3 - 3x^2 + 17x - 10$

34. $f(x) = (x+3)x(x-1)(x-4)$
$= x^4 - 2x^3 - 11x^2 + 12x$

35. $f(x) = (x-3)\left(x - 2 + \sqrt{3}\right)\left(x - 2 - \sqrt{3}\right)$
$= x^3 - 7x^2 + 13x - 3$

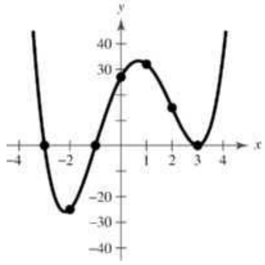
36. $f(x) = (x+7)\left(x - 4 + \sqrt{6}\right)\left(x - 4 - \sqrt{6}\right)$
$= x^3 - x^2 - 46x + 70$

37. (a) The degree of *f* is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b)
$$\begin{aligned} f(x) &= x^4 - 2x^3 - 12x^2 + 18x + 27 \\ &= (x^4 - 12x^2 + 27) - (2x^3 - 18x) \\ &= (x^2 - 9)(x^2 - 3) - 2x(x^2 - 9) \\ &= (x^2 - 9)(x^2 - 3 - 2x) \\ &= (x+3)(x-3)(x^2 - 2x - 3) \\ &= (x+3)(x-3)(x-3)(x+1) \end{aligned}$$

Zeros: $-3, \ 3, \ 3, \ -1$

(c) and (d)

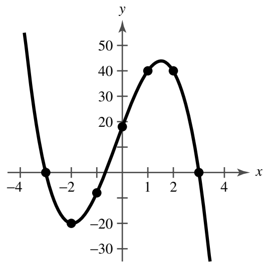


38. (a) The degree of *f* is odd and the leading coefficient is -3. The graph rises to the left and falls to the right.

(b) $f(x) = -3x^3 - 2x^2 + 27x + 18 = -(x-3)(x+3)(3x+2)$

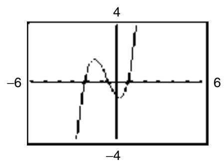
Zeros: $x = \pm 3, \ -\dfrac{2}{3}$

(c) and (d)



39. $f(x) = x^3 + 2x^2 - x - 1$

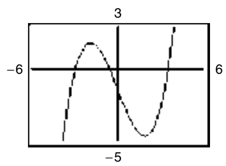
(a) $f(-3) < 0, f(-2) > 0 \Rightarrow$ zero in $(-3, \ -2)$
$f(-1) > 0, f(0) < 0 \Rightarrow$ zero in $(-1, \ 0)$
$f(0) < 0, f(1) > 0 \Rightarrow$ zero in $(0, \ 1)$



(b) Zeros: $-2.247, \ -0.555, \ 0.802$

40. (a) $f(x) = 0.24x^3 - 2.6x - 1.4$

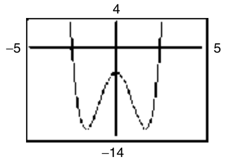
(b) $f(-3) < 0, f(-2) > 0 \Rightarrow$ zero in $(-3, \ -2)$
$f(-1) > 0, f(0) < 0 \Rightarrow$ zero in $(-1, \ 0)$
$f(3) < 0, f(4) > 0 \Rightarrow$ zero in $(3, \ 4)$



(b) Zeros: $-2.979, \ -0.554, \ 3.533$

41. $f(x) = x^4 - 6x^2 - 4$

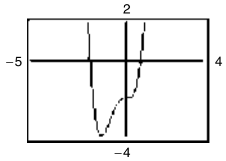
(a) $f(-3) > 0, f(-2) < 0 \Rightarrow$ zero in $(-3, -2)$
$f(2) < 0, f(3) > 0 \Rightarrow$ zero in $(2, 3)$



(b) Zeros: ± 2.570

42. $f(x) = 2x^4 + \dfrac{7}{2}x^3 - 2$

(a) $f(-2) > 0, f(-1) < 0 \Rightarrow$ zero in $(-2, -1)$
$f(0) < 0, f(1) > 0 \Rightarrow$ zero in $(0, 1)$



(b) Zeros: $-1.897, 0.738$

43.

$$\begin{array}{r|l} & 8x + 5 \\ \hline 3x - 2 & 24x^2 - x - 8 \\ & \underline{24x^2 - 16x} \\ & \qquad 15x - 8 \\ & \qquad \underline{15x - 10} \\ & \qquad\qquad 2 \end{array}$$

Thus, $\dfrac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \dfrac{2}{3x - 2}$.

44.

$$\begin{array}{r|l} & \frac{4}{3}x + \frac{8}{9} \\ \hline 3x - 2 & 4x^2 + 0x + 7 \\ & \underline{4x^2 - \frac{8}{3}x} \\ & \qquad \frac{8}{3}x + 7 \\ & \qquad \underline{\frac{8}{3}x - \frac{16}{9}} \\ & \qquad\qquad \frac{79}{9} \end{array}$$

$$\frac{4x^2 + 7}{3x - 2} = \frac{4}{3}x + \frac{8}{9} + \frac{\frac{79}{9}}{3x - 2} = \frac{4}{3}x + \frac{8}{9} + \frac{79}{27x - 18}$$

45.

$$\begin{array}{r|l} & x^2 - 2 \\ \hline x^2 - 1 & x^4 - 3x^2 + 2 \\ & \underline{x^4 - x^2} \\ & \quad -2x^2 + 2 \\ & \quad \underline{-2x^2 + 2} \\ & \qquad\quad 0 \end{array}$$

Thus, $\dfrac{x^4 - 3x^2 + 2}{x^2 - 1} = x^2 - 2, \ (x \pm 1)$.

46.

$$\begin{array}{r|l} & 3x^2 + 4 \\ \hline x^2 - 1 & 3x^4 - 3x^2 - 1 \\ & \underline{3x^4 - 3x^2} \\ & \qquad 4x^2 - 1 \\ & \qquad \underline{4x^2 - 4} \\ & \qquad\qquad 3 \end{array}$$

Thus, $\dfrac{3x^4 + x^2 - 1}{x^2 - 1} = 3x^2 + 4 + \dfrac{3}{x^2 - 1}$.

47.

$$\begin{array}{r|l} & 5x + 2 \\ \hline x^2 - 3x + 1 & 5x^3 - 13x^2 - \ x + 2 \\ & \underline{5x^3 - 15x^2 + 5x} \\ & \qquad 2x^2 - 6x + 2 \\ & \qquad \underline{2x^2 - 6x + 2} \\ & \qquad\qquad\quad 0 \end{array}$$

Thus, $\dfrac{5x^3 - 13x^2 - x + 2}{x^2 - 3x + 1} = 5x + 2,$
$\left(x \neq \dfrac{1}{2}(3 \pm \sqrt{5})\right)$.

48.

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x^2 + 2x & x^4 + x^3 - x^2 + 2x \\ & \underline{x^4 + 2x^3} \\ & \quad -x^3 - x^2 \\ & \quad \underline{-x^3 - 2x^2} \\ & \qquad\quad x^2 + 2x \\ & \qquad\quad \underline{x^2 + 2x} \\ & \qquad\qquad 0 \end{array}$$

Thus, $\dfrac{x^4 + x^3 - x^2 + 2x}{x^2 + 2x} = x^2 - x + 1, \ (x \neq 0, -2)$.

49.
$$\begin{array}{r}
3x^2+5x+8 \\
2x^2+0x-1 \overline{\big) 6x^4+10x^3+13x^2-5x+2} \\
\underline{6x^4+\ 0x^3\ -3x^2\qquad\qquad} \\
10x^3+16x^2-5x\quad \\
\underline{10x^3+\ 0x^2-5x\quad} \\
16x^2-0+2 \\
\underline{16x^2+0-8} \\
10
\end{array}$$

Thus,
$$\frac{6x^4+10x^3+13x^2-5x+2}{2x^2-1}=3x^2+5x+8+\frac{10}{2x^2-1}.$$

50.
$$\begin{array}{r}
x^2-3x+2 \\
x^2+2 \overline{\big) x^4-3x^3+4x^2-6x+3} \\
\underline{x^4\qquad\ +2x^2\qquad\qquad} \\
-3x^3+2x^2-6x\quad \\
\underline{-3x^3\qquad\ -6x\quad} \\
2x^2\qquad+3 \\
\underline{2x^2\qquad+4} \\
-1
\end{array}$$

Thus, $\dfrac{x^4-3x^3+4x^2-6x+3}{x^2+2}=x^2-3x+2+\dfrac{-1}{x^2+2}.$

51.
$$\begin{array}{r|rrrrr}
-2 & 0.25 & -4 & 0 & 0 & 0 \\
 & & -\frac{1}{2} & 9 & -18 & 36 \\ \hline
 & \frac{1}{4} & -\frac{9}{2} & 9 & -18 & 36
\end{array}$$

Thus, $\dfrac{0.25x^4-4x^3}{x+2}=\dfrac{1}{4}x^3-\dfrac{9}{2}x^2+9x-18+\dfrac{36}{x+2}.$

52.
$$\begin{array}{r|rrrr}
5 & 0.1 & 0.3 & 0 & -0.5 \\
 & & 0.5 & 4 & 20 \\ \hline
 & 0.1 & 0.8 & 4 & 19.5
\end{array}$$

Thus, $\dfrac{0.1x^3+0.3x^2-0.5}{x-5}=0.1x^2+0.8x+4+\dfrac{19.5}{x-5}.$

53.
$$\begin{array}{r|rrrrr}
\frac{2}{3} & 6 & -4 & -27 & 18 & 0 \\
 & & 4 & 0 & -18 & 0 \\ \hline
 & 6 & 0 & -27 & 0 & 0
\end{array}$$

Thus, $\dfrac{6x^4-4x^3-27x^2+18x}{x-(2/3)}=6x^3-27x,\ x\neq\dfrac{2}{3}.$

54.
$$\begin{array}{r|rrrr}
\frac{1}{2} & 2 & 2 & -1 & 2 \\
 & & 1 & \frac{3}{2} & \frac{1}{4} \\ \hline
 & 2 & 3 & \frac{1}{2} & \frac{9}{4}
\end{array}$$

Thus, $\dfrac{2x^3+2x^2-x+2}{x-(1/2)}=2x^2+3x+\dfrac{1}{2}+\dfrac{9/4}{x-(1/2)}.$

55.
$$\begin{array}{r|rrrr}
4 & 3 & -10 & 12 & -22 \\
 & & 12 & 8 & 80 \\ \hline
 & 3 & 2 & 20 & 58
\end{array}$$

Thus, $\dfrac{3x^3-10x^2+12x-22}{x-4}=3x^2+2x+20+\dfrac{58}{x-4}.$

56.
$$\begin{array}{r|rrrr}
1 & 2 & 6 & -14 & 9 \\
 & & 2 & 8 & -6 \\ \hline
 & 2 & 8 & -6 & 3
\end{array}$$

Thus, $\dfrac{2x^3+6x^2-14x+9}{x-1}=2x^2+8x-6+\dfrac{3}{x-1}.$

57. (a)
$$\begin{array}{r|rrrrr}
-3 & 1 & 10 & -24 & 20 & 44 \\
 & & -3 & -21 & 135 & -465 \\ \hline
 & 1 & 7 & -45 & 155 & -421
\end{array} = f(-3)$$

(b)
$$\begin{array}{r|rrrrr}
-2 & 1 & 10 & -24 & 20 & 44 \\
 & & -2 & -16 & 80 & -200 \\ \hline
 & 1 & 8 & -40 & 100 & -156
\end{array} = f(-2)$$

58. $g(t)=2t^5-5t^4-8t+20$

(a)
$$\begin{array}{r|rrrrrr}
-4 & 2 & -5 & 0 & 0 & -8 & 20 \\
 & & -8 & 52 & -208 & 832 & -3296 \\ \hline
 & 2 & -13 & 52 & -208 & 824 & -3276
\end{array} = g(-4)$$

(b)
$$\begin{array}{r|rrrrrr}
\sqrt{2} & 2 & -5 & 0 & 0 & -8 & 20 \\
 & & 2\sqrt{2} & 4-5\sqrt{2} & 4\sqrt{2}-10 & 8-10\sqrt{2} & -20 \\ \hline
 & 2 & 2\sqrt{2}-5 & 4-5\sqrt{2} & 4\sqrt{2}-10 & -10\sqrt{2} & 0
\end{array}$$
$=g\left(\sqrt{2}\right)$

59. $f(x)=x^3+4x^2-25x-28$

(a)
$$\begin{array}{r|rrrr}
4 & 1 & 4 & -25 & -28 \\
 & & 4 & 32 & 28 \\ \hline
 & 1 & 8 & 7 & 0
\end{array}$$
$(x-4)$ is a factor.

(b) $x^2+8x+7=(x+1)(x+7)$
Remaining factors: $(x+1),\ (x+7)$

(c) $f(x)=(x-4)(x+1)(x+7)$

(d) Zeros: $4,\ -1,\ -7$

60. $f(x)=2x^3+11x^2-21x-90$

(a)
$$\begin{array}{r|rrrr}
-6 & 2 & 11 & -21 & -90 \\
 & & -12 & 6 & 90 \\ \hline
 & 2 & -1 & -15 & 0
\end{array}$$
$(x+6)$ is a factor.

(b) Remaining factors of
$2x^2-x-15$ are $(2x+5),\ (x-3)$.

(c) $f(x)=(x+6)(2x+5)(x-3)$

(d) Zeros: $-6,\ -\dfrac{5}{2},\ 3$

61. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

(a)
$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$(x+2)$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$(x-3)$ is a factor.

(b) $x^2 - 3x - 4 = (x-4)(x+1)$

Remaining factors: $(x-4)$, $(x+1)$

(c) $f(x) = (x+2)(x-3)(x-4)(x+1)$

(d) Zeros: $-2, 3, 4, -1$

62. $f(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$

(a)
$$\begin{array}{r|rrrrr} 2 & 1 & -11 & 41 & -61 & 30 \\ & & 2 & -18 & 46 & -30 \\ \hline & 1 & -9 & 23 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -9 & 23 & -15 \\ & & 5 & -20 & 15 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$(x-2)$ and $(x-5)$ are factors.

(b) Remaining factors of $x^2 - 4x + 3$ are $(x-3), (x-1)$.

(c) $f(x) = (x-2)(x-5)(x-3)(x-1)$

(d) Zeros: $1, 2, 3, 5$

63. $f(x) = 4x^3 - 11x^2 + 10x - 3$

Possible rational zeros: $\pm 1, \pm 3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{1}{2}, \pm\frac{1}{4}$

Zeros: $1, 1, \dfrac{3}{4}$

64. $f(x) = 10x^3 + 21x^2 - x - 6$

Possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{5}, \pm\frac{1}{10}, \pm\frac{1}{2}, \pm\frac{3}{10}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}, \pm\frac{3}{2}$

Zeros: $-2, \frac{1}{2}, -\frac{3}{5}$

65. $g(x) = 5x^3 - 6x + 9$ has two variation in sign $\Rightarrow 0$ or 2 positive real zeros.

$g(-x) = -5x^3 + 6x + 9$ has one variation in sign $\Rightarrow 1$ negative real zero.

66. $f(x) = 2x^5 - 3x^2 + 2x - 1$ has three variations in sign $\Rightarrow 1$ or 3 positive real zeros.

$f(-x) = -2x^5 - 3x^2 - 2x - 1$ has no variations in sign $\Rightarrow 0$ negative real zeros.

67.
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

All entries positive; $x = 1$ is upper bound.

$$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$$

Alternating signs; $x = -\frac{1}{4}$ is lower bound.

Real zero: $x = \dfrac{3}{4}$

68.
$$\begin{array}{r|rrrr} 8 & 2 & -5 & -14 & 8 \\ & & 16 & 88 & 592 \\ \hline & 2 & 11 & 74 & 600 \end{array}$$

All positive $\Rightarrow x = 8$ is upper bound.

$$\begin{array}{r|rrrr} -4 & 2 & -5 & -14 & 8 \\ & & -8 & 52 & -152 \\ \hline & 2 & -13 & 38 & -144 \end{array}$$

Alternating signs $\Rightarrow x = -4$ is lower bound.

Real zeros: $x = -2, \dfrac{1}{2}, 4$

69. $f(x) = 6x^3 + 31x^2 - 18x - 10$

Possible rational zeros:

$\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

Using synthetic division and a graph check $x = \frac{5}{6}$. .

$$\begin{array}{r|rrrr} \frac{5}{6} & 6 & 31 & -18 & -10 \\ & & 5 & 30 & 10 \\ \hline & 6 & 36 & 12 & 0 \end{array}$$

$x = \frac{5}{6}$ is a real zero.

Rewrite in polynomial form and use the Quadratic Formula:

$$6x^2 + 36x + 12 = 0$$
$$6(x^2 + 6x + 2) = 0$$
$$x^2 + 6x + 2 = 0$$
$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

Real zeros: $x = \frac{5}{6}, -3 \pm \sqrt{7}$

70. $f(x) = x^3 - 1.3x^2 - 1.7x + 0.6$

$= \frac{1}{10}(x-2)(x+1)(10x-3)$

Zeros: $x = -1, 2, \frac{3}{10}$

71. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

Use a graphing utility to see that $x = -1$ and $x = 3$ are probably zeros.

$$\begin{array}{r|rrrrr} -1 & 6 & -25 & 14 & 27 & -18 \\ & & -6 & 31 & -45 & 18 \\ \hline & 6 & -31 & 45 & -18 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 6 & -31 & 45 & -18 \\ & & 18 & -39 & 18 \\ \hline & 6 & -13 & 6 & 0 \end{array}$$

$$\begin{aligned} 6x^4 - 25x^3 + 14x^2 + 27x - 18 &= (x+1)(x-3)(6x^2 - 13x + 6) \\ &= (x+1)(x-3)(3x-2)(2x-3) \end{aligned}$$

Zeros: $x = -1, 3, \frac{2}{3}, \frac{3}{2}$

72. $f(x) = 5x^4 + 126x^2 + 25$
$= (5x^2 + 1)(x^2 + 25)$

No real zeros

73. $6 + \sqrt{-25} = 6 + 5i$

74. $-\sqrt{-12} + 3 = -2\sqrt{3}i + 3 = 3 - 2\sqrt{3}i$

75. $-2i^2 + 7i = 2 + 7i$

76. $-i^2 - 4i = 1 - 4i$

77. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i)$
$= 3 + 7i$

78. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = -\sqrt{2}i$

79. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

80. $(1 + 6i)(5 - 2i) = 5 - 2i + 30i + 12 = 17 + 28i$

81. $(10 - 8i)(2 - 3i) = 20 - 30i - 16i + 24i^2$
$= -4 - 46i$

82. $i(6 + i)(3 - 2i) = i(18 + 3i - 12i + 2)$
$= i(20 - 9i) = 9 + 20i$

83. $(3 + 7i)^2 + (3 - 7i)^2 = (9 + 42i - 49) + (9 - 42i - 49)$
$= -80$

84. $(4 - i)^2 - (4 + i)^2 = (16 - 8i - 1) - (16 + 8i - 1)$
$= -16i$

85. $\left(\sqrt{-16} + 3\right)\left(\sqrt{-25} - 2\right) = \left(4i + 3\right)\left(5i - 2\right)$
$= -20 - 8i + 15i - 6$
$= -26 + 7i$

86. $\left(5 - \sqrt{-4}\right)\left(5 + \sqrt{-4}\right) = \left(5 - 2i\right)\left(5 + 2i\right)$
$= 25 + 4$
$= 29$

87. $\sqrt{-9} + 3 + \sqrt{-36} = 3i + 3 + 6i$
$= 3 + 9i$

88. $7 - \sqrt{-81} + \sqrt{-49} = 7 - 9i + 7i$
$= 7 - 2i$

89. $\frac{6 + i}{i} = \frac{6 + i}{i} \cdot \frac{-i}{-i} = \frac{-6i - i^2}{-i^2}$
$= \frac{-6i + 1}{1} = 1 - 6i$

90. $\frac{4}{-3i} = \frac{-4}{3i} \cdot \frac{-i}{-i} = \frac{4i}{3} = \frac{4}{3}i$

91. $\frac{3 + 2i}{5 + i} \cdot \frac{5 - i}{5 - i} = \frac{15 + 10i - 3i + 2}{25 + 1}$
$= \frac{17}{26} + \frac{7}{26}i$

92. $\frac{1 - 7i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{2 - 21 - 17i}{4 + 9}$
$= -\frac{19}{13} - \frac{17}{13}i$

93. $x^2 + 16 = 0$
$x^2 = -16$
$x = \pm\sqrt{-16}$
$x = \pm 4i$

94. $x^2 + 48 = 0$
$x^2 = -48$
$x = \pm\sqrt{-48}$
$x = \pm 4\sqrt{3}i$

95. $x^2 + 3x + 6 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(6)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{-15}}{2}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}i$$

96. $x^2 + 4x + 8 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(8)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{-16}}{2}$$
$$x = \frac{-4 \pm 4i}{2}$$
$$x = -2 \pm 2i$$

97. $3x^2 - 5x + 6 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-47}}{6}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{47}}{6}i$$

98. $5x^2 - 2x + 4 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(4)}}{2(5)}$$

$$x = \frac{2 \pm \sqrt{-76}}{10}$$

$$x = \frac{2 \pm 2\sqrt{19}i}{10}$$

$$x = \frac{1}{5} \pm \frac{\sqrt{19}}{5}i$$

99. $x^2 + 6x + 9 = 0$

$$(x+3)^2 = 0$$
$$x + 3 = 0$$
$$x = -3$$

100. $x^2 - 10x + 25 = 0$

$$(x-5)^2 = 0$$
$$x - 5 = 0$$
$$x = 5$$

101. $x^3 + 16x = 0$

$$x(x^2 + 16) = 0$$
$$x = 0 \quad x^2 + 16 = 0$$
$$x^2 = -16$$
$$x = \pm 4i$$

102. $x^3 + 144x = 0$

$$x(x^2 + 144) = 0$$
$$x = 0 \quad x^2 + 144 = 0$$
$$x^2 = -144$$
$$x = \pm 12i$$

103. $f(x) = 3x(x-2)^2$

Zeros: $0,\ 2,\ 2$

104. $f(x) = (x-4)(x+9)^2$

Zeros: $4,\ -9,\ -9$

105. $h(x) = x^3 - 7x^2 + 18x - 24$

4	1	−7	18	−24
		4	−12	24
	1	−3	6	0

$x = 4$ is a zero. Applying the Quadratic Formula on $x^2 - 3x + 6$,

$$x = \frac{3 \pm \sqrt{9 - 4(6)}}{2} = \frac{3}{2} \pm \frac{\sqrt{15}}{2}i.$$

Zeros: $4, \frac{3}{2} + \frac{\sqrt{15}}{2}i,\ \frac{3}{2} - \frac{\sqrt{15}}{2}i$

$$h(x) = (x-4)\left(x - \frac{3 + \sqrt{15}i}{2}\right)\left(x - \frac{3 - \sqrt{15}i}{2}\right)$$

106. $f(x) = 2x^3 - 5x^2 + 9x + 40$

$$= (2x+5)(x^2 - 5x + 8)$$

Quadratic Formula: $x = \dfrac{5 \pm \sqrt{25 - 32}}{2} = \dfrac{5 \pm \sqrt{7}i}{2}$

Zeros: $-\frac{5}{2},\ \frac{5}{2} \pm \frac{\sqrt{7}}{2}i$

$$f(x) = (2x+5)\left(x - \frac{5}{2} + \frac{\sqrt{7}}{2}i\right)\left(x - \frac{5}{2} - \frac{\sqrt{7}}{2}i\right)$$

107. $f(x) = 2x^4 - 5x^3 + 10x - 12$

2	2	−5	0	10	−12
		4	−2	−4	12
	2	−1	−2	6	0

$x = 2$ is a zero.

$-\frac{3}{2}$	2	−1	−2	6
		−3	6	−6
	2	−4	4	0

$x = -\frac{3}{2}$ is a zero.

$$f(x) = (x-2)\left(x + \frac{3}{2}\right)(2x^2 - 4x + 4)$$
$$= (x-2)(2x+3)(x^2 - 2x + 2)$$

By the Quadratic Formula, applied to $x^2 - 2x + 2$,

$$x = \frac{2 \pm \sqrt{4 - 4(2)}}{2} = 1 \pm i.$$

Zeros: $2, -\frac{3}{2},\ 1 \pm i$

$$f(x) = (x-2)(2x+3)(x-1+i)(x-1-i)$$

108. $g(x) = 3x^4 - 4x^3 + 7x^2 + 10x - 4$

$= (x+1)(3x-1)(x^2 - 2x + 4)$

Quadratic Formula: $x = \dfrac{2 \pm \sqrt{4-16}}{2} = 1 \pm \sqrt{3}i$

Zeros: $-1, \dfrac{1}{3}, 1 \pm \sqrt{3}i$

$g(x) = (x+1)(3x-1)(x-1+\sqrt{3}i)(x-1-\sqrt{3}i)$

109. $f(x) = x^5 + x^4 + 5x^3 + 5x^2$

$= x^2(x^3 + x^2 + 5x + 5)$

$= x^2\left[x^2(x+1) + 5(x+1)\right]$

$= x^2(x+1)(x^2+5)$

$= x^2(x+1)(x+\sqrt{5}i)(x-\sqrt{5}i)$

Zeros: $0, 0, -1, \pm\sqrt{5}i$

110. $f(x) = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

$= x(x^2-4)(x^2-1)$

$f(x) = x(x-2)(x+2)(x-1)(x+1)$

Zeros : $0, \pm 1, \pm 2$

111. $f(x) = x^3 - 4x^2 + 6x - 4$

(a) $x^3 - 4x^2 + 6x - 4 = (x-2)(x^2 - 2x + 2)$

By the Quadratic Formula for $x^2 - 2x + 2$,

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(2)}}{2} = 1 \pm i$

Zeros: $2, 1+i, 1-i$

(b) $f(x) = (x-2)(x-1-i)(x-1+i)$

(c) x-intercept: $(2, 0)$

112. (a) $f(x) = x^3 - 5x^2 - 7x + 51$

$= (x+3)(x^2 - 8x + 17)$

$x = \dfrac{8 \pm \sqrt{(-8)^2 - 4(17)}}{2} = \dfrac{8 \pm \sqrt{-4}}{2}$

$= 4 \pm i$

Zeros: $-3, 4+i, 4-i$

(b) $f(x) = (x+3)(x-4-i)(x-4+i)$

(c) x-intercept: $(-3, 0)$

113. (a) $f(x) = -3x^3 - 19x^2 - 4x + 12$

$= -(x+1)(3x^2 + 16x - 12)$

$$\begin{array}{r|rrrr} -1 & -3 & -19 & -4 & 12 \\ & & 3 & 16 & -12 \\ \hline & -3 & -16 & 12 & 0 \end{array}$$

$3x^2 + 16x - 12 = 0$

$(3x-2)(x+6) = 0$

$3x - 2 = 0 \Rightarrow x = \dfrac{2}{3}$

$x + 6 = 0 \Rightarrow x = -6$

Zeros: $-1, \dfrac{2}{3}, -6$

(b) $f(x) = -(x+1)(3x-2)(x+6)$

(c) x-intercepts: $(-1, 0), (-6, 0), \left(\dfrac{2}{3}, 0\right)$

114. (a) $f(x) = 2x^3 - 9x^2 + 22x - 30$

$= (2x-5)(x^2 - 2x + 6)$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(6)}}{2} = 1 \pm \sqrt{5}i$

Zeros: $\dfrac{5}{2}, 1+\sqrt{5}i, 1-\sqrt{5}i$

(b) $f(x) = (2x-5)(x-1-\sqrt{5}i)(x-1+\sqrt{5}i)$

(c) x-intercepts: $\left(\dfrac{5}{2}, 0\right)$

115. $f(x) = x^4 + 34x^2 + 225$

(a) $x^4 + 34x^2 + 225 = (x^2+9)(x^2+25)$

Zeros: $\pm 3i, \pm 5i$

(b) $(x+3i)(x-3i)(x+5i)(x-5i)$

(c) No x-intercepts

116. (a), (b)

$f(x) = x^4 + 10x^3 + 26x^2 + 10x + 25$

$= (x^2+1)(x^2 + 10x + 25)$

$= (x^2+1)(x+5)^2 = (x+i)(x-i)(x+5)^2$

Zeros: $\pm i, -5, -5$

(c) x-intercept: $(-5, 0)$

117. Since $5i$ is a zero, so is –5i.

$f(x) = (x-4)(x+2)(x-5i)(x+5i)$

$= (x^2 - 2x - 8)(x^2 + 25)$

$= x^4 - 2x^3 + 17x^2 - 50x - 200$

118. Since $2i$ is a zero, so is $-2i$.

$f(x) = (x-2)(x+2)(x-2i)(x+2i)$

$= (x^2-4)(x^2+4)$

$= x^4 - 16$

119. Since $-3+5i$ is a zero, so is $-3-5i$.

$$\begin{aligned} f(x) &= (x-1)(x+4)(x+3-5i)(x+3+5i) \\ &= (x^2+3x-4)((x+3)^2+25) \\ &= (x^2+3x-4)(x^2+6x+34) \\ &= x^4+9x^3+48x^2+78x-136 \end{aligned}$$

120. Since $1+\sqrt{3}i$ is a zero, so is $1-\sqrt{3}i$.

$$\begin{aligned} f(x) &= (x+4)(x+4)(x-1-\sqrt{3}i)(x-1+\sqrt{3}i) \\ &= (x^2+8x+16)((x-1)^2+3) \\ &= (x^2+8x+16)(x^2-2x+4) \\ &= x^4+6x^3+4x^2+64 \end{aligned}$$

121. $f(x) = x^4-2x^3+8x^2-18x-9$

(a) $f(x) = (x^2+9)(x^2-2x-1)$

(b) For the quadratic

$$x^2-2x-1, x = \frac{2 \pm \sqrt{(-2)^2-4(-1)}}{2} = 1 \pm \sqrt{2}.$$

$$f(x) = \left(x^2+9\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$$

(c) $f(x) = \left(x+3i\right)\left(x-3i\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

122. $f(x) = x^4-4x^3+3x^2+8x-16$

(a) $f(x) = (x^2-x-4)(x^2-3x+4)$

(b) $x = \dfrac{1 \pm \sqrt{(-1)^2-4(-4)}}{2} = \dfrac{1}{2} \pm \dfrac{\sqrt{17}}{2}$

$$f(x) = \left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)\left(x^2-3x+4\right)$$

(c) $x = \dfrac{3 \pm \sqrt{(-3)^2-4(4)}}{2} = \dfrac{3}{2} \pm \dfrac{\sqrt{7}}{2}i$

$$f(x) = \left(x-\frac{1}{2}-\frac{\sqrt{17}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{17}}{2}\right)\left(x-\frac{3}{2}+\frac{\sqrt{7}}{2}i\right)\left(x-\frac{3}{2}-\frac{\sqrt{7}}{2}i\right)$$

123. Zeros: $-2i$, 2i

$(x+2i)(x-2i) = x^2+4$ is a factor.

$f(x) = (x^2+4)(x+3)$

Zeros: $\pm 2i, -3$

124. Zeros: $2+\sqrt{5}i$, $2-\sqrt{5}i$

$\left(x-2-\sqrt{5}i\right)\left(x-2-\sqrt{5}i\right) = \left(x-2\right)^2+5 = x^2-4x+9$ is a factor.

$f(x) = \left(x^2-4x+9\right)\left(2x+1\right)$

Zeros: $x = -\dfrac{1}{2}$, $2 \pm \sqrt{5}i$

125. $f(x) = \dfrac{2-x}{x+3}$

(a) Domain: all $x \neq -3$

(b) Not continuous

(c) Horizontal asymptote: $y = -1$

Vertical asymptote: $x = -3$

126. $f(x) = \dfrac{4x}{x-8}$

(a) Domain: all $x \neq 8$

(b) Not continuous

(c) Horizontal asymptote: $y = 4$

Vertical asymptote: $x = 8$

127. $f(x) = \dfrac{2}{x^2-3x-18} = \dfrac{2}{(x-6)(x+3)}$

(a) Domain: all $x \neq 6, -3$

(b) Not continuous

(c) Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = 6$, $x = -3$

128. $f(x) = \dfrac{2x^2+3}{x^2+x+3}$

The denominator x^2+x+3 has no real zeros.

(a) Domain: all x

(b) Continuous

(c) Horizontal asymptote: $y = 2$

Vertical asymptote: none

129. $f(x) = \dfrac{7+x}{7-x}$

(a) Domain: all $x \neq 7$

(b) Not continuous

(c) Horizontal asymptote: $y = -1$

Vertical asymptote: $x = 7$

130. $f(x) = \dfrac{6x}{x^2-1} = \dfrac{6x}{(x+1)(x-1)}$

(a) Domain: all $x \neq \pm 1$

(b) Not continuous

(c) Horizontal asymptote: $y = 0$

Vertical asymptote: $x = \pm 1$

131. $f(x) = \dfrac{4x^2}{2x^2-3}$

(a) Domain: all $x \neq \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

(b) Not continuous

(c) Horizontal asymptote: $y = 2$

Vertical asymptote: $x = \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

132. $f(x) = \dfrac{3x^2-11x-4}{x^2+2}$

(a) Domain: all x

(b) Continuous

(c) Horizontal asymptote: $y = 3$

No vertical asymptote

133. $f(x) = \dfrac{2x-10}{x^2-2x-15} = \dfrac{2(x-5)}{(x-5)(x+3)} = \dfrac{2}{x+3}$, $x \neq 5$

(a) Domain: all $x \neq 5, -3$

(b) Not continuous

(c) Vertical asymptote: $x = -3$

(There is a hole at $x = 5.$)

Horizontal asymptote: $y = 0$

134. $f(x)=\dfrac{x^3-4x^2}{x^2+3x+2}=\dfrac{x^2(x-4)}{(x+2)(x+1)}$

(a) Domain: all $x\neq -1,\ -2$
(b) Not continuous
(c) Vertical asymptote: $x=-2,\ x=-1$
No horizontal asymptotes

135. $f(x)=\dfrac{x-2}{|x|+2}$

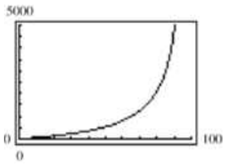
(a) Domain: all real numbers
(b) Continuous
(c) No vertical asymptotes
Horizontal asymptotes: $y=1,\ y=-1$

136. $f(x)=\dfrac{2x}{|2x-1|}$

(a) Domain: all $x\neq\dfrac{1}{2}$
(b) Not continuous
(c) Vertical asymptote: $x=\dfrac{1}{2}$
Horizontal asymptotes: $y=1$ (to the right)
$y=-1$ (to the left)

137. $C=\dfrac{528p}{100-p},\ 0\le p<100$

(a) When $p=25,\ C=\dfrac{528(25)}{100-25}=\$176$ million.
When $p=50,\ C=\dfrac{528(50)}{100-50}=\$528$ million.
When $p=75,\ C=\dfrac{528(75)}{100-75}=\$1584$ million.

(b) 

Answers will vary.

(c) No. As $p\to 100$, C approaches infinity.

138. $y=\dfrac{1.568x-0.001}{6.360x+1},\ x>0$

The moth will be satiated at the horizontal asymptote,
$y=\dfrac{1.568}{6.360}\approx 0.247$ mg.

139. $f(x)=\dfrac{x^2-5x+4}{x^2-1}$
$=\dfrac{(x-4)(x-1)}{(x-1)(x+1)}$
$=\dfrac{x-4}{x+1},\ x\neq 1$

Vertical asymptote: $x=-1$
Horizontal asymptote: $y=1$
No slant asymptotes
Hole at $x=1$

140. $f(x)=\dfrac{2x^2-7x+3}{2x^2-3x-9}$
$=\dfrac{(x-3)(2x-1)}{(x-3)(2x+3)}$
$=\dfrac{2x-1}{2x+3},\ x\neq 3$

Vertical asymptote: $x=-\dfrac{3}{2}$
Horizontal asymptote: $y=1$
No slant asymptotes
Hole at $x=3$

141. $f(x)=\dfrac{3x^2+5x-2}{x+1}$
$=\dfrac{(3x-1)(x+2)}{x+1}$

Vertical asymptote: $x=-1$
Horizontal asymptote: none
Long division gives:

$$\begin{array}{r} 3x+2 \\ x+1\overline{)3x^2+5x-2} \\ \underline{3x^2+3x} \\ 2x-2 \\ \underline{2x+2} \\ -4 \end{array}$$

Slant asymptote: $y=3x+2$

142. $f(x)=\dfrac{2x^2+5x+3}{x-2}$
$=\dfrac{(2x+3)(x+1)}{x-2}$

Vertical asymptote: $x=2$
Horizontal asymptote: none
Long division gives:

$$\begin{array}{r} 2x+9 \\ x-2\overline{)2x^2+5x+3} \\ \underline{2x^2-4x} \\ 9x+3 \\ \underline{9x-18} \\ 21 \end{array}$$

Slant asymptote: $y=2x+9$

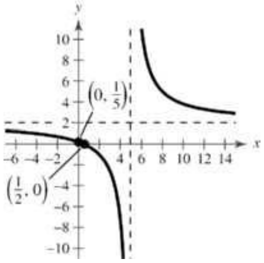
143. $f(x) = \dfrac{2x-1}{x-5}$

Intercepts: $\left(0, \dfrac{1}{5}\right), \left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 2$

x	-4	-1	0	$\frac{1}{2}$	1	6	8
y	1	$\frac{1}{2}$	$\frac{1}{5}$	0	$-\frac{1}{4}$	11	5



144. $f(x) = \dfrac{x-3}{x-2}$

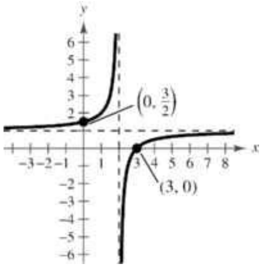
x-intercept: $(3, 0)$

y-intercept: $\left(0, \dfrac{3}{2}\right)$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

x	-1	0	1	3	4	5
y	$\frac{4}{3}$	$\frac{3}{2}$	2	0	$\frac{1}{2}$	$\frac{2}{3}$



145. $f(x) = \dfrac{2x^2}{x^2-4}$

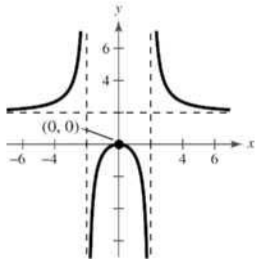
Intercept: $(0, 0)$

y-axis symmetry

Vertical asymptotes: $x = \pm 2$

Horizontal asymptote: $y = 2$

	-6	-4	-1	0	1	4	6
y	$\frac{9}{4}$	$\frac{8}{3}$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{8}{3}$	$\frac{9}{4}$



146. $f(x) = \dfrac{5x}{x^2+1}$

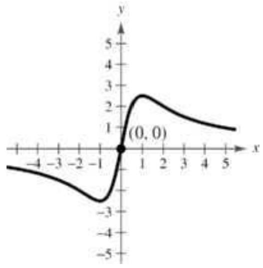
Intercept: $(0, 0)$

Symmetry: origin

Horizontal asymptote: $y = 0$

No vertical asymptotes

x	-3	-2	-1	0	1	2	3
y	$-\frac{3}{2}$	-2	$-\frac{5}{2}$	0	$\frac{5}{2}$	2	$\frac{3}{2}$



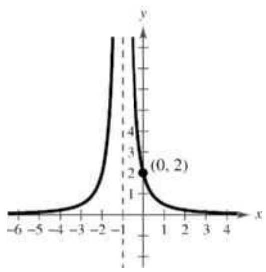
147. $f(x) = \dfrac{2}{(x+1)^2}$

Intercept: $(0, 2)$

Horizontal asymptote: $y = 0$

Vertical asymptote: $x = -1$

x	-4	-3	-2	0	1	2
y	$\frac{2}{9}$	$\frac{1}{2}$	2	2	$\frac{1}{2}$	$\frac{2}{9}$



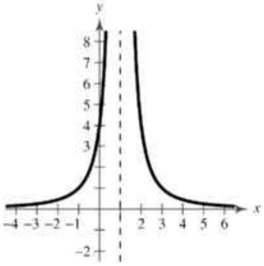
148. $h(x) = \dfrac{4}{(x-1)^2}$

y-intercept: $(0, 4)$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 0$

x	-2	-1	0	2	3	4
y	$\frac{4}{9}$	1	4	4	1	$\frac{4}{9}$



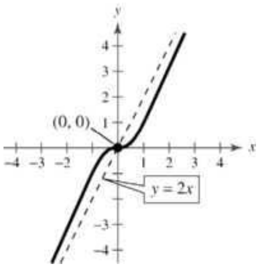
149. $f(x) = \dfrac{2x^3}{x^2+1} = 2x - \dfrac{2x}{x^2+1}$

Intercept: $(0, 0)$

Origin symmetry

Slant asymptote: $y = 2x$

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



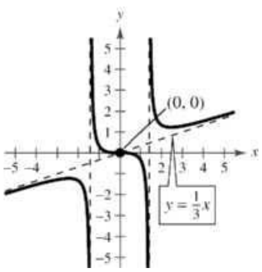
150. $f(x) = \dfrac{x^3}{3x^2-6} = \dfrac{1}{3}x + \dfrac{2x}{3x^2-6} = \dfrac{1}{3}\left[x + \dfrac{2x}{x^2-2}\right]$

Intercepts: $(0, 0)$

Vertical asymptotes: $x = \pm\sqrt{2}$

Slant asymptote: $y = \dfrac{1}{3}x$

x	-4	-2	-1	0	1	2	4
y	-1.52	-1.33	0.33	0	-0.33	1.33	1.52



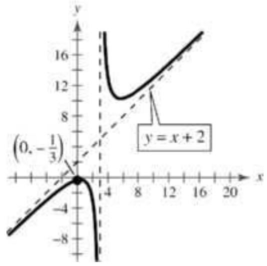
151. $f(x) = \dfrac{x^2-x+1}{x-3} = x + 2 + \dfrac{7}{x-3}$

Intercept: $\left(0, -\dfrac{1}{3}\right)$

Vertical asymptote: $x = 3$

Slant asymptote: $y = x + 2$

x	-4	0	2	4	5
y	-3	$-\frac{1}{3}$	-3	13	10.5



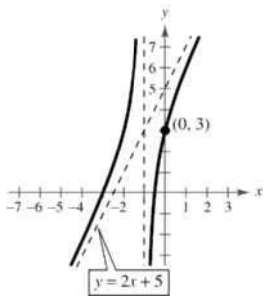
152. $f(x) = \dfrac{2x^2+7x+3}{x+1} = 2x + 5 - \dfrac{2}{x+1}$

Intercepts: $(0, 3)$, $(-3, 0)$, $\left(-\dfrac{1}{2}, 0\right)$

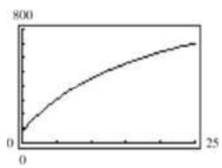
Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x + 5$

x	-5	-4	-3	-2	0	1
y	$-\frac{9}{2}$	$-\frac{7}{3}$	0	3	3	6



153. (a)



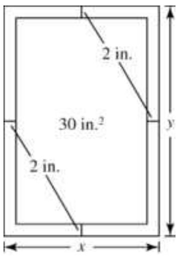
5 years: $N(5) = \dfrac{20(4+3(5))}{1+0.05(5)} = 304$ thousand fish

10 years: $N(10) = \dfrac{20(4+3(10))}{1+0.05(10)} = 453.\overline{3}$ thousand fish

25 years: $N(25) = \dfrac{20(4+3(25))}{1+0.05(25)} = 702.\overline{2}$ thousand fish

(b) The maximum number of fish is $N = 1{,}200{,}000$.
The graph of N has a horizontal asymptote at $N = 1200$ or 1,200,000 fish.

154. (a)

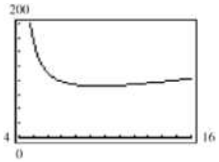


(b) $(x-4)(y-4) = 30 \Rightarrow y = 4 + \dfrac{30}{x-4}$

$$\text{Area} = A = xy = x\left[4 + \frac{30}{x-4}\right]$$
$$= x\left[\frac{4x - 16 + 30}{x-4}\right]$$
$$= \frac{2x(2x+7)}{x-4}$$

(c) Domain: $x > 4$

(d)



9.48 by 9.48

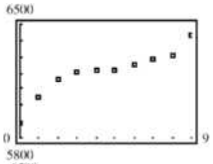
155. Quadratic model

156. Neither

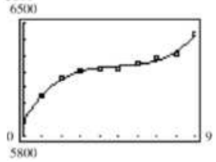
157. Linear model

158. Quadratic model

159. (a)

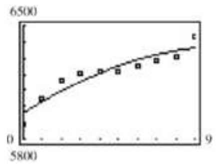


(b)



(c) $S = -3.49t^2 + 76.3t + 5958$; $R^2 \approx 0.8915$

(d)



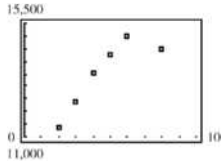
(e) The cubic model is a better fit because it more closely follows the pattern of the data.

(f) For 2012, let $t = 12$.

$$S(12) = 2.520(12)^3 - 37.51(12)^2 + 192.4(12) + 5895$$
$$= 7157 \text{ stations}$$

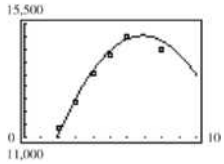
160.

(a)



(b) $S = -161.65t^2 + 2230.9t + 7333$

(c)



The models fits the data well.

(d) Set $S = 11$, and solve for t.

$-161.65t^2 + 2230.9t + 7333 = 11000$

$-161.65t^2 + 2230.9t - 3667 = 0$

Using the Quadratic Formula,
$t \approx 11.9$ or 2012.

(e) Answers will vary.

161. False. The degree of the numerator is two more than the degree of the denominator.

162. False. A fourth degree polynomial with real coefficients can have at most four zeros. Since $-8i$ and $4i$ are zeros, so are $8i$ and $-4i$.

163. False. $(1+i)+(1-i) = 2$, a real number

164. The domain will still restrict any value that makes the denominator equal to zero.

165. Not every rational function has a vertical asymptote. For example,

$$y = \frac{x}{x^2+1}.$$

166. $\sqrt{-6}\sqrt{-6} \neq \sqrt{(-6)(-6)}$

In fact, $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = -6$.

167. The error is $\sqrt{-4} \neq 4i$. In fact,

$-i\left(\sqrt{-4} - 1\right) = -i(2i - 1) = 2 + i$.

168.

(a) $i^{40} = (i^4)^{10} = 1^{10} = 1$

(b) $i^{25} = i(i^{24}) = i(1) = i$

(c) $i^{50} = i^2(i^{48}) = (-1)(1) = -1$

(d) $i^{67} = i^3(i^{64}) = -i(1) = -i$

Chapter 2 Test

1. $y = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1$

Vertex: $(-2, -1)$

$x = 0 \Rightarrow y = 3$

$y = 0 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow (x+3)(x+1) = 0 \Rightarrow x = -1, -3$

Intercepts: $(0, 3)$, $(-1, 0)$, $(-3, 0)$

2. Let $y = a(x-h)^2 + k$. The vertex $(3, -6)$ implies that $y = a(x-3)^2 - 6$. For $(0, 3)$ you obtain

$3 = a(0-3)^2 - 6 = 9a - 6 \Rightarrow a = 1.$

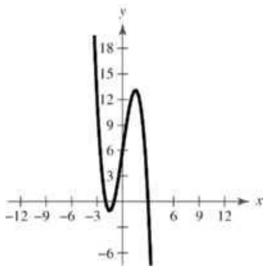
Thus, $y = (x-3)^2 - 6 = x^2 - 6x + 3.$

3. $f(x) = 4x^3 + 4x^2 + x = x(4x^2 + 4x + 1) = x(2x+1)^2$

Zeros: 0 (multiplicity 1)

$-\frac{1}{2}$ (multiplicity 2)

4. $f(x) = -x^3 + 7x + 6$



5.

$$\begin{array}{r} 3x \\ x^2+1 \overline{\smash{)}\, 3x^3 + 0x^2 + 4x - 1} \\ \underline{3x^3 + 3x} \\ x - 1 \end{array}$$

$3x + \dfrac{x-1}{x^2+1}$

6.

$$\begin{array}{r|rrrrr} 2 & 2 & 0 & -5 & 0 & -3 \\ & & 4 & 8 & 6 & 12 \\ \hline & 2 & 4 & 3 & 6 & 9 \end{array}$$

$2x^3 + 4x^2 + 3x + 6 + \dfrac{9}{x-2}$

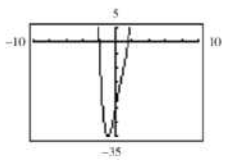
7.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -6 & 5 & -1 \\ & & -6 & 12 & -12 & 14 \\ \hline & 3 & -6 & 6 & -7 & 13 \end{array}$$

$f(-2) = 13$

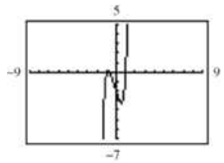
8. Possible rational zeros:

$\pm 24, \pm 12, \pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1, \pm \frac{3}{2}, \pm \frac{1}{2}$



Rational zeros: $-2, \frac{3}{2}$

9. Possible rational zeros: $\pm 2, \pm 1, \pm \frac{2}{3}, \pm \frac{1}{3}$



Rational zeros: $\pm 1, -\frac{2}{3}$

10. $f(x) = x^3 - 7x^2 + 11x + 19$

$= (x+1)(x^2 - 8x + 19)$

For the quadratic,

$x = \dfrac{8 \pm \sqrt{64 - 4(19)}}{2} = 4 \pm \sqrt{3}i.$

Zeros: $-1, 4 \pm \sqrt{3}\, i$

$f(x) = (x+1)(x - 4 + \sqrt{3}i)(x - 4 - \sqrt{3}i)$

11. $(-8 - 3i) + (-1 - 15i) = -9 - 18i$

12. $\left(10 + \sqrt{-20}\right) - \left(4 - \sqrt{-14}\right) = 6 + 2\sqrt{5}i + \sqrt{14}i = 6 + \left(2\sqrt{5} + \sqrt{14}\right)i$

13. $(2+i)(6-i) = 12 + 6i - 2i + 1 = 13 + 4i$

14. $(4+3i)^2 - (5+i)^2 = (16 + 24i - 9) - (25 + 10i - 1) = -17 + 14i$

15. $\dfrac{8+5i}{6-i} \cdot \dfrac{6+i}{6+i} = \dfrac{48 + 30i + 8i - 5}{36+1} = \dfrac{43}{37} + \dfrac{38}{37}i$

16. $\dfrac{5i}{2+i} \cdot \dfrac{2-i}{2-i} = \dfrac{10i + 5}{4+1} = 1 + 2i$

17. $\dfrac{(2i-1)}{(3i+2)} \cdot \dfrac{2-3i}{2-3i} = \dfrac{6 - 2 + 4i + 3i}{4+9}$

$= \dfrac{4}{13} + \dfrac{7}{13}i$

18. $x^2 + 75 = 0$

$x^2 = -75$

$x = \pm\sqrt{-75}$

$= \pm 5\sqrt{3}i$

19. $x^2 - 2x + 8 = 0$

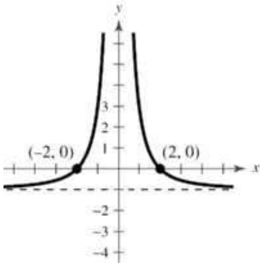
$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{-28}}{2}$

$x = \dfrac{2 \pm 2\sqrt{7}i}{2}$

$x = 1 \pm \sqrt{7}i$

20.



Vertical asymptote: $x = 0$

Intercepts: $(2, 0)$, $(-2, 0)$

Symmetry: y-axis

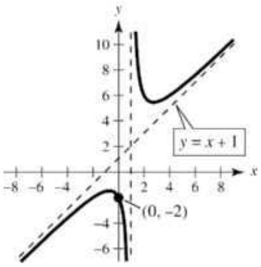
Horizontal asymptote: $y = -1$

21. $g(x) = \dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$

Vertical asymptote: $x = 1$

Intercept: $(0, -2)$

Slant asymptote: $y = x + 1$

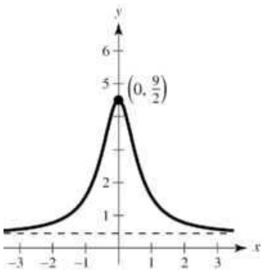


22. $f(x) = \dfrac{2x^2 + 9}{5x^2 + 2}$

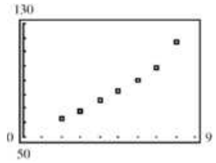
Horizontal asymptote: $y = \dfrac{2}{5}$

y-axis symmetry

Intercept: $\left(0, \dfrac{9}{2}\right)$

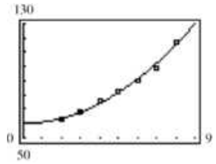


23. (a)



(b) $A = 0.861t^2 + 0.03t + 60.0$

(c)



The model fits the data well.

(d) For 2010, let $t = 10$.

$$A(10) = 0.861(10)^2 + 0.03(10) + 60.0$$
$$= \$146.4 \text{ million}$$

For 2012, let $t = 12$.

$$A(12) = 0.861(12)^2 + 0.03(12) + 60.0$$
$$\approx \$184.3 \text{ billion}$$

(e) Answers will vary.

Chapter 2 Polynomial and Rational Functions

Section 2.1 Quadratic Functions

Course/Section
Lesson Number
Date

Section Objectives: Students will know how to sketch and analyze graphs of quadratic functions.

I. The Graph of a Quadratic Function (pp. 92–94) Pace: 10 minutes

- Define a polynomial function to be a function of the form
  $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$

  where each a_I is a real number and n is a nonnegative integer. Note that we have already dealt with two forms of this equation, for the cases $n = 0$ and $n = 1$. In this section we focus on $n = 2$. Functions of this form are called **quadratic functions,** and we simplify the notation to
  $f(x) = ax^2 + bx + c$, with $a \neq 0$.
  The graph of a quadratic function is a parabola.

- Draw the graph of $y = x^2$ and identify the vertex (at the origin) and the axis of symmetry. Direct the students' attention to the "Library of Functions" feature on page 93 and review with them the characteristics of a quadratic function. Emphasize that the parabola opens up if $a > 0$ and down if $a < 0$.

II. The Standard Form of a Quadratic Function (pp. 95–96)
Pace: 10 minutes

- Apply the material covered in Section 1.4 to produce the following.
  The quadratic function

  $f(x) = a(x - h)^2 + k, a \neq 0$

  is in standard form. The graph of f is a parabola with a vertical axis $x = h$ and vertex at (h, k). If $a > 0$, the parabola opens upward, and if $a < 0$, the parabola opens downward. Work the *Exploration* on page 95 of the text.

Example 1. Find the vertex of the following parabola.

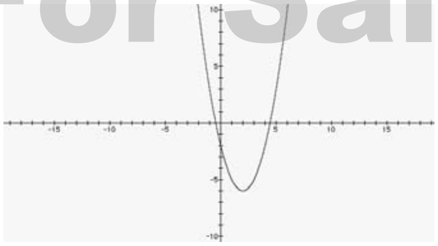
$$\begin{aligned} f(x) &= -2x^2 - 4x + 1 \\ &= -2(x^2 + 2x) + 1 \\ &= -2(x^2 + 2x + 1) + 1 + 2 \\ &= -2(x^2 + 1)^2 + 3 \end{aligned}$$

The vertex is (–1, 3).

Example 2. Graph the following quadratic function.

$$\begin{aligned} f(x) &= x^2 - 4x - 2 \\ &= (x^2 - 4x + 4) + 4 - 2 \\ &= (x - 2)^2 + 2 \end{aligned}$$

The vertex is at (2, 2), the parabola opens upward, and there is no change in the width.



Example 3. Find the standard form of the equation of the parabola that has vertex (1, – 2) and passes through the point (3, 6).

From the vertex, we have this much of the equation: $f(x) = a(x - 1)^2 - 2$. To find a, we substitute the point (3, 6) and solve for a.

$$6 = a(3-1)^2 - 2$$
$$6 = 4a - 2$$
$$8 = 4a$$
$$2 = a$$

The equation is $f(x) = 2(x - 1)^2 - 2$.

III. Finding Minimum and Maximum Values (pp. 97– 98)
Pace: 5 minutes

- Graph $y = (x - 4)^2 + 5$. Ask the class, "What is the minimum value of y?"

Example 4. The daily cost of manufacturing a particular product is given by

$$C(x) = 1200 - 7x + 0.1x^2$$

where x is the number of units produced each day. Determine how many units should be produced daily to minimize cost.

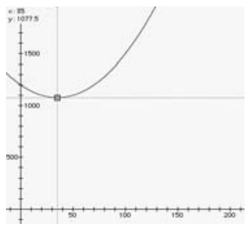
Algebraic Solution

We need to find h.

$$\begin{aligned} C(x) &= 1200 - 7x + 0.1x^2 \\ &= 0.1(x^2 - 70x) + 1200 \\ &= 0.1(x^2 - 70x + 35^2) + 1200 + 122.5 \\ &= 0.1(x - 35)^2 + 1322.5 \end{aligned}$$

Producing 35 units per day will minimize cost.

Graphical Solution



Course/Section Lesson Number Date

Chapter 2 Polynomial and Rational Functions

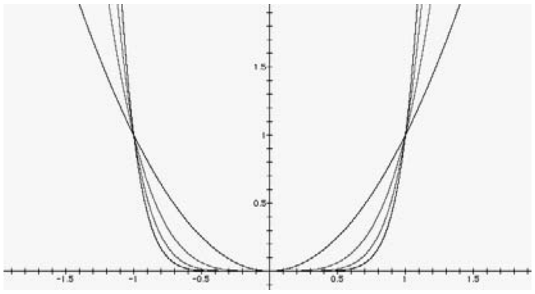
Section 2. 2 Polynomial Functions of Higher Degree

Section Objectives: Students will know how to sketch and analyze graphs of polynomial functions.

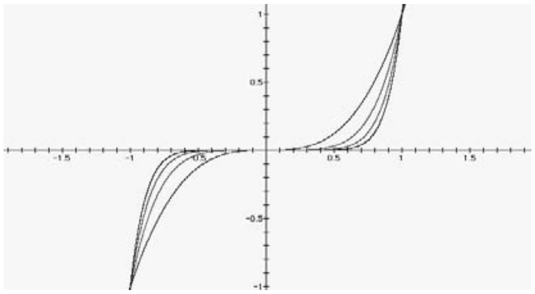
I. Graphs of Polynomial Functions (pp. 103–104) Pace: 10 minutes

- Discuss these characteristics of the graphs of polynomial functions.
  1. Polynomial functions are **continuous.** This means that the graphs of polynomial functions have no breaks, holes, or gaps.
  2. The graphs of polynomial functions have only nice smooth turns and bends. There are no sharp turns, as in the graph of $y = |x|$.

- We will first look at the simplest polynomials, $f(x) = x^n$. We can break these functions into two cases, n is even and n is odd.



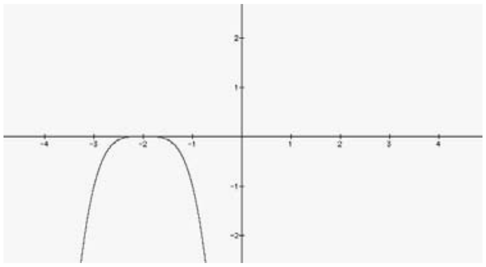
Here n is even. Note how the graph flattens at the origin as n increases. Note also that for n even, the graph touches, but does not cross, the axis at the x-intercept.



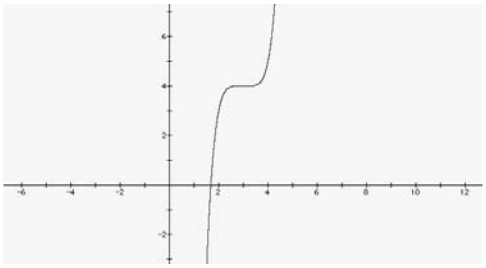
Here n is odd. Note how the graph flattens at the origin as n increases. Note also that for n odd, the graph crosses the axis at the x-intercept.

Example 1. Sketch the graphs of the following.

a) $f(x) = -(x + 2)^4$



b) $f(x) = (x - 3)^5 + 4$



II. The Leading Coefficient Test (pp. 105–106) Pace: 10 minutes

- Work the *Exploration* on page 105 of the text. Then summarize with the following chart.

$f(x) = a_n x^n + \ldots$	$a_n > 0$	$a_n < 0$
n even	$\nwarrow \quad \nearrow$	$\swarrow \quad \searrow$
n odd	$\swarrow \quad \nearrow$	$\nwarrow \quad \searrow$

Example 2. Describe the right-hand and left-hand behavior of the graph of each function.

a) $f(x) = -x^4 + 7x^3 - 14x - 9$
Falls to the left and right

b) $g(x) = 5x^5 + 2x^3 - 14x^2 + 6$
Falls to the left and rises to the right

III. Zeros of Polynomial Functions (pp. 106–110) Pace: 15 minutes

- State the following as being equivalent statements, where f is a polynomial function and a is a real number.
  1. $x = a$ is a zero of f.
  2. $x = a$ is a solution of the equation $f(x) = 0$.
  3. $(x - a)$ is a factor of $f(x)$.
  4. $(a, 0)$ is an x-intercept of the graph of f.

Example 3. Find the x-intercept(s) of the graph of $f(x) = x^3 - x^2 - x + 1$.

Algebraic Solution

$$f(x) = x^3 - x^2 - x + 1$$
$$0 = x^2(x - 1) - 1(x - 1)$$
$$0 = (x^2 - 1)(x - 1)$$
$$0 = (x - 1)^2(x + 1)$$
$$(x - 1)^2 = 0 \Rightarrow x = 1$$
$$x + 1 = 0 \Rightarrow x = -1$$

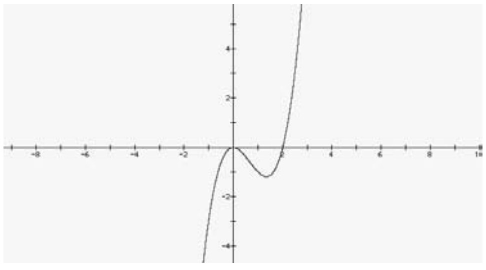
The x-intercepts are $(-1, 0)$ and $(1, 0)$.

Graphical Solution

- Note that in the above example, 1 is a repeated zero. In general, a factor $(x - a)^k$, $k > 1$, yields a repeated zero $x = a$ of multiplicity k. If k is odd, the graph *crosses* the x-axis at $x = a$. If k is even, the graph only *touches* the x-axis at $x = a$.

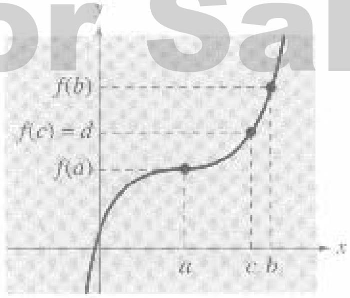
Example 4. Sketch the graph of $f(x) = x^3 - 2x^2$.

1. Since $f(x) = x^2(x - 2)$, the x-intercepts are $(0, 0)$ and $(2, 0)$. Also, 0 has multiplicity 2, therefore the graph will just touch at $(0, 0)$.
2. The graph will rise to right and fall to the left.
3. Additional points on the graph are $(1, -1)$, $(-1, -3)$, and $(-2, -16)$.



IV. The Intermediate Value Theorem (p. 111) Pace: 10 minutes

- Draw and label a picture similar to the one on page 111 of the text, which has been included here.



- Show how the Intermediate Value Theorem, which follows here, applies.

  Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

- We can use this theorem to approximate zeros of polynomial functions if $f(a)$ and $f(b)$ have different signs.

Example 5. Use the Intermediate Value Theorem to approximate the real zero of $f(x) = 4x^3 - 7x^2 - 21x + 18$ on $[0, 1]$.

Note that $f(0) = 18$ and $f(1) = -6$. Therefore, by the Intermediate Value Theorem, there must be a zero between 0 and 1.

Note further that $f(0.7) = 1.242$ and $f(0.8) = -1.232$. Therefore, by the Intermediate Value Theorem, there must be a zero between 0.7 and 0.8.

This process can be repeated until the desired accuracy is obtained.

Chapter 2 Polynomial and Rational Functions

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Section 2.3 Real Zeros of Polynomial Functions

Section Objectives: Students will know how to use long division and synthetic division to divide polynomials by other polynomials, how to determine the number of zeros of a polynomial, and how to find real zeros of a polynomial function.

I. Long Division of Polynomials (pp. 116−118) Pace: 10 minutes

- Review long division of integers. Use 7213 y 61 as an illustration. Now divide polynomials the same way.

Tip: Many students have trouble with the subtraction; they want to add. They will need to be reminded to subtract.

Example 1. Divide $2x^3 - 5x^2 + x - 8$ by $x - 3$.

$$\begin{array}{r} 2x^2 + x + 4 \\ x-3\overline{)\,2x^3 - 5x^2 + x - 8} \\ \underline{2x^2 - 6x^2} \\ x^2 + x \\ \underline{x^2 - 3x} \\ 4x - 8 \\ \underline{4x - 12} \\ 4 \end{array}$$

The result is $2x^2 + x + 4 + \dfrac{4}{x-3}$.

- State the **Division Algorithm:**
For all polynomials $f(x)$ and $d(x)$ such that the degree of d is less than or equal to the degree of f and $d(x) \neq 0$, there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x) = d(x)q(x) + r(x)$, where $r(x) = 0$ *or* the degree of r is less than the degree of d.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

This is why we write the remainder in the way that we do.

Example 2. Divide $3x^3 - x^2 + 5x - 3$ by $x - 2$.

$$\begin{array}{r} 3x^2 + 5x + 12 \\ x-2\overline{)\,3x^3 - x^2 + 2x - 3} \\ \underline{3x^2 - 6x^2} \\ 5x^2 + 2x \\ \underline{5x^2 - 10x} \\ 12x - 3 \\ \underline{12x - 24} \\ 21 \end{array}$$

The result is $3x^2+5x+12+\dfrac{21}{x-2}$.

II. Synthetic Division (p. 119) Pace: 10 minutes

- Use the above example to develop synthetic division as follows.
  1. State that the following only applies when the divisor is $x - c$, and when every descending power of x has a place in the dividend.
  2. Eliminate all the variables, since we are keeping everything in nice columns.

$$\begin{array}{rrrrrr}
 & & 3 & 5 & 12 & \\
\cline{3-6}
1-2 \,) & 3 & -1 & 2 & -3 & \\
 & \underline{3} & \underline{-6} & & & \\
 & & 5 & 2 & & \\
 & & \underline{5} & \underline{-10} & & \\
 & & & 12 & -3 & \\
 & & & \underline{12} & \underline{-24} & \\
 & & & & 21 &
\end{array}$$

3. Now eliminate the 1 in the divisor, since, to do this, that coefficient must always be 1. Also, eliminate the numbers that are, by design, the same as the number directly above them.

$$\begin{array}{rrrrr}
 & & 3 & 5 & 12 \\
\cline{2-5}
-2 \,) & 3 & -1 & 2 & -3 \\
 & & \underline{-6} & & \\
 & & 5 & 2 & \\
 & & & \underline{-10} & \\
 & & & 12 & -3 \\
 & & & & \underline{-24} \\
 & & & & 21
\end{array}$$

4. Next eliminate the numbers that we “bring down.”

$$\begin{array}{rrrrr}
 & & 3 & 5 & 12 \\
\cline{2-5}
-2 \,) & 3 & -1 & 2 & -3 \\
 & & \underline{-6} & & \\
 & & 5 & & \\
 & & & \underline{-10} & \\
 & & & 12 & \\
 & & & & \underline{-24} \\
 & & & & 21
\end{array}$$

5. Now each column has a number, a line, and another number. Let us merge all of the lines.

$$\begin{array}{r|rrrr}
\multicolumn{1}{r}{} & & 3 & 5 & 12 \\
\cline{2-5}
-2 & 3 & -1 & 2 & -3 \\
 & & -6 & -10 & -24 \\
\cline{2-5}
\multicolumn{1}{r}{} & & 5 & 12 & 21
\end{array}$$

6. Note that if we place a 3 at the start of the last row and isolate the 21, the top and bottom rows will look the same. So, eliminate the top row.

$$\begin{array}{r|rrr|r}
\cline{2-5}
-2 & 3 & -1 & 2 & -3 \\
 & & -6 & -10 & -24 \\
\cline{2-5}
\multicolumn{1}{r}{} & 3 & 5 & 12 & \multicolumn{1}{|r}{21} \\
\cline{5-5}
\end{array}$$

7. The last thing to do is to eliminate the subtraction, since we prefer to add. We need to change the sign of everything in the second row. We can achieve this by changing the sign of the divisor, since everything in the second row is a multiple of the divisor.

$$\begin{array}{r|rrrr} 2 & 3 & -1 & 2 & -3 \\ & & 6 & 10 & 24 \\ \hline & 3 & 5 & 12 & \boxed{21} \end{array}$$

Example 3. Use synthetic division to divide $4x^4 - 2x^2 - x + 1$ by $x + 2$.

$$\begin{array}{r|rrrrr} -2 & 4 & 0 & -2 & -1 & 1 \\ & & -8 & 16 & -28 & 58 \\ \hline & 4 & -8 & 14 & -29 & \boxed{59} \end{array}$$

The result is $4x^3 - 8x^2 + 14x - 29 + \dfrac{59}{x+2}$.

III. The Remainder and Factor Theorems (pp. 120–121) Pace: 15 minutes

- State the **Remainder Theorem:**
  If a polynomial $f(x)$ is divided by $x - k$, the remainder is $r = f(k)$.

Tip: If you have time you should prove this theorem. A proof is given on page 180.

Example 4. Use the Remainder Theorem to find $f(1)$.

$f(x) = x^3 - 2x^2 - 4x + 1$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -4 & 1 \\ & & 1 & -1 & -5 \\ \hline & 1 & -1 & -5 & \boxed{-4} \end{array}$$

$f(1) = -4$.

- State the **Factor Theorem:**
  A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$. (A proof of this theorem is given on page 180.)

Example 5. Show that $x - 1$ is a factor of $f(x) = x^4 - 1$.

$f(1) = 1^4 - 1 = 0$

Since $f(1) = 0$, by the Factor Theorem, $x - 1$ is a factor of $f(x)$.

IV. The Rational Zero Test (pp. 122–124) Pace: 15 minutes

- Ask the class how they would solve $x^3 + 6x - 7 = 0$. Then ask them how they would solve the same equation if they knew that the rational zeros (if they existed) would have to be in the list $\pm 1, \pm 2, \pm 3, \pm 6$. Now state the **Rational Zero Test:**

  If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients with $a_n \neq 0$ and $a_0 \neq 0$, then any rational zero of f will be of the form p/q, where p is a factor of a_0 and q is a factor of a_n.

Example 6. Solve $x^3 - 7x - 6 = 0$.

p: $\pm 1, \pm 2, \pm 3, \pm 6$
q: ± 1
p/q: $\pm 1, \pm 2, \pm 3, \pm 6$

Use synthetic division to find a number from the list that is a solution.

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

We now have $(x + 1)(x^2 - x - 6) = 0$. $x - 1 = 0 \Rightarrow x = 1$.
$x^2 - x - 6 = (x - 3)(x + 2) = 0 \Rightarrow x = -2$ or $x = 3$.

Example 7. Find all real zeros of $3x^3 - 20x^2 + 23x + 10$.

p: $\pm 1, \pm 2, \pm 5, \pm 10$
q: $\pm 1, \pm 3$
p/q: $\pm 1, \pm 2, \pm 5, \pm 10, \pm 1/3, \pm 2/3, \pm 5/3, \pm 10/3$

$$\begin{array}{r|rrrr} 2 & 3 & -20 & 23 & 10 \\ & & 6 & -28 & -10 \\ \hline & 3 & -14 & -5 & \boxed{0} \end{array}$$

One zero is 2. Two more come from solving

$3x^2 - 14x - 5 = 0$
$(3x + 1)(x - 5) = 0$
$x - 5 = 0 \Rightarrow \quad x = 5$

$$3x + 1 = 0 \Rightarrow \quad x = -\frac{1}{3}$$

V. Other Tests for Zeros of Polynomial s (pp. 124–126)
Pace: 10 minutes

- **Descartes' Rule of Signs** can give us information about the real zeros of a polynomial function:

For the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with real coefficients and $a_0 \neq 0$,

1. The number of *positive* real zeros of f is either equal to the number of variations in sign of $f(x)$ or less than that number by an even integer.
2. The number of *negative* real zeros of f is either equal to the number of variations in sign of $f(-x)$ or less than that number by an even integer.

A **variation in sign** means that two consecutive nonzero coefficients have opposite signs.

Example 8. Describe the possible real zeros of $f(x) = 2x^3 - x^2 + x + 4$.

The original polynomial has two variations in sign. The polynomial $f(-x) = 2(-x)^3 - (-x)^2 + (-x) + 4 = -2x^3 - x^2 - x + 4$ has one variation in sign. Therefore, by Descartes' Rule of Signs, the original polynomial has either two positive or no positive real zeros, and one negative real zero.

- A way of dealing with a very large list generated by the Rational Zero Test is the **Upper and Lower Bound Rule**. Before you state this rule, define upper and lower bounds. A real number b is an **upper bound** for the real zeros of f if there no zeros of f greater than b. A real number b is a **lower bound** for the real zeros of f if there no zeros of f less than b.

Let f be a polynomial function with real coefficients and a positive leading coefficient. Suppose $f(x)$ is divided by $x - c$ using synthetic division.

1. If $c > 0$ and each number in the last row is either positive or zero, then c is an upper bound for the real zeros of f.
2. If $c < 0$ and each number in the last row is alternately positive and negative (zero entries count as either positive or negative), then c is a lower bound for the real zeros of f.

Example 9. Find all real zeros of $f(x) = x^4 - 6x^3 + 6x^2 + 10x - 3$.
p: $\pm1, \pm3$
q: ±1
p/q: $\pm1, \pm3$

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 6 & 10 & -3 \\ & & -1 & 7 & -13 & 3 \\ \hline & 1 & -7 & 13 & -3 & \boxed{0} \end{array}$$

We look for the positive zero, since no other negative numbers can be zeros. Testing 1 does not work, so 3 should be a zero.

$$\begin{array}{r|rrrr} 3 & 1 & -7 & 13 & -3 \\ & & 3 & -12 & 3 \\ \hline & 1 & -4 & 1 & \boxed{0} \end{array}$$

Now we solve $x^2 - 4x + 1 = 0$ using the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}i$$

Therefore, the four zeros of f are –1, 3, and $2 \pm \sqrt{3}i$.

Chapter 2 Polynomial and Rational Functions

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Section 2.4 Complex Numbers

Section Objectives: Students will know how to perform operations with complex numbers and plot complex numbers in the complex plane.

I. The Imaginary Unit *i* (p. 131) Pace: 5 minutes

- We need to define a new set of numbers because simple equations such as $x^2 + 1 = 0$ do not have real solutions. We need a number whose square is –1. So we define $i^2 = -1$. i is called the **imaginary unit.** By adding real numbers to multiples of the imaginary unit, we get the set of **complex numbers,** defined as $\{a + bi \mid a \text{ is real, } b \text{ is real, and } i^2 = -1\}$. $a + bi$ is the **standard form** of a complex number. a is called the **real part** and bi is the **imaginary part**.

- Two complex numbers $a + bi$ and $c + di$ are equal to each other if and only if $a = c$ and $b = d$.

II. Operations with Complex Numbers (pp.132–133) Pace: 10 minutes

- To add two complex numbers, we add the two real parts and then add the two imaginary parts. That is,
$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

- Also note that the additive identity of the complex numbers is zero. Furthermore, the additive inverse of $a + bi$ is $-(a + bi) = -a - bi$.

Example 1. Add or subtract the following complex numbers.

a) $(6 - 3i) + (1 + 2i) = (6 + 1) + (-3 + 2)i = 7 - i$

b) $(5 - i) - (2 - 4i) = 5 - i - 2 + 4i = 3 + 3i$

- Many of the properties of the real numbers are valid for the complex numbers also, such as both of the Associative and Commutative Properties and both Distributive Properties.

- Using these properties (much as we did with polynomials) is the best method for multiplying two complex numbers.

- Now is a good time to examine the *Exploration* on page 133 of the text.

Example 2. Multiply the following complex numbers.

a) $2(17 - 5i) = 2(17) - 2(5i) = 34 - 10i$

b)
$$\begin{aligned}(3 - i)(5 + 4i) &= 15 + 12i - 5i - 4i^2\\ &= 15 + 12i - 5i + 4\\ &= 19 + 7i\end{aligned}$$

c) $(1 + 7i)^2 + 1^2 + 2(1)(7i) + (7i)^2 = 1 + 14i - 49 = -48 + 14i$

d) $(4 + 5i)(4 - 5i) = 4^2 - (5i)^2 = 16 + 25 = 41$

Tip: Note that in the last example, we took the product of two complex numbers and got a real number. This leads to our next topic.

III. Complex Conjugates (p. 134) Pace: 5 minutes

- Two complex numbers of the forms $a + bi$ and $a - bi$ are called **complex conjugates**. Note how their product is a real number:
$$(a + bi)(a - bi) = a^2 - (bi)^2 = a^2 + b^2$$
To find the quotient of two complex numbers, multiply the numerator and denominator by the complex conjugate of the *denominator*.

Example 3. Find the quotient of the following complex numbers.

a) $$\frac{2}{1-i} = \frac{2}{1-i}\cdot\frac{1+i}{1+i} = \frac{2(1+i)}{1^2+1^2} = \frac{2(1+i)}{2} = 1+i$$

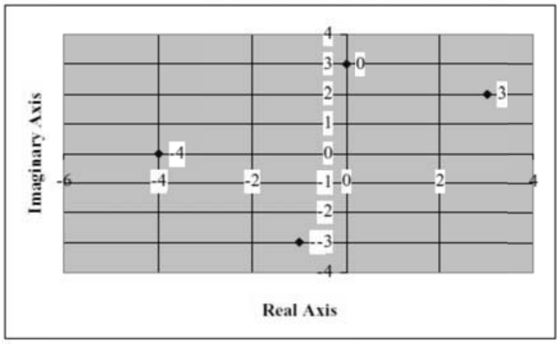
b)
$$\begin{aligned}\frac{2-i}{4+3i} &= \frac{2-i}{4+3i}\cdot\frac{4-3i}{4-3i}\\ &= \frac{8-6i-4i+3i^2}{4^2+3^2}\\ &= \frac{8-3-6i-4i}{16+9}\\ &= \frac{5-10i}{25}\\ &= \frac{1}{5}-\frac{2}{5}i\end{aligned}$$

IV. Fractals and the Mandelbrot Set (pp. 135–136) Pace: 5 minutes

- State that each complex number corresponds to a point in the complex plane.
- Transform the rectangular coordinate system into the **complex plane** by stating that the y-axis becomes the **imaginary axis** and the x-axis becomes the **real axis.** The complex number $a + bi$ is plotted just as the point (a, b).

Example 4. Plot each of the complex numbers in the complex plane.

a) $3 + 2i$
b) $3i$
c) -4
d) $-1 - 3i$



Chapter 2 Polynomial and Rational Functions

Section 2.5 The Fundamental Theorem of Algebra

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Section Objectives: Students will know how to find zeros of a polynomial function.

I. The Fundamental Theorem of Algebra (pp. 139– 140) Pace: 5 minutes

- State the **Fundamental Theorem of Algebra:**
  If $f(x)$ is a polynomial function of degree n, where $n > 0$, then f has at least one zero in the complex number system.
- State the **Linear Factorization Theorem:**
  If $f(x)$ is a polynomial function of degree n, where $n > 0$, then f has precisely n linear factors
  $f(x) = a_n(x - c_1)\,(x - c_2)\ldots(x - c_n)$
  where $c_1, c_2, \ldots, c_n$ are complex zeros.

Tip: It should be pointed out that the zeros in the above theorem may not be distinct.

Example 1. Solve $x^3 + 6x - 7 = 0$.
p: $\pm1, \pm7$
q: ±1
p/q: $\pm1, \pm7$

Use synthetic division to find a number from the list that is a solution.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 6 & -7 \\ & & 1 & 1 & 7 \\ \hline & 1 & 1 & 7 & \boxed{0} \end{array}$$

We know have $(x - 1)(x^2 + x + 7) = 0$. $x - 1 = 0 \Rightarrow x = 1$.

$$x^2 + x + 7 = 0 \Rightarrow x = -\frac{1}{2} \pm \frac{3\sqrt{3}}{2}i.$$

Example 2. Find all real zeros of $f(x) = x^4 - 3x^3 + x - 3$.
p: $\pm1, \pm3$
q: ±1
p/q: $\pm1, \pm3$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & 0 & 1 & -3 \\ & & -1 & 4 & -4 & 3 \\ \hline & 1 & -4 & 4 & -3 & \boxed{0} \end{array}$$

Now we look for the positive zero, since no other negative numbers can be zeros. Testing 1 does not work, so 3 must be a zero.

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 4 & -3 \\ & & 3 & -3 & 3 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

Now we solve $x^2 - x + 1 = 0$ using the quadratic formula.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

Therefore all four of the zeros of f are -1, 3, and $\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

II. Conjugate Pairs (p. 141) Pace: 5 minutes

- Note that in Example 1, the two complex zeros were conjugates. State that if f is a polynomial function with real coefficients, then whenever $a + bi$ is a zero of f, $a - bi$ is also a zero of f.

Example 3. Find a fourth-degree polynomial function with real coefficients that has 0, 1, and i as zeros.

Since i is a zero, $-i$ is also a zero.

$$f(x) = x(x-1)(x-i)(x+i) = x^4 - x^3 + x^2 - x.$$

III. Factoring a Polynomial (pp. 141–143) Pace: 15 minutes

- State that the **Linear Factorization Theorem,** together with the above statement regarding complex zeros and conjugate pairs, leads to the following statement regarding factoring a polynomial over the reals:

Every polynomial of degree $n > 0$ with real coefficients can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros.

Example 4. Factor $f(x) = x^4 - 12x^2 - 13$

a) as the product of factors that re irreducible over the rationals.

$(x^2 - 13)(x^2 + 1)$

b) as the product of factors that re irreducible over the reals.

$$(x+\sqrt{13})(x-\sqrt{13})(x^2+1)$$

c) completely.

$$(x+\sqrt{13})(x-\sqrt{13})(x+i)(x-i)$$

Example 5. Find all zeros of $f(x) = x^4 - 4x^3 + 12x^2 + 4x - 13$, given that $2 + 3i$ is a zero.

Since $2 + 3i$ is a zero, $2 - 3i$ is also a zero. That means that $x^2 - 4x + 13$ is a factor of $f(x)$.

$$\begin{array}{r}
x^2 - 1 \\
x^2 - 4x + 13 \overline{\smash{\big)}\, x^4 - 4x^3 + 12x^2 + 4x - 13} \\
\underline{x^4 - 4x^3 + 13x^2} \\
-x^2 + 4x - 13 \\
\underline{-x^2 + 4x - 13} \\
0
\end{array}$$

All the zeros of f are -1, 1, $2 + 3i$, and $2 - 3i$.

Chapter 2 Polynomial and Rational Functions

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Section 2.6 Rational Functions and Asymptotes

Section Objectives: Students will know how to determine the domains and find the asymptotes of rational functions.

I. Introduction to Rational Functions (pp. 146–147) Pace: 5 minutes

- State the following definition:
  A **rational function** is a function of the form $f(x) = N(x)/D(x)$, where N and D are both polynomials. The domain of f is all x such that $D(x) \neq 0$.

  Example 1. Find the domain of $f(x) = \dfrac{2x+1}{x^2-4}$.

  $$x^2 - 4 = 0$$
  $$(x+2)(x-2) = 0$$
  $$x + 2 = 0 \Rightarrow x = -2$$
  $$x - 2 = 0 \Rightarrow x = 2$$

  The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

- Use the Library of Functions feature at the top of page 147 to discuss the characteristics of a rational function.

II. Horizontal and Vertical Asymptotes (pp. 147–150) Pace: 15 minutes

- Use the graph of $y = \dfrac{2x-5}{x-2}$ to discuss vertical and horizontal asymptotes.

- State the following definition of asymptotes:
  1. The line $x = a$ is a *vertical asymptote* of the graph of f if $f(x) \to \pm\infty$ as $x \to a$, either from the right or from the left.
  2. The line $y = b$ is a *horizontal asymptote* of the graph of f if $f(x) \to$ b as $x \to \pm\infty$.

- State the following **Asymptotes of Rational Functions Rules:**
  Let f be a rational function given by

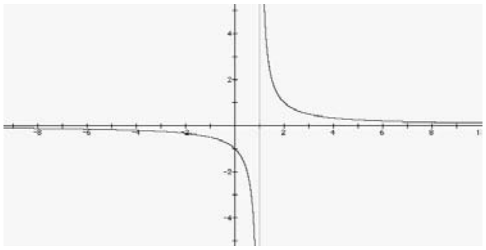
  $$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots b_1 x + b_0}.$$

  1. The graph of f has a vertical asymptote at $x = a$ if $D(a) = 0$ and $N(a) \neq 0$.
  2. The graph of f has one horizontal asymptote or no horizontal asymptote, depending on the degree of N and D.
     a. If $n < m$, then $y = 0$ is the horizontal asymptote of the graph of f.
     b. If $n = m$, then $y = a_n/b_m$ is the horizontal asymptote of the graph of f.
     c. If $n > m$, then there is no horizontal asymptote of the graph of f.

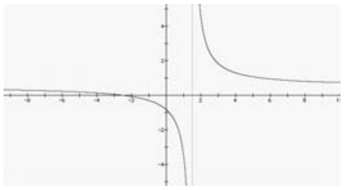
  Example 2. Find any horizontal and vertical asymptotes of the following.

a) $f(x) = \dfrac{x+1}{x^2 - 1}$

The horizontal asymptote is $y = 0$. The only vertical asymptote is $x = 1$. There will be a hole in the graph at $x = -1$.

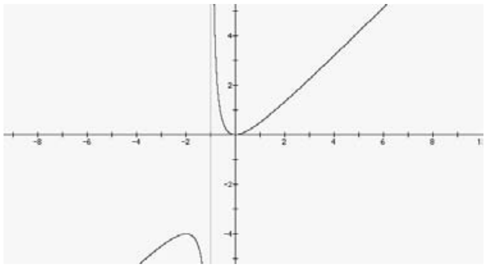


b) $g(x) = \dfrac{2x+5}{4x-6}$. The horizontal asymptote is at $y = 1/2$ and the vertical asymptote is at $x = 3/2$.



c) $h(x) = \dfrac{x^2}{x+1}$. No horizontal asymptote, and a vertical

asymptote at $x = -1$

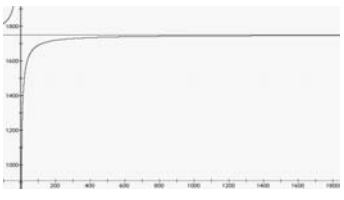


III. Applications (p. 150–151) Pace: 5 minutes

Example 3. A game commission has determined that if 500 deer are introduced into a preserve, the population at any time t (in months) is given by

$$N = \frac{500 + 350t}{1 + 0.2t}.$$

What is the carrying capacity of the preserve?



The carrying capacity will be the horizontal asymptote, $y = 1750$.

Chapter 2 Polynomial and Rational Functions

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Section 2.7 Graphs of Rational Functions

Section Objectives: Students will know how to analyze and sketch the graph of a rational function.

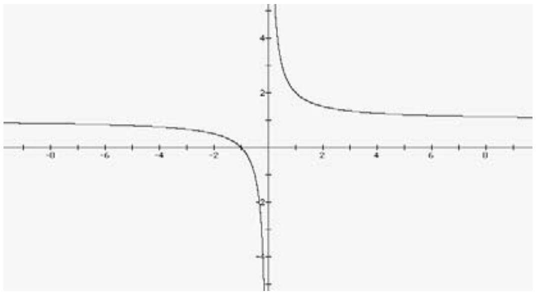
I. The Graph of a Rational Function (pp. 156–158) Pace: 15 minutes

- Draw attention to the **Guidelines for Graphing Rational Functions** on page 156 of the text.

Example 1. Sketch the graph of each of the following functions.

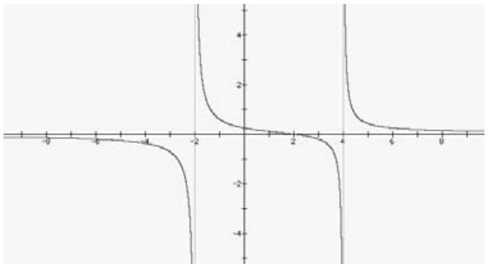
a) $f(x) = \dfrac{x+1}{x}$

y-Intercept:	None
x-Intercept:	(–1, 0)
Vertical asymptote:	$x = 0$
Horizontal asymptote:	$y = 1$
Additional points:	(–2, 0.5), (–1.5, 1/3), (1, 2)



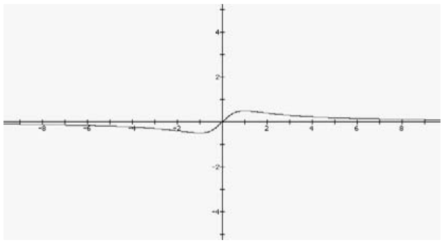
b) $g(x) = \dfrac{x-2}{x^2 - 2x - 8}$

y-Intercept:	(0, –0.25)
x-Intercept:	(2, 0)
Vertical asymptote:	$x = -2$ and $x = 4$
Horizontal asymptote:	$y = 0$
Additional points:	(–4, –0.375), (0, 1/4), (6, 1/4)



c) $h(x) = \dfrac{x}{x^2 + 1}$

y-Intercept:	(0, 0)
x-Intercept:	(0, 0)
Vertical asymptote:	none
Horizontal asymptote:	$y = 0$
Additional points:	(–2, –0.4), (–1, –1/2), (1, 1/2)

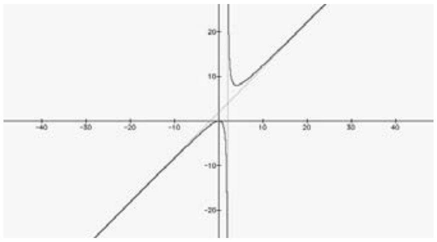


II. Slant Asymptotes (p. 159) Pace: 10 minutes

- Add one more rule to the **Asymptotes of a Rational Function Rules** from Section 2.6:
  If $n = m + 1$, then the graph of f has a slant asymptote at $y = q(x)$, where $q(x)$ is the quotient from the division algorithm.

Example 2. Sketch the graph of $y = \dfrac{x^2}{x-2} = x + 2 + \dfrac{4}{x-2}$.

y-Intercept:	(0, 0)
x-Intercept:	(0, 0)
Vertical asymptote:	$x = 2$
Slant asymptote:	$y = x + 2$
Additional points:	(–1/2, –0.1), (1, –1), (3, 9)

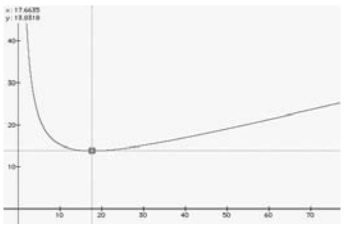


III. Application (p. 160) Pace: 5 minutes

Example 3. The cost of producing x units is $C = 0.25x^2 + 5x + 78$. The average cost per unit is

$$\overline{C} = \frac{0.25x^2 + 5x + 78}{x} = 0.25x + 5 + \frac{78}{x}.$$

Find the number of units that should be produced to minimize the average cost. Graph this function on a graphing utility, then use the "minimum" command. $x \approx 17.66$



Chapter 2 Polynomial and Rational Functions

Course/Section Lesson Number Date

Section 2.8 Quadratic Models

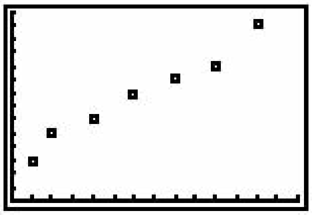
Section Objectives: Students will know how to use scatter plots and a graphing utility to find quadratic models for data.

I. Classifying Scatter Plots (p. 165) Pace: 10 minutes

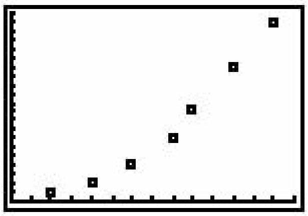
- Discuss the fact that real-life data can sometimes be modeled by a quadratic equation. Scatter plots and graphing utilities can be used to help determine whether a linear model or a quadratic model best fits a particular set of data.

Example 1. Decide whether each data set would best be modeled by a linear model or a quadratic model.

a) (1, 3), (2, 5), (4, 6), (6, 8), (8, 9), (10, 10), (12, 13)
b) (2, 1), (4, 2), (6, 4), (8, 7), (9, 10), (11, 15), (13, 20)
Enter both sets of data into a graphing utility and display the scatter plots.



a)



b)

From the scatter plots, it appears that the data in part (a) follow a linear pattern and thus can be modeled by a linear function. The data in part (b) follow a parabolic pattern and thus can be modeled by a quadratic equation.

II. Fitting a Quadratic Model to Data (pp. 166–167) Pace: 10 minutes

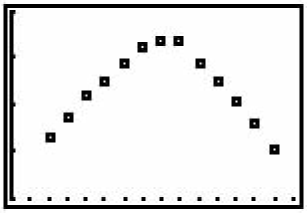
Example 2. The following table gives the mileage *y,* in miles per gallon, of a certain car at various speeds *x* (in miles per hour).

a) Use a graphing utility to create a scatter plot of the data.
b) Use the regression feature of a graphing utility to find a quadratic model that best fits the data.
c) Use the model to predict the speed that gives the greatest mileage.

Speed, x	Mileage, y
10	21.3
15	23.7

20	25.9
25	27.6
30	29.4
35	31.0
40	31.7
45	31.9
50	29.5
55	27.6
60	25.3
65	23.0
70	20.0

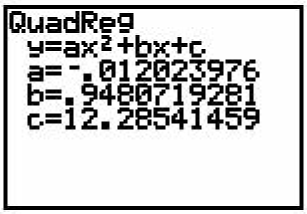
a) Enter the data into a graphing utility and display the scatter plot, as shown in the graph below.



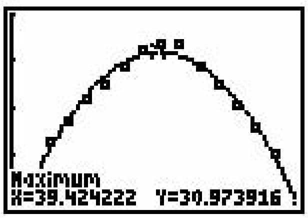
From the scatter plot, students should see that the graph of the data has a parabolic shape and thus can be modeled by a quadratic equation.

b) Use the *regression* feature of the graphing utility to find the model that best fits the data. The model is

$y = -0.012x^2 + 0.9481x + 12.2854.$



c) Graph the data and the model in the same viewing window, as shown below. Use the *maximum* feature or the *zoom* and *trace* features to approximate the speed at which the mileage is greatest. You should obtain a maximum of approximately (39.424, 30.974), as shown in the figure.



So the mileage is greatest (≈ 31 mpg) at a speed of approximately 39.4 miles per hour.

III. Choosing a Model (pp. 167–168) Pace: 10 minutes

Emphasize to students that it is not always obvious from a scatter plot which type of model, linear or quadratic, best fits a particular data set. One way to find the best-fitting odel is to generate more than one type of model and then compare the y-values given by each model with the actual y-values from the data set. Or, compare correlation coefficients. The model with a correlation coefficient that is closer to 1 better fits the data.

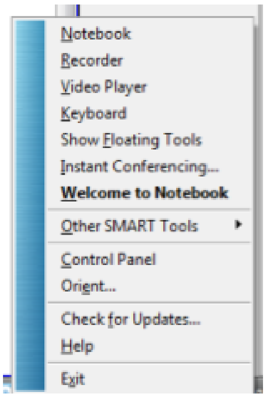
Example 3. For the data points below, determine whether a linear model or a quadratic model best fits the data.

(1, 5)	(6, 10)
(2, 6)	(7, 11)
(3, 8)	(8, 12)
(4, 9)	(9, 14)
(5, 11)	(10, 16)

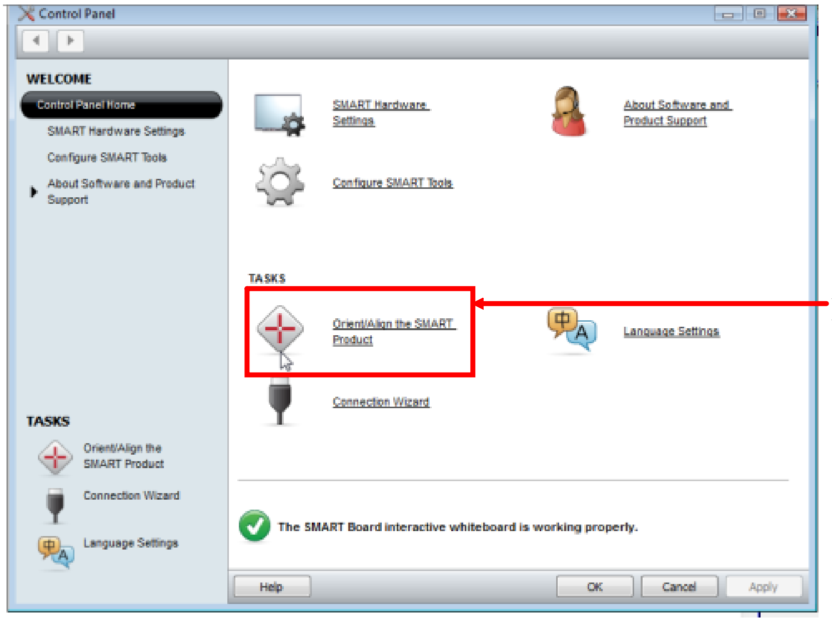
Using a graphing utility, a linear model for the data is $y = 1.091x + 4.2$. For this linear model the correlation coefficient r^2 is 0.9477. Using a graphing utility, a quadratic model for the data is $y = 0.00757x^2 + 1.00757x + 4.36667$. For this quadratic model the correlation coefficient is 0.948. Therefore the quadratic model is a *slightly* better fit for this data set.

Orienting

In order to orient the Smartboard you will need to click on the symbol for Smart Notebook located in the lower right of your screen. 
If you are not connected to a Smartboard the symbol will appear as a red box with a  white "x."



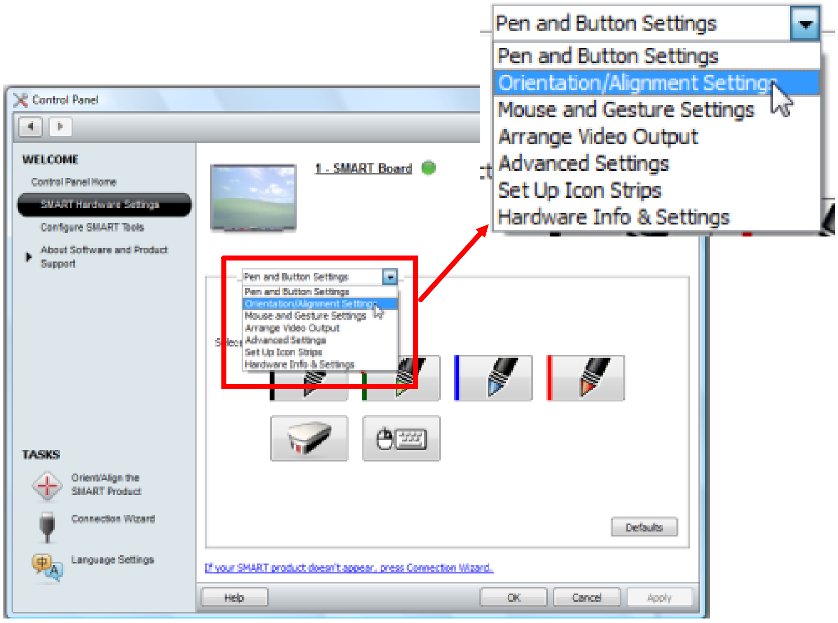
You will get a menu from which you will want to choose **Orient...** If this is the first time using the Smartboard, you might want to choose **Control Panel** and set your own orientation style. If you are not connected to a Smartboard, the option to "Orient..." will only be grayed out.



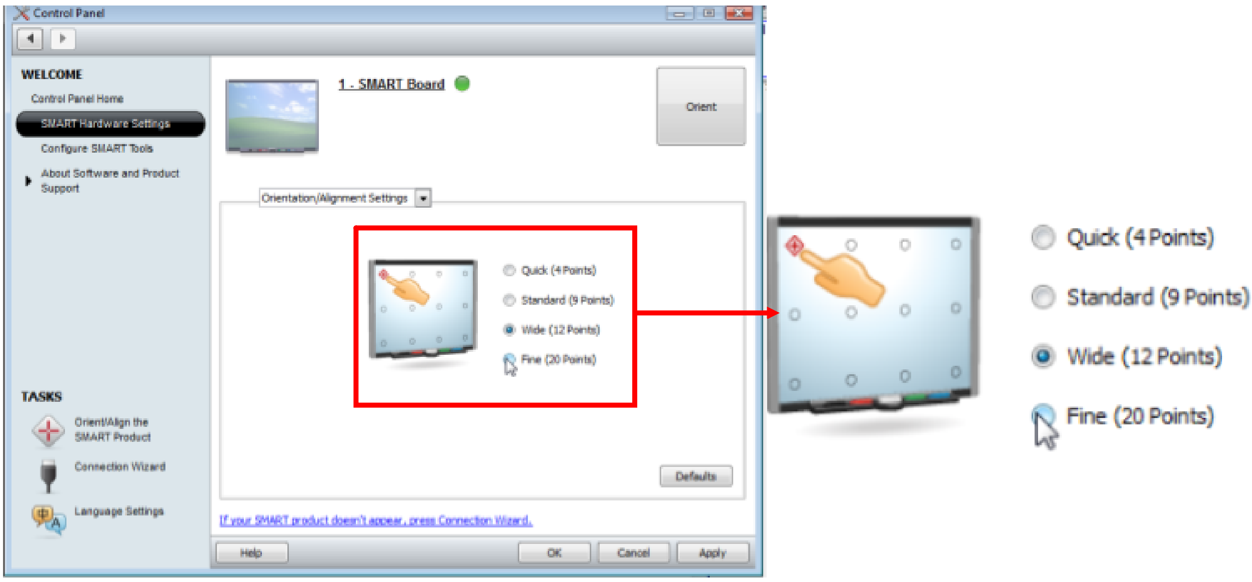
If you choose **Orient/Align the SMART Product** will be taken straight to the orienting screen.

This is what the orienting screen looks like with nine points. To orient, just follow the directions on the screen. (Points have been enhanced here to make them more visible.)

If you choose **SMART Hardware Settings** you will get the page shown to the right. Click on the pull down menu and choose **Orientation/Alignment Settings**



On this screen you have the option to set the number of orientation points. Once you have chosen how many points you want to use for aligning the Smartboard, click on the "Orient" square in the upper right to start the process of orienting the Smartboard.



C H A P T E R 2

Section 2.1

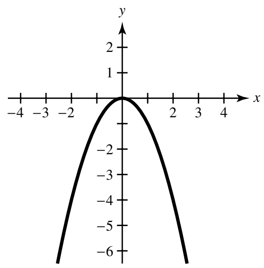
1. nonnegative integer, real

3. Yes, $f(x) = (x-2)^2 + 3$ is in the form $f(x) = a(x-h)^2 + k$. The vertex is $(2, 3)$.

5. $f(x) = (x-2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (c).

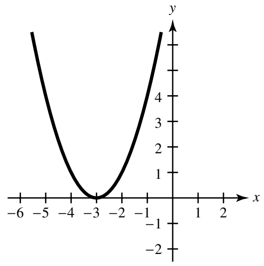
7. $f(x) = x^2 + 3$ opens upward and has vertex $(0, 3)$. Matches graph (b).

9.



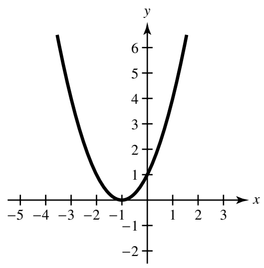
The graph of $y = -x^2$ is a reflection of $y = x^2$ in the x-axis.

11.



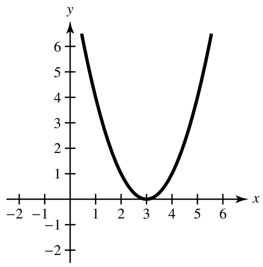
The graph of $y = (x+3)^2$ is a horizontal shift three units to the left of $y = x^2$.

13.



The graph of $y = (x+1)^2$ is a horizontal shift one unit to the left of $y = x^2$.

15.



The graph of $y = (x-3)^2$ is a horizontal shift three units to the right of $y = x^2$.

17. $f(x) = 25 - x^2$

$$= -x^2 + 25$$

A parabola opening downward with vertex $(0, 25)$

19. $f(x) = \frac{1}{2}x^2 - 4$

A parabola opening upward with vertex $(0, -4)$

21. $f(x) = (x+4)^2 - 3$

A parabola opening upward with vertex $(-4, -3)$

23. $h(x) = x^2 - 8x + 16$

$$= (x-4)^2$$

A parabola opening upward with vertex $(4, 0)$

25. $f(x) = x^2 - x + \frac{5}{4}$

$$= (x^2 - x) + \frac{5}{4}$$

$$= \left(x^2 - x + \frac{1}{4}\right) + \frac{5}{4} - \frac{1}{4}$$

$$= \left(x - \frac{1}{2}\right)^2 + 1$$

A parabola opening upward with vertex $\left(\frac{1}{2}, 1\right)$

27. $f(x) = -x^2 + 2x + 5$

$$= -(x^2 - 2x) + 5$$

$$= -(x^2 - 2x + 1) + 5 + 1$$

$$= -(x-1)^2 + 6$$

A parabola opening downward with vertex $(1, 6)$

29. $h(x) = 4x^2 - 4x + 21$
$= 4(x^2 - x) + 21$
$= 4\left(x^2 - x + \frac{1}{4}\right) + 21 - 4\left(\frac{1}{4}\right)$
$= 4\left(x - \frac{1}{2}\right)^2 + 20$

A parabola opening upward with vertex $\left(\frac{1}{2},\ 20\right)$

31. $f(x) = -(x^2 + 2x - 3)$
$= -(x^2 + 2x) + 3$
$= -(x^2 + 2x + 1) + 3 + 1$
$= -(x + 1)^2 + 4$
$-(x^2 + 2x - 3) = 0$
$-(x + 3)(x - 1) = 0$
$x + 3 = 0 \Rightarrow x = -3$
$x - 1 = 0 \Rightarrow x = 1$

A parabola opening downward with vertex $(-1,\ 4)$ and x-intercepts $(-3,\ 0)$ and $(1,\ 0)$.

33. $g(x) = x^2 + 8x + 11$
$= (x^2 + 8x) + 11$
$= (x^2 + 8x + 16) + 11 - 16$
$= (x + 4)^2 - 5$
$x^2 + 8x + 11 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(11)}}{2(1)} = \frac{-8 \pm \sqrt{64 - 44}}{2} = \frac{-8 \pm \sqrt{20}}{2} = \frac{-8 \pm 2\sqrt{5}}{2} = -4 \pm \sqrt{5}$$

A parabola opening upward with vertex $(-4,\ -5)$ and x-intercepts $(-4 \pm \sqrt{5},\ 0)$.

35. $f(x) = -2x^2 + 16x - 31$
$= -2(x^2 - 8x) - 31$
$= -2(x^2 - 8x + 16) - 31 + 32$
$= -2(x - 4)^2 + 1$
$-2x^2 + 16x - 31 = 0$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-31)}}{2(-2)} = \frac{-16 \pm \sqrt{256 - 248}}{-4} = \frac{-16 \pm \sqrt{8}}{-4} = \frac{-16 \pm 2\sqrt{2}}{-4} = 4 \pm \frac{1}{2}\sqrt{2}$$

A parabola opening downward with vertex (4, 1) and x-intercepts $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$

37. $(-1,\ 4)$ is the vertex.
$f(x) = a(x + 1)^2 + 4$

Since the graph passes through the point $(1,\ 0)$, we have:
$0 = a(1 + 1)^2 + 4$
$0 = 4a + 4$
$-1 = a$

Thus, $f(x) = -(x + 1)^2 + 4.$ Note that $(-3,\ 0)$ is on the parabola.

39. $(-2,5)$ is the vertex.
$f(x) = a(x + 2)^2 + 5$

Since the graph passes through the point $(0,\ 9)$, we have:
$9 = a(0 + 2)^2 + 5$
$4 = 4a$
$1 = a$

Thus, $f(x) = (x + 2)^2 + 5.$

41. $(1,\ -2)$ is the vertex.
$f(x) = a(x - 1)^2 - 2$

Since the graph passes through the point $(-1,\ 14)$, we have:

$14 = a(-1 - 1)^2 - 2$
$14 = 4a - 2$
$16 = 4a$
$4 = a$

Thus, $f(x) = 4(x - 1)^2 - 2.$

43. $\left(\frac{1}{2}, 1\right)$ is the vertex.

$$f(x) = a\left(x - \frac{1}{2}\right)^2 + 1$$

Since the graph passes through the point $\left(-2, -\frac{21}{5}\right)$, we have:

$$-\frac{21}{5} = a\left(-2 - \frac{1}{2}\right)^2 + 1$$
$$-\frac{21}{5} = \frac{25}{4}a + 1$$
$$-\frac{26}{5} = \frac{25}{4}a$$
$$-\frac{104}{125} = a$$

Thus, $f(x) = -\frac{104}{125}\left(x - \frac{1}{2}\right)^2 + 1.$

45. $y = x^2 - 4x - 5$

x-intercepts: $(5, 0)$, $(-1, 0)$

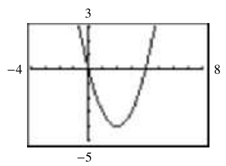
$$0 = x^2 - 4x - 5$$
$$0 = (x - 5)(x + 1)$$
$$x = 5 \text{ or } x = -1$$

47. $y = x^2 + 8x + 16$

x-intercept: $(-4, 0)$

$$0 = x^2 + 8x + 16$$
$$0 = (x + 4)^2$$
$$x = -4$$

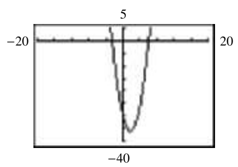
49. $y = x^2 - 4x$



x-intercepts: $(0, 0)$, $(4, 0)$

$$0 = x^2 - 4x$$
$$0 = x(x - 4)$$
$$x = 0 \text{ or } x = 4$$

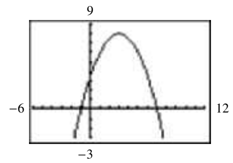
51. $y = 2x^2 - 7x - 30$



x-intercepts: $\left(-\frac{5}{2}, 0\right)$, $(6, 0)$

$$0 = 2x^2 - 7x - 30$$
$$0 = (2x + 5)(x - 6)$$
$$x = -\frac{5}{2} \text{ or } x = 6$$

53. $y = -\frac{1}{2}(x^2 - 6x - 7)$



x-intercepts: $(-1, 0)$, $(7, 0)$

$$0 = -\frac{1}{2}(x^2 - 6x - 7)$$
$$0 = x^2 - 6x - 7$$
$$0 = (x + 1)(x - 7)$$
$$x = -1, 7$$

55. $f(x) = [x - (-1)](x - 3)$, opens upward

$$= (x + 1)(x - 3)$$
$$= x^2 - 2x - 3$$

$g(x) = -[x - (-1)](x - 3)$, opens downward

$$= -(x + 1)(x - 3)$$
$$= -(x^2 - 2x - 3)$$
$$= -x^2 + 2x + 3$$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

57. $f(x) = [x - (-3)]\left[x - \left(-\frac{1}{2}\right)\right](2)$, opens upward

$$= (x + 3)\left(x + \frac{1}{2}\right)(2)$$
$$= (x + 3)(2x + 1)$$
$$= 2x^2 + 7x + 3$$

$g(x) = -(2x^2 + 7x + 3)$, opens downward

$$= -2x^2 - 7x - 3$$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$ and $\left(-\frac{1}{2}, 0\right)$ for all real numbers $a \neq 0$.

59. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x + y = 110 \Rightarrow y = 110 - x$.

The product is

$$P(x) = xy = x(110 - x) = 110x - x^2.$$
$$P(x) = -x^2 + 110x$$

$$= -(x^2 - 110x + 3025 - 3025)$$
$$= -[(x-55)^2 - 3025]$$
$$= -(x-55)^2 + 3025$$

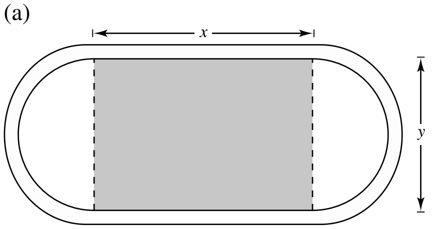
The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

61. Let x be the first number and y be the second number. Then $x + 2y = 24 \Rightarrow x = 24 - 2y$. The product is
$P = xy = (24-2y)y = 24y - 2y^2$.
Completing the square,

$$P = -2y^2 + 24y$$
$$= -2(y^2 - 12y + 36) + 72$$
$$= -2(y-6)^2 + 72.$$

The maximum value of the product P occurs at the vertex of the parabola and equals 72. This happens when $y = 6$ and $x = 24 - 2(6) = 12$.

63. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

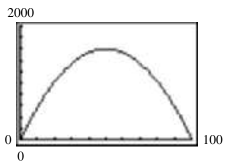
Distance around two semicircular parts of track:
$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$

(c) Distance traveled around track in one lap:

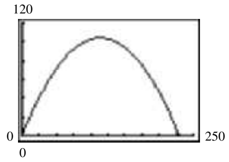
$$d = \pi y + 2x = 200$$
$$\pi y = 200 - 2x$$
$$y = \frac{200 - 2x}{\pi}$$

(d) Area of rectangular region: $A = xy = x\left(\frac{200-2x}{\pi}\right)$

(e) The area is maximum when $x = 50$ and
$y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}$.



65. (a)



(b) When $x = 0$, $y = \frac{3}{2}$ feet.

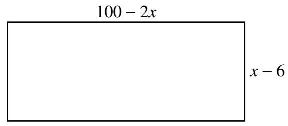
(c) The vertex occurs at
$x = \frac{-b}{2a} = \frac{-9/5}{2(-16/2025)} = \frac{3645}{32} \approx 113.9.$

The maximum height is

$$y = \frac{-16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + \frac{3}{2}$$
≈ 104.0 feet.

(d) Using a graphing utility, the zero of y occurs at $x \approx 228.6$, or 228.6 feet from the punter.

67. (a)



$$A = lw$$
$$A = (100 - 2x)(x - 6)$$
$$A = -2x^2 + 112x - 600$$

(b) $Y_1 = -2x^2 + 112x - 600$

X	Y
25	950
26	960
27	966
28	968
29	966
30	960

The area is maximum when $x = 28$ inches.

69. $R(p) = -10p^2 + 1580p$

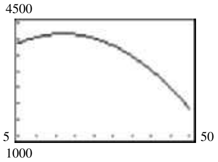
(a) When $p = \$50$, $R(50) = \$54{,}000$.
When $p = \$70$, $R(70) = \$61{,}600$.
When $p = \$90$, $R(90) = \$61{,}200$.

(b) The maximum R occurs at the vertex,
$$p = \frac{-b}{2a}$$
$$p = \frac{-1580}{2(-10)} = \$79$$

(c) When $p = \$79$, $R(79) = \$62{,}410$.

(d) Answers will vary.

71. (a)



(b) Using the graph, during 1966 the maximum average annual consumption of cigarettes appears to have occurred and was 4155 cigarettes per person.

Yes, the warning had an effect because the maximum consumption occurred in 1966 and consumption decreased from then on.

(c) In 2000, $C(50) = 1852$ cigarettes per person.

$\frac{1852}{365} \approx 5$ cigarettes per day

73. True.

$$-12x^2 - 1 = 0$$
$$12x^2 = -1, \text{ impossible}$$

75. The parabola opens downward and the vertex is $(-2, -4)$. Matches (c) and (d).

77. The graph of $f(x) = (x - z)^2$ would be a horizontal shift z units to the right of $g(x) = x^2$.

79. The graph of $f(x) = z(x-3)^2$ would be a vertical stretch $(z > 1)$ and horizontal shift three units to the right of $g(x) = x^2$. The graph of $f(x) = z(x-3)^2$ would be a vertical shrink $(0 < z < 1)$ and horizontal shift three units to the right of $g(x) = x^2$.

81. For $a < 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a maximum when $x = \frac{-b}{2a}$. In this case, the maximum value is $c - \frac{b^2}{4a}$. Hence,

$$25 = -75 - \frac{b^2}{4(-1)}$$
$$-100 = 300 - b^2$$
$$400 = b^2$$
$$b = \pm 20.$$

83. For $a > 0$, $f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$ is a minimum when $x = \frac{-b}{2a}$. In this case, the minimum value is $c - \frac{b^2}{4a}$. Hence,

$$10 = 26 - \frac{b^2}{4}$$
$$40 = 104 - b^2$$
$$b^2 = 64$$
$$b = \pm 8.$$

85. Let x = first number and y = second number.
Then $x + y = s$ or $y = s - x$.
The product is given by $P = xy$ or $P = x(s - x)$.

$$P = x(s - x)$$
$$P = sx - x^2$$

The maximum P occurs at the vertex when $x = \frac{-b}{2a}$.

$$x = \frac{-s}{2(-1)} = \frac{s}{2}$$

When $x = \frac{s}{2}$, $y = s - \frac{s}{2} = \frac{s}{2}$.

So, the numbers x and y are both $\frac{s}{2}$.

87. $y = ax^2 + bx - 4$
$(1, 0)$ on graph: $0 = a + b - 4$
$(4, 0)$ on graph: $0 = 16a + 4b - 4$
From the first equation, $b = 4 - a$.
Thus, $0 = 16a + 4(4 - a) - 4 = 12a + 12 \Rightarrow a = -1$ and hence $b = 5$, and $y = -x^2 + 5x - 4$.

89. $x + y = 8 \Rightarrow y = 8 - x$

$$-\frac{2}{3}x + 8 - x = 6$$
$$-\frac{5}{3}x + 8 = 6$$
$$-\frac{5}{3}x = -2$$
$$x = 1.2$$
$$y = 8 - 1.2 = 6.8$$

The point of intersection is $(1.2, 6.8)$.

91.
$$y = x + 3 = 9 - x^2$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3, \quad x = 2$$
$$y = -3 + 3 = 0$$
$$y = 2 + 3 = 5$$

Thus, $(-3, 0)$ and $(2, 5)$ are the points of intersection.

93. Answers will vary. (Make a Decision)

Section 2.2

1. continuous

3. (a) solution
(b) $(x - a)$
(c) $(a, 0)$

5. No. If f is an even-degree fourth-degree polynomial function, its left and right end behavior is either that it rises left and right or falls left and right.

7. Because f is a polynomial, it is a continuous on $[x_1, x_2]$ and $f(x_1) < 0$ and $f(x_2) > 0$. Then $f(x) = 0$ for some value of x in $[x_1, x_2]$.

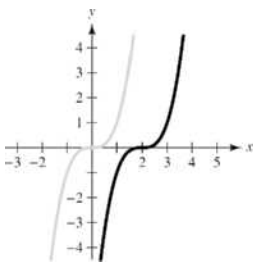
9. $f(x) = -2x + 3$ is a line with y-intercept $(0, 3)$. Matches graph (f).

11. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (c).

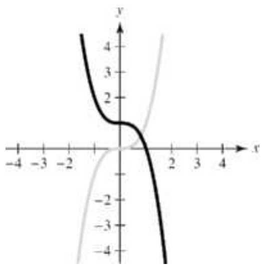
13. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $\left(\pm 2\sqrt{3}, 0\right)$. Matches graph (e).

15. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (g).

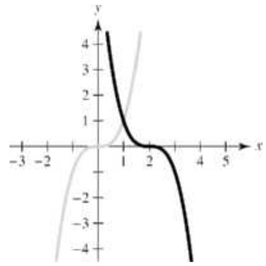
17. The graph of $f(x) = (x - 2)^3$ is a horizontal shift two units to the right of $y = x^3$.



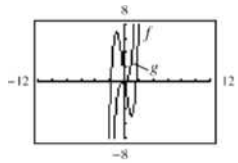
19. The graph of $f(x) = -x^3 + 1$ is a reflection in the x-axis and a vertical shift one unit upward of $y = x^3$.



21. The graph of $f(x) = -(x - 2)^3$ is a horizontal shift two units to the right and a reflection in the x-axis of $y = x^3$.

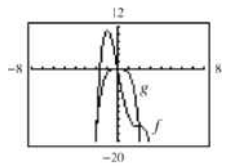


23.



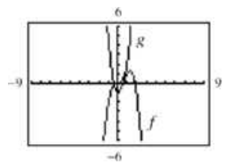
Yes, because both graphs have the same leading coefficient.

25.



Yes, because both graphs have the same leading coefficient.

27.



No, because the graphs have different leading coefficients.

29. $f(x) = 2x^4 - 3x + 1$

Degree: 4

Leading coefficient: 2

The degree is even and the leading coefficient is positive. The graph rises to the left and right.

31. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: –3

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

33. $f(x) = \dfrac{6x^5 - 2x^4 + 4x^2 - 5x}{3}$

Degree: 5

Leading coefficient: $\frac{6}{3} = 2$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

35. $h(t) = -\frac{2}{3}\left(t^2 - 5t + 3\right)$

Degree: 2

Leading coefficient: $-\frac{2}{3}$

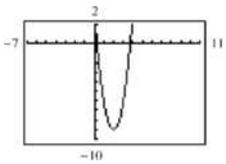
The degree is even and the leading coefficient is negative. The graph falls to the left and right.

37. (a) $f(x) = 3x^2 - 12x + 3$

$$= 3\left(x^2 - 4x + 1\right) = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

(b)



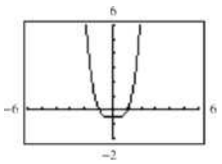
(c) $x \approx 3.732, 0.268$; the answers are approximately the same.

39. (a) $g(t) = \frac{1}{2}t^4 - \frac{1}{2}$

$$= \frac{1}{2}\left(t + 1\right)(t - 1)(t^2 + 1) = 0$$

$t = \pm 1$

(b)



(c) $t = \pm 1$; the answers are the same.

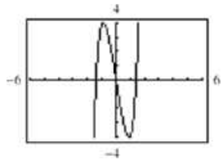
41. (a) $f(x) = x^5 + x^3 - 6x$

$$= x\left(x^4 + x^2 - 6\right)$$

$$= x\left(x^2 + 3\right)(x^2 - 2) = 0$$

$x = 0, \ \pm\sqrt{2}$

(b)



(c) $x = 0, 1.414, -1.414$; the answers are approximately the same.

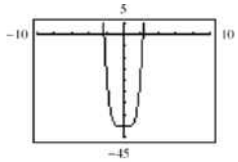
43. (a) $f(x) = 2x^4 - 2x^2 - 40$

$$= 2\left(x^4 - x^2 - 20\right)$$

$$= 2\left(x^2 + 4\right)\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right) = 0$$

$x = \pm\sqrt{5}$

(b)



(c) $x = 2.236, -2.236$; the answers are approximately the same.

45. (a) $f(x) = x^3 - 4x^2 - 25x + 100$

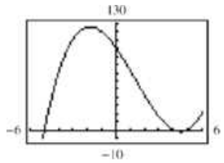
$$= x^2(x - 4) - 25(x - 4)$$

$$= (x^2 - 25)(x - 4)$$

$$= (x - 5)(x + 5)(x - 4) = 0$$

$x = \pm 5, \ 4$

(b)



(c) $x = 4, 5, -5$; the answers are the same.

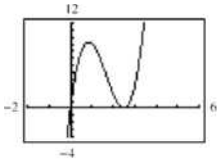
47. (a) $y = 4x^3 - 20x^2 + 25x$

$0 = 4x^3 - 20x^2 + 25x$

$0 = x(2x-5)^2$

$x = 0, \ \frac{5}{2}$

(b)



(c) $x = 0, \ \frac{5}{2}$; the answers are the same.

49. $f(x) = x^2 - 25$

$= (x+5)(x-5)$

$x = \pm 5$ (multiplicity 1)

51. $h(t) = t^2 - 6t + 9$

$= (t-3)^2$

$t = 3$ (multiplicity 2)

53. $f(x) = x^2 + x - 2$

$= (x+2)(x-1)$

$x = -2, 1$ (multiplicity 1)

55. $f(t) = t^3 - 4t^2 + 4t$

$= t(t-2)^2$

$t = 0$ (multiplicity 1), 2 (multiplicity 2)

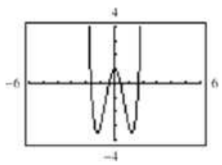
57. $f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2}$

$= \frac{1}{2}(x^2 + 5x - 3)$

$x = \frac{-5 \pm \sqrt{25 - 4(-3)}}{2} = -\frac{5}{2} \pm \frac{\sqrt{37}}{2}$

$\approx 0.5414, \ -5.5414$ (multiplicity 1)

59. $f(x) = 2x^4 - 6x^2 + 1$

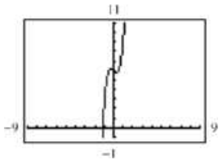


Zeros: $x \approx \pm 0.421, \pm 1.680$

Relative maximum: $(0, 1)$

Relative minima: $(1.225, \ -3.5)$, $(-1.225, \ -3.5)$

61. $f(x) = x^5 + 3x^3 - x + 6$

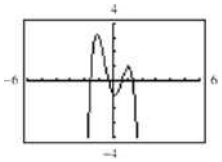


Zero: $x \approx -1.178$

Relative maximum: $(-0.324, \ 6.218)$

Relative minimum: $(0.324, \ 5.782)$

63. $f(x) = -2x^4 + 5x^2 - x - 1$



Zeros: $-1.618, \ -0.366, \ 0.618, 1.366$

Relative minimum: $(0.101, \ -1.050)$

Relative maxima: $(-1.165, \ 3.267)$, $(1.064, \ 1.033)$

65. $f(x) = (x-0)(x-4) = x^2 - 4x$

Note: $f(x) = a(x-0)(x-4) = ax(x-4)$ has zeros 0 and 4 for all nonzero real numbers a.

67. $f(x) = (x-0)(x+2)(x+3) = x^3 + 5x^2 + 6x$

Note: $f(x) = ax(x+2)(x+3)$ has zeros $0, \ -2,$ and -3 for all nonzero real numbers a.

69. $f(x) = (x-4)(x+3)(x-3)(x-0)$

$= (x-4)(x^2-9)x$

$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, \ -3, \ 3,$ and 0 for all nonzero real numbers a.

71. $f(x) = \left[x - \left(1+\sqrt{3}\right)\right]\left[x - \left(1-\sqrt{3}\right)\right]$

$= \left[\left(x-1\right) - \sqrt{3}\right]\left[\left(x-1\right) + \sqrt{3}\right]$

$= (x-1)^2 - \left(\sqrt{3}\right)^2$

$= x^2 - 2x + 1 - 3$

$= x^2 - 2x - 2$

Note: $f(x) = a(x^2 - 2x - 2)$ has zeros $1 + \sqrt{3}$ and $1 - \sqrt{3}$ for all nonzero real numbers a.

73. $f(x) = (x-2)\left[x-\left(4+\sqrt{5}\right)\right]\left[x-\left(4-\sqrt{5}\right)\right]$

$$= (x-2)\left[\left(x-4\right)-\sqrt{5}\right]\left[\left(x-4\right)+\sqrt{5}\right]$$
$$= (x-2)[(x-4)^2-5]$$
$$= x^3-10x^2+27x-22$$

Note: $f(x) = a(x-2)[(x-4)^2-5]$ has zeros $2,\ 4+\sqrt{5},$ and $4-\sqrt{5}$ for all nonzero real numbers a.

75. $f(x) = (x+2)^2(x+1) = x^3+5x^2+8x+4$

Note: $f(x) = a(x+2)^2(x+1)$ has zeros $-2,\ -2,$ and -1 for all nonzero real numbers a.

77. $f(x) = (x+4)^2(x-3)^2$

$$= x^4+2x^3-23x^2-24x+144$$

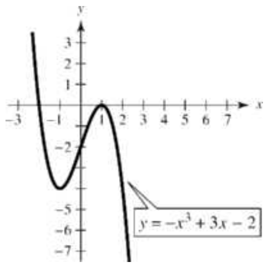
Note: $f(x) = a(x+4)^2(x-3)^2$ has zeros $-4,\ -4,\ 3,\ 3$ for all nonzero real numbers a.

79. $f(x) = -(x+1)^2(x+2)$

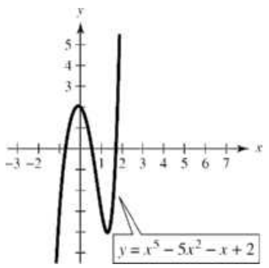
$$= -x^3-4x^2-5x-2$$

Note: $f(x) = a(x+1)^2(x+2)^2,\ a<0,$ has zeros $-1,\ -1,\ -2,$ rises to the left, and falls to the right.

81.



83.

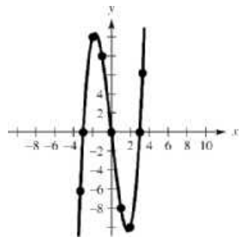


85. (a) The degree of *f* is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3-9x = x(x^2-9) = x(x-3)(x+3)$

Zeros: $0,\ 3,\ -3$

(c) and (d)

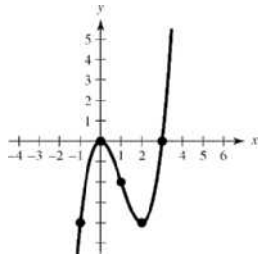


87. (a) The degree of *f* is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f\left(x\right) = x^3-3x^2 = x^2(x-3)$

Zeros: $0,\ 3$

(c) and (d)

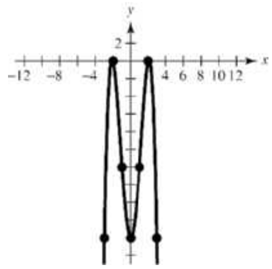


89. (a) The degree of *f* is even and the leading coefficient is -1. The graph falls to the left and falls to the right.

(b) $f\left(x\right) = -x^4+9x^2-20 = -(x^2-4)(x^2-5)$

Zeros: $\pm 2,\ \pm\sqrt{5}$

(c) and (d)



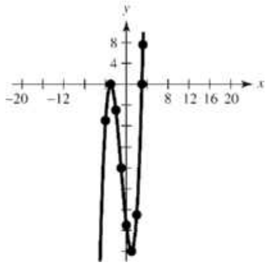
91. (a) The degree of f is odd and the leading coefficient is 1. The graph falls to the left and rises to the right.

(b) $f(x) = x^3 + 3x^2 - 9x - 27 = x^2(x+3) - 9(x+3)$

$$= (x^2 - 9)(x+3)$$
$$= (x-3)(x+3)^2$$

Zeros: $3, \ -3$

(c) and (d)

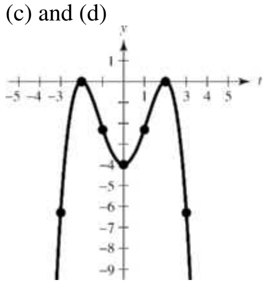


93. (a) The degree of g is even and the leading coefficient is $-\dfrac{1}{4}$, The graph falls to the left and falls to the right.

(b) $g(t) = -\dfrac{1}{4}(t^4 - 8t^2 + 16) = -\dfrac{1}{4}(t^2 - 4)^2$

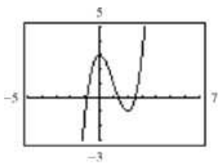
Zeros: $-2, \ -2, 2, 2$

(c) and (d)



95. $f(x) = x^3 - 3x^2 + 3$

(a)



The function has three zeros. They are in the intervals $(-1, 0), (1, 2)$ and $(2, 3)$.

(b) Zeros: $-0.879, 1.347, 2.532$

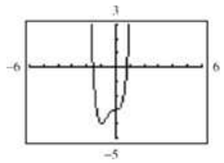
x	y
–0.9	–0.159
–0.89	–0.0813
–0.88	–0.0047
–0.87	0.0708
–0.86	0.14514
–0.85	0.21838
–0.84	0.2905

x	y
1.3	0.127
1.31	0.09979
1.32	0.07277
1.33	0.04594
1.34	0.0193
1.35	–0.0071
1.36	–0.0333

x	y
2.5	–0.125
2.51	–0.087
2.52	–0.0482
2.53	–0.0084
2.54	0.03226
2.55	0.07388
2.56	0.11642

97. $g(x) = 3x^4 + 4x^3 - 3$

(a)



The function has two zeros. They are in the intervals $(-2, -1)$ and $(0, 1)$.

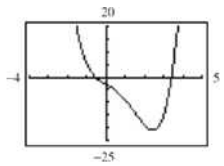
(b) Zeros: $-1.585, 0.779$

x	y_1
–1.6	0.2768
–1.59	0.09515
–1.58	–0.0812
–1.57	–0.2524
–1.56	–0.4184
–1.55	–0.5795
–1.54	–0.7356

x	y_1
0.75	–0.3633
0.76	–0.2432
0.77	–0.1193
0.78	0.00866
0.79	0.14066
0.80	0.2768
0.81	0.41717

99. $f(x) = x^4 - 3x^3 - 4x - 3$

(a)



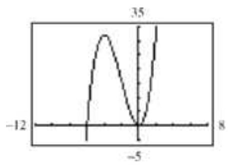
The function has two zeros. They are in the intervals $(-1, 0)$ and $(3, 4)$.

(b) Zeros: $-0.578, 3.418$

x	y_1
–0.61	0.2594
–0.60	0.1776
–0.59	0.09731
–0.58	0.0185
–0.57	–0.0589
–0.56	–0.1348
–0.55	–0.2094

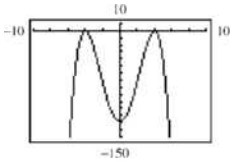
x	y_1
3.39	–1.366
3.40	–0.8784
3.41	–0.3828
3.42	0.12071
3.43	0.63205
3.44	1.1513
3.45	1.6786

101. $f(x) = x^2(x+6)$



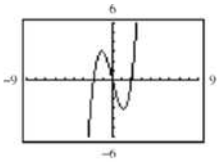
No symmetry
Two x-intercepts

103. $g(t) = -\frac{1}{2}(t-4)^2(t+4)^2$



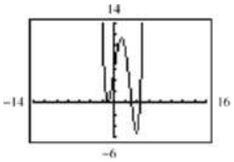
Symmetric with respect to the y-axis
Two x-intercepts

105. $f(x) = x^3 - 4x = x(x+2)(x-2)$



Symmetric with respect to the origin
Three x-intercepts

107. $g(x) = \frac{1}{5}(x+1)^2(x-3)(2x-9)$



No symmetry
Three x-intercepts

109. (a) Volume = length $\times$ width $\times$ height

Because the box is made from a square, length = width.

Thus: Volume = $(\text{length})^2 \times$ height = $(36-2x)^2\,x$

(b) Domain: $0 < 36 - 2x < 36$

$-36 < -2x < 0$

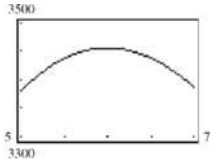
$18 > x > 0$

(c)

Height, x	Length and Width	Volume, V
1	$36-2(1)$	$1\left[36-2(1)\right]^2 = 1156$
2	$36-2(2)$	$2\left[36-2(2)\right]^2 = 2048$
3	$36-2(3)$	$3\left[36-2(3)\right]^2 = 2700$
4	$36-2(4)$	$4\left[36-2(4)\right]^2 = 3136$
5	$36-2(5)$	$5\left[36-2(5)\right]^2 = 3380$
6	$36-2(6)$	$6\left[36-2(6)\right]^2 = 3456$
7	$36-2(7)$	$7\left[36-2(7)\right]^2 = 3388$

Maximum volume is in the interval $5 < x < 7$.

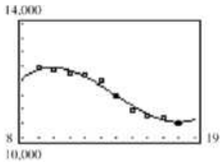
(d)



$x = 6$ when $V(x)$ is maximum.

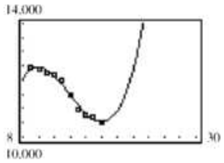
111. The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$ The point is (200, 160) which corresponds to spending \$2,000,000 on advertising to obtain a revenue of \$160 million.

113. (a)



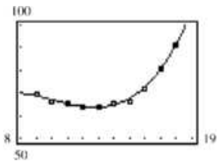
The model fits the data well.

(b)

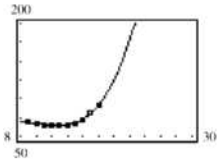


Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

(c)



The model fits the data well.

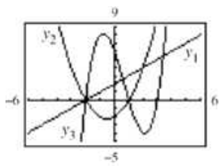


Answers will vary. Sample answer: You could use the model to estimate production in 2010 because the result is somewhat reasonable, but you would not use the model to estimate the 2020 production because the result is unreasonably high.

115. True. The degree is odd and the leading coefficient is –1.

117. False. The graph crosses the x-axis at $x = -3$ and $x = 0$.

119.



The graph of y_3 will fall to the left and rise to the right. It will have another x-intercept at $(3,\ 0)$ of odd multiplicity (crossing the x-axis).

121. $(f+g)(-4) = f(-4) + g(-4)$

$$= -59 + 128 = 69$$

123. $(f \circ g)\left(-\frac{4}{7}\right) = f\left(-\frac{4}{7}\right)g\left(-\frac{4}{7}\right) = (-11)\left(\frac{8 \cdot 16}{49}\right)$

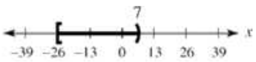
$$= -\frac{1408}{49} \approx -28.7347$$

125. $(f \circ g)(-1) = f(g(-1)) = f(8) = 109$

127. $3(x-5) < 4x - 7$

$$3x - 15 < 4x - 7$$
$$-8 < x$$

129. $\dfrac{5x-2}{x-7} \le 4$



$$\frac{5x-2}{x-7} - 4 \le 0$$
$$\frac{5x-2-4(x-7)}{x-7} \le 0$$
$$\frac{x+26}{x-7} \le 0$$

$[x + 26 \ge 0 \text{ and } x - 7 < 0]$ or $[x + 26 \le 0 \text{ and } x - 7 > 0]$

$[x \ge -26 \text{ and } x < 7]$ or $[x \le -26 \text{ and } x > 7]$

$-26 \le x < 7$ impossible

Section 2.3

1. $f(x)$ is the dividend, $d(x)$ is the divisor, $q(x)$ is the quotient, and $r(x)$ is the remainder.

3. constant term, leading coefficient

5. upper, lower

7. According to the Remainder Theorem, if you divide $f(x)$ by $x - 4$ and the remainder is 7, then $f(4) = 7$.

9.
$$\begin{array}{r} 2x+4 \\ x+3 \overline{)2x^2+10x+12} \\ \underline{2x^2+6x} \\ 4x+12 \\ \underline{4x+12} \\ 0 \end{array}$$

$$\frac{2x^2+10x+12}{x+3} = 2x + 4,\ x \ne -3$$

11.
$$\begin{array}{r} x^3+3x^2\qquad -1 \\ x+2\overline{\big)x^4+5x^3+6x^2-x-2} \\ \underline{x^4+2x^3}\qquad\qquad\quad \\ 3x^3+6x^2\qquad\quad \\ \underline{3x^3+6x^2}\qquad\quad \\ -x-2 \\ \underline{-x-2} \\ 0 \end{array}$$

$$\frac{x^4+5x^3+6x^2-x-2}{x+2}=x^3+3x^2-1,\ x\neq -2$$

13.
$$\begin{array}{r} x^2-\ 3x+1 \\ 4x+5\overline{\big)4x^3-7x^2-11x+5} \\ \underline{-4x^3+5x^2}\qquad\qquad\quad \\ -12x^2-11x\qquad \\ \underline{-12x^2-15x}\qquad \\ 4x+5 \\ -\underline{4x+5} \\ 0 \end{array}$$

$$\frac{4x^3-7x^2-11x+5}{4x+5}=x^2-3x+1,\ x\neq -\frac{5}{4}$$

15.
$$\begin{array}{r} 7x^2\ \ -14x+28 \\ x+2\overline{\big)7x^3+\ 0x^2\ \ +0x\ +\ 3} \\ \underline{7x^3+14x^2}\qquad\qquad\qquad \\ -14x^2\qquad\qquad\quad \\ \underline{-14x^2-28x}\qquad\quad \\ 28x+3 \\ \underline{28x+56} \\ -53 \end{array}$$

$$\frac{7x^3+3}{x+2}=7x^2-14x+28-\frac{53}{x+2}$$

17.
$$\begin{array}{r} 3x+5 \\ 2x^2+0x+1\overline{\big)6x^3+10x^2+\ x+8} \\ \underline{6x^3+\ 0x^2\ +3x}\quad \\ 10x^2-2x\ +8 \\ \underline{10x^2+0x\ +5} \\ -2x+3 \end{array}$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1}=3x+5-\frac{2x-3}{2x^2+1}$$

19.
$$\begin{array}{r} x \\ x^2+1\overline{\big)x^3+0x^2+0x-9} \\ \underline{x^3\qquad\ \ +\ x}\quad \\ -x-9 \end{array}$$

$$\frac{x^3-9}{x^2+1}=x-\frac{x+9}{x^2+1}$$

21.
$$\begin{array}{r} 2x \\ x^2-2x+1\overline{\big)2x^3-4x^2-15x+5} \\ \underline{2x^3-4x^2+\ 2x}\quad \\ -17x+5 \end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2}=2x-\frac{17x-5}{(x-1)^2}$$

23.
$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$\frac{3x^3-17x^2+15x-25}{x-5}=3x^2-2x+5,\ x\neq 5$$

25.
$$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3+7x^2-x+26}{x-3}=6x^2+25x+74+\frac{248}{x-3}$$

27.
$$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3-18x^2-16x+32}{x-2}=9x^2-16,\ x\neq 2$$

29.
$$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3+512}{x+8}=x^2-8x+64,\ x\neq -8$$

31.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3+16x^2-23x-15}{x+\frac{1}{2}}=4x^2+14x-30,\ x\neq -\frac{1}{2}$$

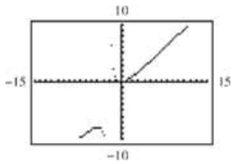
33. $y_2 = x - 2 + \dfrac{4}{x+2}$

$= \dfrac{(x-2)(x+2)+4}{x+2}$

$= \dfrac{x^2-4+4}{x+2}$

$= \dfrac{x^2}{x+2}$

$= y_1$



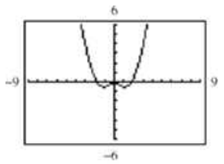
35. $y_2 = x^2 - 8 + \dfrac{39}{x^2+5}$

$= \dfrac{(x^2-8)(x^2+5)+39}{x^2+5}$

$= \dfrac{x^4-8x^2+5x^2-40+39}{x^2+5}$

$= \dfrac{x^4-3x^2-1}{x^2+5}$

$= y_1$



37. $f(x) = x^3 - x^2 - 14x + 11,\ k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x-4)(x^2+3x-2)+3$

$f(4) = (0)(26)+3 = 3$

39.

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2+3\sqrt{2} & 6 \\ \hline & 1 & 3+\sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = \left(x-\sqrt{2}\right)\left(x^2+\left(3+\sqrt{2}\right)x+3\sqrt{2}\right)-8$

$f\left(\sqrt{2}\right) = 0\left(4+6\sqrt{2}\right)-8 = -8$

41.

$$\begin{array}{r|rrrr} 1-\sqrt{3} & 4 & -6 & -12 & -4 \\ & & 4-4\sqrt{3} & 10-2\sqrt{3} & 4 \\ \hline & 4 & -2-4\sqrt{3} & -2-2\sqrt{3} & 0 \end{array}$$

$f(x) = (x-1+\sqrt{3})[4x^2-(2+4\sqrt{3})x-(2+2\sqrt{3})]$

$f(1-\sqrt{3}) = 0$

43. $f(x) = 2x^3 - 7x + 3$

(a)
$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array} \quad = f(1)$$

(b)
$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array} \quad = f(-2)$$

(c)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array} \quad = f\left(\frac{1}{2}\right)$$

(d)
$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array} \quad = f(2)$$

45. $h(x) = x^3 - 5x^2 - 7x + 4$

(a)
$$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array} \quad = h(3)$$

(b)
$$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array} \quad = h(2)$$

(c)
$$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array} \quad = h(-2)$$

(d)
$$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array} \quad = h(-5)$$

47.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^3 - 7x + 6 = (x-2)(x^2+2x-3)$

$= (x-2)(x+3)(x-1)$

Zeros: 2, −3, 1

49.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$2x^3 - 15x^2 + 27x - 10$

$= (x - \frac{1}{2})(2x^2 - 14x + 20)$

$= (2x-1)(x-2)(x-5)$

Zeros: $\frac{1}{2}$, 2, 5

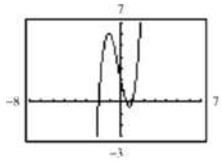
51. (a) $\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$

(b) $2x^2 - 3x + 1 = (2x-1)(x-1)$

Remaining factors: $(2x-1)$, $(x-1)$

(c) $f(x) = (x+2)(2x-1)(x-1)$

(d) Real zeros: $-2, \frac{1}{2}, 1$



53. (a) $\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$

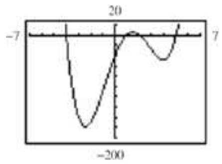
$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$

(b) $x^2 - 3x + 2 = (x-2)(x-1)$

Remaining factors: $(x-2)$, $(x-1)$

(c) $f(x) = (x-5)(x+4)(x-2)(x-1)$

(d) Real zeros: $5, -4, 2, 1$



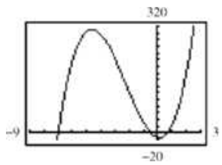
55. (a) $\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$

(b) $6x^2 + 38x - 28 = (3x-2)(2x+14)$

Remaining factors: $(3x-2)$, $(x+7)$

(c) $f(x) = (2x+1)(3x-2)(x+7)$

(d) Real zeros: $-\frac{1}{2}, \frac{2}{3}, -7$



57. $f(x) = x^3 + 3x^2 - x - 3$

p = factor of -3
q = factor of 1

Possible rational zeros: $\pm 1, \pm 3$

$f(x) = x^2(x+3) - (x+3) = (x+3)(x^2-1)$

Rational zeros: $\pm 1, -3$

59. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

p = factor of -45
q = factor of 2

Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Using synthetic division, -1, 3, and 5 are zeros.

$f(x) = (x+1)(x-3)(x-5)(2x-3)$

Rational zeros: $-1, 3, 5, \frac{3}{2}$

61. $f(x) = 2x^4 - x^3 + 6x^2 - x + 5$

4 variations in sign $\Rightarrow$ 4, 2, or 0 positive real zeros
$f(-x) = 2x^4 + x^3 + 6x^2 + x + 5$
0 variations in sign $\Rightarrow$ 0 negative real zeros

63. $g(x) = 4x^3 - 5x + 8$

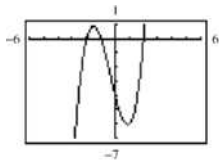
2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros
$g(-x) = -4x^3 + 5x + 8$
1 variation in sign $\Rightarrow$ 1 negative real zero

65. $f(x) = x^3 + x^2 - 4x - 4$

(a) $f(x)$ has 1 variation in sign $\Rightarrow$ 1 positive real zero.
$f(-x) = -x^3 + x^2 + 4x - 4$ has 2 variations in sign $\Rightarrow$ 2 or 0 negative real zeros.

(b) Possible rational zeros: $\pm 1, \pm 2, \pm 4$

(c)



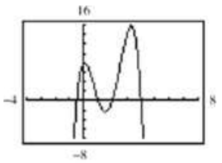
(d) Real zeros: $-2, -1, 2$

67. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) $f(x)$ has variations in sign $\Rightarrow$ 3 or 1 positive real zeros.
$f(-x) = -2x^4 - 13x^3 - 21x^2 - 2x + 8$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros: $\pm\frac{1}{2}, \ \pm 1, \ \pm 2, \ \pm 4, \ \pm 8$

(c)



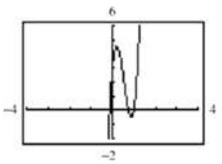
(d) Real zeros: $-\frac{1}{2}, 1, 2, 4$

69. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) $f(x)$ has 2 variations in sign $\Rightarrow$ 2 or 0 positive real zeros.
$f(-x) = -32x^3 - 52x^2 - 17x + 3$ has 1 variation in sign $\Rightarrow$ 1 negative real zero.

(b) Possible rational zeros:
$\pm\frac{1}{32}, \pm\frac{1}{16}, \pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{32}, \pm\frac{3}{16}, \pm\frac{3}{8}, \pm\frac{3}{4}, \pm\frac{3}{2}, \pm 3$

(c)



(d) Real zeros: $1, \frac{3}{4}, -\frac{1}{8}$

71. $f(x) = x^4 - 4x^3 + 15$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$$

4 is an upper bound.

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$$

-1 is a lower bound.
Real zeros: 1.937, 3.705

73. $f(x) = x^4 - 4x^3 + 16x - 16$

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 25 & 105 & 525 & 2705 \\ \hline & 5 & 21 & 105 & 541 & 2689 \end{array}$$

5 is an upper bound.

$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

-3 is a lower bound.

Real zeros: $-2, 2$

75. $P(x) = x^4 - \frac{25}{4}x^2 + 9$

$$= \frac{1}{4}(4x^4 - 25x^2 + 36)$$
$$= \frac{1}{4}(4x^2 - 9)(x^2 - 4)$$
$$= \frac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$$

The rational zeros are $\pm\frac{3}{2}$ and ± 2.

77. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$$= \frac{1}{4}(4x^3 - x^2 - 4x + 1)$$
$$= \frac{1}{4}[x^2(4x - 1) - (4x - 1)]$$
$$= \frac{1}{4}(4x - 1)(x^2 - 1)$$
$$= \frac{1}{4}(4x - 1)(x + 1)(x - 1)$$

The rational zeros are $\frac{1}{4}$ and ± 1.

79. $f(x) = x^3 - 1$
$= (x - 1)(x^2 + x + 1)$

Rational zeros: $1 \ (x = 1)$

Irrational zeros: 0

Matches (d).

81. $f(x) = x^3 - x = x(x+1)(x-1)$

Rational zeros: $3\ (x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

83. $y = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

Using the graph and synthetic division, $-\frac{1}{2}$ is a zero.

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -9 & 5 & 3 & -1 \\ & & -1 & 5 & -5 & 1 \\ \hline & 2 & -10 & 10 & -2 & 0 \end{array}$$

$y = \left(x + \frac{1}{2}\right)(2x^3 - 10x^2 + 10x - 2)$

$x = 1$ is a zero of the cubic, so

$y = (2x+1)(x-1)(x^2 - 4x + 1)$.

For the quadratic term, use the Quadratic Formula.

$$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

The real zeros are $-\frac{1}{2}, 1, 2 \pm \sqrt{3}$.

85. $y = -2x^4 + 17x^3 - 3x^2 - 25x - 3$

Using the graph and synthetic division, -1 and $\frac{3}{2}$ are zeros.

$y = -(x+1)(2x-3)(x^2 - 8x - 1)$

For the quadratic term, use the Quadratic Formula.

$$x = \frac{8 \pm \sqrt{64+4}}{2} = 4 \pm \sqrt{17}$$

The real zeros are $-1, \frac{3}{2}, 4 \pm \sqrt{17}$.

87. $3x^4 - 14x^2 - 4x = 0$

$x(3x^3 - 14x - 4) = 0$

$x = 0$ is a real zero.

Possible ratoinal zeros: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

$$\begin{array}{r|rrrr} -2 & 3 & 0 & -14 & -4 \\ & & -6 & 12 & 4 \\ \hline & 3 & -6 & -2 & 0 \end{array}$$

$x = -2$ is a real zero.

Use the Quadratic Formula. $3x^2 - 6x - 2 = 0$

$$x = \frac{3 \pm \sqrt{15}}{3}$$

Real zeros: $x = 0, -2, \frac{3 \pm \sqrt{15}}{3}$

89. $z^4 - z^3 - 2z - 4 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 0 & -2 & -4 \\ & & -1 & 2 & -2 & 4 \\ \hline & 1 & -2 & 2 & -4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 2 & -4 \\ & & 2 & 0 & 4 \\ \hline & 1 & 0 & 2 & 0 \end{array}$$

$z = -1$ and $z = 2$ are real zeros.

$z^4 - z^3 - 2z - 4 = (z+1)(z-2)(z^2+2) = 0$

The only real zeros are –1 and 2. You can verify this by graphing the function $f(z) = z^4 - z^3 - 2z - 4$.

91. $2y^4 + 7y^3 - 26y^2 + 23y - 6 = 0$

Possible rational zeros:

$\pm\frac{1}{2},\ \pm 1,\ \pm\frac{3}{2},\ \pm 2,\ \pm 3,\ \pm 6$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & 7 & -26 & 23 & -6 \\ & & 1 & 4 & -11 & 6 \\ \hline & 2 & 8 & -22 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 8 & -22 & 12 \\ & & 2 & 10 & -12 \\ \hline & 2 & 10 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrr} -6 & 2 & 10 & -12 \\ & & -12 & 12 \\ \hline & 2 & -2 & 0 \end{array}$$

$$\begin{array}{r|rr} 1 & 2 & -2 \\ & & 2 \\ \hline & 2 & 0 \end{array}$$

$x = \frac{1}{2},\ x = 1, x = -6,$ and $x = 1$ are real zeros.

$(y+6)(y-1)^2(2y-1) = 0$

Real zeros: $x = -6,\ 1,\ \frac{1}{2}$

93. $4x^4 - 55x^2 - 45x + 36 = 0$

Possible rational zeros:

$\pm\frac{1}{2},\ \pm\frac{3}{4},\ \pm 1,\ \pm\frac{3}{2},\ \pm 2,\ \pm\frac{9}{4},\ \pm 3,\ \pm 4,\ \pm\frac{9}{2},\ \pm 6,\ \pm 9,\ \pm 18$

$$\begin{array}{r|rrrrr} 4 & 4 & 0 & -55 & -45 & 36 \\ & & 16 & 64 & 36 & -36 \\ \hline & 4 & 16 & 9 & -9 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 4 & 16 & 9 & -9 \\ & & -12 & -12 & 9 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2} & 4 & 4 & -3 \\ & & 2 & 3 \\ \hline & 4 & 6 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{3}{2} & 4 & 6 \\ & & -6 \\ \hline & 4 & 0 \end{array}$$

$x = 4,\ x = -3,\ x = \frac{1}{2},$ and $x = -\frac{3}{2}$ are real zeros.

$(x-4)(x+3)(2x-1)(2x+3) = 0$

Real zeros: $x = 4, -3, \frac{1}{2}, -\frac{3}{2}$

95. $8x^4 + 28x^3 + 9x^2 - 9x = 0$

$x(8x^3 + 28x^2 + 9x - 9) = 0$

$x = 0$ is a real zero.

Possible rational zeros:

$\pm 1,\ \pm 3,\ \pm 9,\ \pm\frac{1}{8},\ \pm\frac{3}{8},\ \pm\frac{9}{8},\ \pm\frac{1}{2},\ \pm\frac{3}{2},\ \pm\frac{9}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{4},\ \pm\frac{9}{4}$

$$\begin{array}{r|rrrr} -3 & 8 & 28 & 9 & -9 \\ & & -24 & -12 & 9 \\ \hline & 8 & 4 & -3 & 0 \end{array}$$

$x = -3$ is a real zeros

Use the Quadratic Formula.

$8x^2 + 4x - 3 = 0 \qquad x = \frac{- \pm\sqrt{7}}{4}$

Real zeros: $x = 0,\ -3,\ \frac{-1\pm\sqrt{7}}{4}$

97. $4x^5 + 12x^4 - 11x^3 - 42x^2 + 7x + 30 = 0$

Possible rational zeros:

$\pm 1,\ \pm 2,\ \pm 3,\ \pm 5,\ \pm 6,\ \pm 10,\ \pm 15,\ \pm 30,$

$\pm\frac{1}{2},\ \pm\frac{1}{4},\ \pm\frac{3}{2},\ \pm\frac{3}{4},\ \pm\frac{5}{2},\ \pm\frac{5}{4},\ \pm\frac{15}{2},\ \pm\frac{15}{4}$

$$\begin{array}{r|rrrrrr} 1 & 4 & 12 & -11 & -42 & 7 & 30 \\ & & 4 & 16 & 5 & -37 & -30 \\ \hline & 4 & 16 & 5 & -37 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 16 & 5 & -37 & -30 \\ & & -4 & -12 & 7 & 30 \\ \hline & 4 & 12 & -7 & -30 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 4 & 12 & -7 & -30 \\ & & -8 & -8 & 30 \\ \hline & 4 & 4 & -15 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{2} & 4 & 4 & -15 \\ & & 6 & 15 \\ \hline & 4 & 10 & 0 \end{array}$$

$$\begin{array}{r|rr} -\frac{5}{2} & 4 & 10 \\ & & -10 \\ \hline & 4 & 0 \end{array}$$

$x = 1,\ x = -1,\ x = -2,\ x = \frac{3}{2},$ and $x = -\frac{5}{2}$ are real zeros.

$(x-1)(x+1)(x+2)(2x-3)(2x+5) = 0$

Real zeros: $x = 1,\ -1,\ -2,\ \frac{3}{2},\ -\frac{5}{2}.$

99. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) Zeros: -2, 3.732, 0.268

(b)
$$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array} \qquad t = -2 \text{ is a zero.}$$

(c) $h(t) = (t+2)(t^2 - 4t + 1)$

$= (t+2)\left[t - (\sqrt{3}+2)\right]\left[t + (\sqrt{3}-2)\right]$

101. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $x = 0,\ 3,\ 4,\ \pm 1.414$

(b)
$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

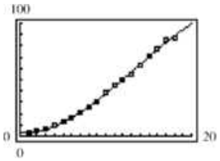
$x = 3$ is a zero.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array} \qquad x = 4 \text{ is a zero.}$$

(c) $h(x) = x(x-3)(x-4)(x^2 - 2)$

$= x(x-3)(x-4)(x - \sqrt{2})(x + \sqrt{2})$

103. (a)



(b) The model fits the data well.

(c) $S = -0.0135t^3 + 0.545t^2 - 0.71t + 3.6$

$$\begin{array}{r|rrrr} 25 & -0.0135 & 0.545 & -0.71 & 3.6 \\ & & -0.3375 & 5.1875 & 111.9375 \\ \hline & -0.0135 & 0.2075 & 4.4775 & 115.5375 \end{array}$$

In 2015 (t = 25), the model predicts approximately 116 subscriptions per 100 people, obviously not a reasonable prediction because you cannot have more subscriptions than people.

105. (a) Combined length and width:

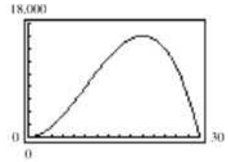
$4x + y = 120 \Rightarrow y = 120 - 4x$

$\text{Volume} = l \cdot w \cdot h = x^2 y$

$= x^2(120 - 4x)$

$= 4x^2(30 - x)$

(b)



Dimension with maximum volume: $20 \times 20 \times 40$

(c) $13{,}500 = 4x^2(30 - x)$

$4x^3 - 120x^2 + 13{,}500 = 0$

$x^3 - 30x^2 + 3375 = 0$

$$\begin{array}{r|rrrr} 15 & 1 & -30 & 0 & 3375 \\ & & 15 & -225 & -3375 \\ \hline & 1 & -15 & -225 & 0 \end{array}$$

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15$ or $\dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

107. False, $-\dfrac{4}{7}$ is a zero of f.

109. The zeros are 1, 1, and –2. The graph falls to the right.

$y = a(x-1)^2(x+2),\ a < 0$

Since $f(0) = -4,\ a = -2.$

$y = -2(x-1)^2(x+2)$

111. $f(x) = -(x+1)(x-1)(x+2)(x-2)$

113. (a) $\dfrac{x^2-1}{x-1}=x+1,\ x\neq 1$

(b) $\dfrac{x^3-1}{x-1}=x^2+x+1,\ x\neq 1$

(c) $\dfrac{x^4-1}{x-1}=x^3+x^2+x+1,\ x\neq 1$

In general,

$$\frac{x^n-1}{x-1}=x^{n-1}+x^{n-2}+\cdots+x+1,\ x\neq 1.$$

115. $9x^2-25 = 0$

$(3x+5)(3x-5)=0$

$x=-\dfrac{5}{3},\ \dfrac{5}{3}$

117. $2x^2+6x+3=0$

$$x=\frac{-6\pm\sqrt{6^2-4(2)(3)}}{2(2)}$$
$$=\frac{-6\pm\sqrt{12}}{4}$$
$$=\frac{-3\pm\sqrt{3}}{2}$$
$$x=-\frac{3}{2}+\frac{\sqrt{3}}{2},-\frac{3}{2}-\frac{\sqrt{3}}{2}$$

Section 2.4

1. (a) ii
(b) iii
(c) i

3. $(7+6i)+(8+5i)=(7+8)+(6+5)i$
$=15+11i$
The real part is 15 and the imaginary part is $11i$.

5. The additive inverse of $2-4i$ is $-2+4i$ so that $(2-4i)+(-2+4i)=0$.

7. $a+bi=-9+4i$
$a=-9$
$b=4$

9. $3a+(b+3)i=9+8i$
$3a=9 \quad b+3=8$
$a=3 \quad b=5$

11. $5+\sqrt{-16}=5+\sqrt{16(-1)}$
$=5+4i$

13. $-6=-6+0i$

15. $-5i+i^2=-5i-1=-1-5i$

17. $\left(\sqrt{-75}\right)^2=-75$

19. $\sqrt{-0.09}=\sqrt{0.09}i=0.3i$

21. $(4+i)-(7-2i)=(4-7)+(1+2)i$
$=-3+3i$

23. $(-1+8i)+(8-5i)=(-1+8)+(8-5)i$
$=7+3i$

25. $13i-(14-7i)=13i-14+7i=-14+20i$

27. $$\left(\frac{3}{2}+\frac{5}{2}i\right)+\left(\frac{5}{3}+\frac{11}{3}i\right)=\left(\frac{3}{2}+\frac{5}{3}\right)+\left(\frac{5}{2}+\frac{11}{3}\right)i$$
$$=\frac{9+10}{6}+\frac{15+22}{6}i$$
$$=\frac{19}{6}+\frac{37}{6}i$$

29. $(1.6+3.2i)+(-5.8+4.3i)=-4.2+7.5i$

31. $4(3+5i)=12+20i$

33. $(1+i)(3-2i)=3-2i+3i-2i^2$
$=3+i+2$
$=5+i$

35. $4i(8+5i)=32i+20i^2$
$=32i+20(-1)$
$=-20+32i$

37. $\left(\sqrt{14}+\sqrt{10}\,i\right)\left(\sqrt{14}-\sqrt{10}\,i\right)=14-10i^2=14+10=24$

39. $\left(6+7i\right)^2=36+42i+42i+49i^2$
$=36+84i-49$
$=-13+84i$

41. $(4+5i)^2-(4-5i)^2$
$=\left[(4+5i)+(4-5i)\right]\left[(4+5i)-(4-5i)\right]=8(10i)=80i$

43. $4-3i$ is the complex conjugate of $4+3i$.
$(4+3i)(4-3i)=16+9=25$

45. $-6+\sqrt{5}i$ is the complex conjugate of $-6-\sqrt{5}i$.
$\left(-6-\sqrt{5}i\right)\left(-6+\sqrt{5}i\right)=36+5=41$

47. $-\sqrt{20}i$ is the complex conjugate of $\sqrt{-20}=\sqrt{20}i$.

$$\left(\sqrt{20}i\right)\left(-\sqrt{20}i\right)=20$$

49. $3+\sqrt{2}i$ is the complex conjugate of

$$3-\sqrt{-2}=3-\sqrt{2}\,i$$
$$(3-\sqrt{2}i)(3+\sqrt{2}i)=9+2=11$$

51. $\dfrac{6}{i}=\dfrac{6}{i}\cdot\dfrac{-i}{-i}=\dfrac{-6i}{-i^2}=\dfrac{-6i}{1}=-6i$

53. $\dfrac{2}{4-5i}=\dfrac{2}{4-5i}\cdot\dfrac{4+5i}{4+5i}=\dfrac{8+10i}{16+25}=\dfrac{8}{41}+\dfrac{10}{41}i$

55.
$$\begin{aligned}\frac{2+i}{2-i}&=\frac{2+i}{2-i}\cdot\frac{2+i}{2+i}\\&=\frac{4+4i+i^2}{4+1}\\&=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i\end{aligned}$$

57.
$$\begin{aligned}\frac{i}{(4-5i)^2}&=\frac{i}{16-25-40i}\\&=\frac{i}{-9-40i}\cdot\frac{-9+40i}{-9+40i}\\&=\frac{-40-9i}{81+40^2}\\&=-\frac{40}{1681}-\frac{9}{1681}i\end{aligned}$$

59.
$$\begin{aligned}\frac{2}{1+i}-\frac{3}{1-i}&=\frac{2(1-i)-3(1+i)}{(1+i)(1-i)}\\&=\frac{2-2i-3-3i}{1+1}\\&=\frac{-1-5i}{2}\\&=-\frac{1}{2}-\frac{5}{2}i\end{aligned}$$

61.
$$\begin{aligned}\frac{i}{3-2i}+\frac{2i}{3+8i}&=\frac{3i+8i^2+6i-4i^2}{(3-2i)(3+8i)}\\&=\frac{-4+9i}{9+18i+16}\\&=\frac{-4+9i}{25+18i}\cdot\frac{25-18i}{25-18i}\\&=\frac{-100+72i+225i+162}{25^2+18^2}\\&=\frac{62+297i}{949}\\&=\frac{62}{949}+\frac{297}{949}i\end{aligned}$$

63.
$$\begin{aligned}\sqrt{-18}-\sqrt{-54}&=3\sqrt{2}i-3\sqrt{6}i\\&=3\left(\sqrt{2}-\sqrt{6}\right)i\end{aligned}$$

65.
$$\begin{aligned}\left(-3+\sqrt{-24}\right)+\left(7-\sqrt{-44}\right)&=\left(-3+2\sqrt{6}i\right)+\left(7-2\sqrt{11}i\right)\\&=4+\left(2\sqrt{6}-2\sqrt{11}\right)i\\&=4+2\left(\sqrt{6}-\sqrt{11}\right)i\end{aligned}$$

67.
$$\begin{aligned}\sqrt{-6}\cdot\sqrt{-2}&=\left(\sqrt{6}i\right)\left(\sqrt{2}i\right)\\&=\sqrt{12}\,i^2=\left(2\sqrt{3}\right)(-1)=-2\sqrt{3}\end{aligned}$$

69. $\left(\sqrt{-10}\right)^2=\left(\sqrt{10}i\right)^2=10i^2=-10$

71.
$$\begin{aligned}\left(2-\sqrt{-6}\right)^2&=\left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)\\&=4-2\sqrt{6}i-2\sqrt{6}i+6i^2\\&=4-2\sqrt{6}i-2\sqrt{6}i+6(-1)\\&=4-6-4\sqrt{6}i\\&=-2-4\sqrt{6}i\end{aligned}$$

73.
$$\begin{aligned}x^2+25&=0\\x^2&=-25\\x&=\pm5i\end{aligned}$$

75. $x^2-2x+2=0;\ a=1,\ b=-2,\ c=2$

$$\begin{aligned}x&=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}\\&=\frac{2\pm\sqrt{-4}}{2}\\&=\frac{2\pm2i}{2}\\&=1\pm i\end{aligned}$$

77. $4x^2+16x+17=0;\ a=4,\ b=16,\ c=17$

$$\begin{aligned}x&=\frac{-16\pm\sqrt{(16)^2-4(4)(17)}}{2(4)}\\&=\frac{-16\pm\sqrt{-16}}{8}\\&=\frac{-16\pm4i}{8}\\&=-2\pm\frac{1}{2}i\end{aligned}$$

79. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$
$$= \frac{4 \pm \sqrt{-176}}{32}$$
$$= \frac{4 \pm 4\sqrt{11}\,i}{32}$$
$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

81. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{-72}}{6}$$
$$= \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

83. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0;\ a = 7,\ b = -10,\ c = -50$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)}$$
$$= \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14}$$
$$= \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

85. $-6i^3 + i^2 = -6i^2 i + i^2 = -6(-1)i + (-1) = 6i - 1 = -1 + 6i$

87. $\left(\sqrt{-75}\right)^3 = \left(5\sqrt{3}i\right)^3 = 5^3\left(\sqrt{3}\right)^3 i^3 = 125\left(3\sqrt{3}\right)(-i)$
$= -375\sqrt{3}i$

89. $\frac{1}{i^3} = \frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = \frac{i}{1} = i$

91. (a) $(2)^3 = 8$

(b) $\left(-1+\sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$
$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$
$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$
$= 8$

(c) $\left(-1-\sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$
$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$
$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$
$= 8$

The three numbers are cube roots of 8.

93. (a) $i^{20} = (i^4)^5 = (1)^5 = 1$

(b) $i^{45} = (i^4)^{11} i = (1)^{11} i = i$

(c) $i^{67} = (i^4)^{16} i^3 = (1)^{16}(-i) = -i$

(d) $i^{114} = (i^4)^{28} i^2 = (1)^{28}(-1) = -1$

95. False. A real number $a + 0i = a$ is equal to its conjugate.

97. False. For example, $(1+2i) + (1-2i) = 2,$ which is not an imaginary number.

99. True. Let $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i.$ Then

$$\overline{z_1 z_2} = \overline{(a_1 + b_1 i)(a_2 + b_2 i)}$$
$$= \overline{(a_1 a_2 - b_1 b_2) + (a_1 b_2 + b_1 a_2)i}$$
$$= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + b_1 a_2)i$$
$$= (a_1 - b_1 i)\ (a_2 - b_2 i)$$
$$= \overline{a_1 + b_1 i}\ \overline{a_2 + b_2 i}$$
$$= \overline{z_1}\,\overline{z_2}.$$

101. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}\,i = 6i^2 = -6$

103. $(4x-5)(4x+5) = 16x^2 - 20x + 20x - 25 = 16x^2 - 25$

105. $\left(3x - \frac{1}{2}\right)(x+4) = 3x^2 - \frac{1}{2}x + 12x - 2 = 3x^2 + \frac{23}{2}x - 2$

Section 2.5

1. Fundamental Theorem, Algebra

3. The Linear Factorization Theorem states that a polynomial function f of degree $n,\ n > 0$, has exactly n linear factors

$$f(x) = a(x - c_1)(x - c_2)\ldots(x - c_n).$$

5. $f(x) = x^3 + x$ has exactly 3 zeros. Matches (c).

7. $f(x) = x^5 + 9x^3$ has exactly 5 zeros. Matches (d).

9. $f(x) = x^2 + 25$

$$x^2 + 25 = 0$$
$$x^2 = -25$$
$$x = \pm\sqrt{-25}$$
$$x = \pm 5i$$

11. $f(x) = x^3 + 9x$

$$x^3 + 9x = 0$$
$$x(x^2 + 9) = 0$$
$$x = 0 \quad x^2 + 9 = 0$$
$$x^2 = -9$$
$$x = \pm\sqrt{-9}$$
$$x = \pm 3i$$

13. $f(x) = x^3 - 4x^2 + x - 4 = x^2(x-4) + 1(x-4) = (x-4)(x^2+1)$

Zeros: $4, \pm i$

The only real zero of *f(x)* is $x = 4$. This corresponds to the *x*-intercept of $(4, 0)$ on the graph.

15. $f(x) = x^4 + 4x^2 + 4 = (x^2 + 2)^2$

Zeros: $\pm\sqrt{2}i, \ \pm\sqrt{2}i$

f(*x*) has no real zeros and the graph of *f*(*x*) has no *x*-intercepts.

17. $h(x) = x^2 - 4x + 1$

h has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}.$$

$$h(x) = \left[x - \left(2 + \sqrt{3}\right)\right]\left[x - \left(2 - \sqrt{3}\right)\right]$$
$$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$$

19. $f(x) = x^2 - 12x + 26$

f has no rational zeros. By the Quadratic Formula, the zeros are

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(26)}}{2} = 6 \pm \sqrt{10}.$$

$$f(x) = \left[x - (6 + \sqrt{10})\right]\left[x - (6 - \sqrt{10})\right]$$
$$= (x - 6 - \sqrt{10})(x - 6 + \sqrt{10})$$

21. $f(x) = x^2 + 25$

Zeros: $\pm 5i$

$f(x) = (x + 5i)(x - 5i)$

23. $f(x) = 16x^4 - 81$

$$= (4x^2 - 9)(4x^2 + 9)$$
$$= (2x - 3)(2x + 3)(2x + 3i)(2x - 3i)$$

Zeros: $\pm\frac{3}{2}, \ \pm\frac{3}{2}i$

25. $f(z) = z^2 - z + 56$

$$z = \frac{1 \pm \sqrt{1 - 4(56)}}{2}$$
$$= \frac{1 \pm \sqrt{-223}}{2}$$
$$= \frac{1}{2} \pm \frac{\sqrt{223}}{2}i$$

Zeros: $\frac{1}{2} \pm \frac{\sqrt{223}}{2}i$

$$f(z) = \left(z - \frac{1}{2} + \frac{\sqrt{223}i}{2}\right)\left(z - \frac{1}{2} - \frac{\sqrt{223}i}{2}\right)$$

27. $f(x) = x^4 + 10x^2 + 9$

$$= (x^2 + 1)(x^2 + 9)$$
$$= (x + i)(x - i)(x + 3i)(x - 3i)$$

The zeros of *f*(*x*) are $x = \pm i$ and $x = \pm 3i$.

29. $f(x) = 3x^3 - 5x^2 + 48x - 80$

Using synthetic division, $\frac{5}{3}$ is a zero:

$$\begin{array}{r|rrrr} \frac{5}{3} & 3 & -5 & 48 & -80 \\ & & 5 & 0 & 80 \\ \hline & 3 & 0 & 48 & 0 \end{array}$$

$$f(x) = \left(x - \frac{5}{3}\right)\left(3x^2 + 48\right)$$
$$= (3x - 5)(x^2 + 16)$$
$$= (3x - 5)(x + 4i)(x - 4i)$$

The zeros are $\frac{5}{3}, \ 4i, \ -4i$.

31. $f(t) = t^3 - 3t^2 - 15t + 125$

Possible rational zeros: $\pm 1, \ \pm 5, \ \pm 25, \ \pm 125$

$$\begin{array}{r|rrrr} -5 & 1 & -3 & -15 & 125 \\ & & -5 & 40 & -125 \\ \hline & 1 & -8 & 25 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $t^2 - 8t + 25$ are

$$t = \frac{8 \pm \sqrt{64 - 100}}{2} = 4 \pm 3i.$$

The zeros of *f*(*t*) are $t = -5$ and $t = 4 \pm 3i$.

$$f(t) = \left[t - (-5)\right]\left[t - (4 + 3i)\right]\left[t - (4 - 3i)\right]$$
$$= \left(t + 5\right)\left(t - 4 - 3i\right)\left(t - 4 + 3i\right)$$

33. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros: $\pm 6, \pm \frac{6}{5}, \pm 3, \pm \frac{3}{5}, \pm 2, \pm \frac{2}{5}, \pm 1, \pm \frac{1}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30$ are those of $x^2 - 2x + 6$:

$$x = \frac{2 \pm \sqrt{4 - 4(6)}}{2} = 1 \pm \sqrt{5}i$$

Zeros: $-\frac{1}{5}, 1 \pm \sqrt{5}i$

$$\begin{aligned} f(x) &= 5\left(x + \frac{1}{5}\right)\left[x - \left(1 + \sqrt{5}i\right)\right]\left[x - \left(1 - \sqrt{5}i\right)\right] \\ &= (5x + 1)\left(x - 1 - \sqrt{5}i\right)\left(x - 1 + \sqrt{5}i\right) \end{aligned}$$

35. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline 2 & 1 & -2 & 4 & -8 & 0 \\ & & 2 & 0 & 8 & \\ \hline & 1 & 0 & 4 & 0 & \end{array}$$

$$\begin{aligned} g(x) &= (x - 2)(x - 2)\left(x^2 + 4\right) \\ &= (x - 2)^2 (x + 2i)(x - 2i) \end{aligned}$$

The zeros of g are $2, 2$, and $\pm 2i$.

37. (a) $f(x) = x^2 - 14x + 46.$

By the Quadric Formula,

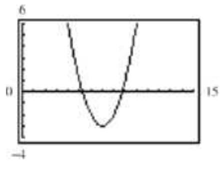
$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(46)}}{2} = 7 \pm \sqrt{3}.$$

The zeros are $7 + \sqrt{3}$ and $7 - \sqrt{3}$.

(b) $f(x) = \left[x - (7 + \sqrt{3})\right]\left[x - (7 - \sqrt{3})\right]$

$= (x - 7 - \sqrt{3})(x - 7 + \sqrt{3})$

(c) x-intercepts: $\left(7 + \sqrt{3}, 0\right)$ and $\left(7 - \sqrt{3}, 0\right)$



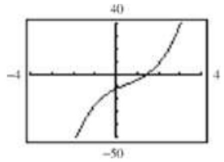
39. (a) $f(x) = 2x^3 - 3x^2 + 8x - 12$

$= (2x - 3)\left(x^2 + 4\right)$

The zeros are $\frac{3}{2}$ and $\pm 2i$.

(b) $f(x) = (2x - 3)(x + 2i)(x - 2i)$

(c) x-intercept: $\left(\frac{3}{2}, 0\right)$



41. (a) $f(x) = x^3 - 11x + 150$

$= (x + 6)\left(x^2 - 6x + 25\right)$

Use the Quadratic Formula to find the zeros of $x^2 - 6x + 25$.

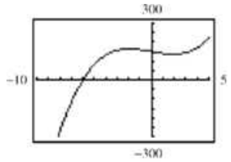
$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(25)}}{2} = 3 \pm 4i.$$

The zeros are $-6, 3 + 4i$, and $3 - 4i$.

(b)

$f(x) = (x + 6)(x - 3 + 4i)(x - 3 - 4i)$

(c) x-intercept: $(-6, 0)$



43. (a) $f(x) = x^4 + 25x^2 + 144$

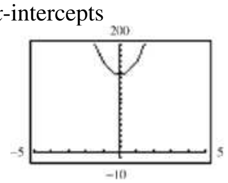
$= \left(x^2 + 9\right)\left(x^2 + 16\right)$

The zeros are $\pm 3i, \pm 4i$.

(b) $f(x) = \left(x^2 + 9\right)\left(x^2 + 16\right)$

$= (x + 3i)(x - 3i)(x + 4i)(x - 4i)$

(c) No x-intercepts



45. $f(x) = (x - 2)(x - i)(x + i)$

$= (x - 2)\left(x^2 + 1\right)$

Note that $f(x) = a(x^3 - 2x^2 + x - 2)$, where a is any nonzero real number, has zeros $2, \pm i$.

47. $f(x) = (x-2)^2(x-4-i)(x-4+i)$

$= (x-2)^2(x-8x+16+1)$

$= (x^2-4x+4)(x^2-8x+17)$

$= x^4-12x^3+53x^2-100x+68$

Note that $f(x) = a(x^4-12x^3+53x^2-100x+68)$, where a is any nonzero real number, has zeros $2,\ 2,\ 4 \pm i$.

49. Because $1+\sqrt{2}i$ is a zero, so is $1-\sqrt{2}i$.

$f(x) = (x-0)(x+5)(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)$

$= (x^2+5x)(x^2-2x+1+2)$

$= (x^2+5x)(x^2-2x+3)$

$= x^4+3x^3-7x^2-15x$

Note that $f(x) = a(x^4+3x^3-7x^2+15x)$, where a is any nonzero real number, has zeros $0,\ -5,\ 1 \pm \sqrt{2}i$.

51. (a) $f(x) = a(x-1)(x+2)(x-2i)(x+2i)$

$= a(x-1)(x+2)(x^2+4)$

$f(1) = 10 = a(-2)(1)(5) \Rightarrow a = -1$

$f(x) = -(x-1)(x+2)(x-2i)(x+2i)$

(b) $f(x) = -(x-1)(x+2)(x^2+4)$

$= -(x^2+x-2)(x^2+4)$

$= -x^4-x^3-2x^2-4x+8$

53. (a) $f(x) = a(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$

$= a(x+1)(x^2-4x+4+5)$

$= a(x+1)(x^2-4x+9)$

$f(-2) = 42 = a(-1)(4+8+9) \Rightarrow a = -2$

$f(x) = -2(x+1)(x-2-\sqrt{5}i)(x-2+\sqrt{5}i)$

(b) $f(x) = -2(x+1)(x^2-4x+9)$

$= -2x^3+6x^2-10x-18$

55. $f(x) = x^4-6x^2-7$

(a) $f(x) = (x^2-7)(x^2+1)$

(b) $f(x) = (x-\sqrt{7})(x+\sqrt{7})(x^2+1)$

(c) $f(x) = (x-\sqrt{7})(x+\sqrt{7})(x+i)(x-i)$

57. $f(x) = x^4-2x^3-3x^2+12x-18$

(a) $f(x) = (x^2-6)(x^2-2x+3)$

(b) $f(x) = (x+\sqrt{6})(x-\sqrt{6})(x^2-2x+3)$

(c) $f(x) = (x+\sqrt{6})(x-\sqrt{6})(x-1-\sqrt{2}i)(x-1+\sqrt{2}i)$

59. $f(x) = 2x^3+3x^2+50x+75$

Since is $5i$ a zero, so is $-5i$.

$$\begin{array}{r|rrrr} 5i & 2 & 3 & 50 & 75 \\ & & 10i & -50+15i & -75 \\ \hline & 2 & 3+10i & 15i & 0 \end{array}$$

$$\begin{array}{r|rrr} -5i & 2 & 3+10i & 15i \\ & & -10i & -15i \\ \hline & 2 & 3 & 0 \end{array}$$

The zero of $2x+3$ is $x = -\frac{3}{2}$. The zeros of f are $x = -\frac{3}{2}$ and $x = \pm 5i$.

Alternate solution

Since $x = \pm 5i$ are zeros of $f(x)$, $(x+5i)(x-5i) = x^2+25$ is a factor of $f(x)$. By long division we have:

$$\begin{array}{r|l} & \phantom{2x^3+{}}2x+3 \\ \hline x^2+0x+25 & 2x^3+3x^2+50x+75 \\ & \underline{2x^3+0x^2+50x} \\ & \phantom{2x^3+{}}3x^2+0x+75 \\ & \phantom{2x^3+{}}\underline{3x^2+0x+75} \\ & 0 \end{array}$$

Thus, $f(x) = (x^2+25)(2x+3)$ and the zeros of f are $x = \pm 5i$ and $x = -\frac{3}{2}$.

61. $g(x) = x^3-7x^2-x+87$. Since $5+2i$ is a zero, so is $5-2i$.

$$\begin{array}{r|rrrr} 5+2i & 1 & -7 & -1 & 87 \\ & & 5+2i & -14+6i & -87 \\ \hline & 1 & -2+2i & -15+6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5-2i & 1 & -2+2i & -15+6i \\ & & 5-2i & 15-6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x+3$ is $x = -3$.
The zeros of f are $-3,\ 5 \pm 2i$. .

63. $h(x) = 3x^3 - 4x^2 + 8x + 8$ Since $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of h are

$x = -\frac{2}{3}, 1 \pm \sqrt{3}i.$

65. $h(x) = 8x^3 - 14x^2 + 18x - 9.$ Since $\frac{1}{2}(1 - \sqrt{5}i)$ is a zero, so is $\frac{1}{2}(1 + \sqrt{5}i).$

$$\begin{array}{r|rrrr} \frac{1}{2}\left(1-\sqrt{5}i\right) & 8 & -14 & 18 & -9 \\ & & 4-4\sqrt{5}i & -15+3\sqrt{5}i & 9 \\ \hline & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{1}{2}\left(1+\sqrt{5}i\right) & 8 & -10-4\sqrt{5}i & 3+3\sqrt{5}i \\ & & 4+4\sqrt{5}i & -3-3\sqrt{5}i \\ \hline & 8 & -6 & 0 \end{array}$$

The zero of $8x - 6$ is $x = \frac{3}{4}$. The zeros of h are

$x = \frac{3}{4},\ \frac{1}{2}(1 \pm \sqrt{5}i).$

67. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

(a) The *root* feature yields the real roots 1 and 2, and the complex roots $-3 \pm 1.414i$.

(b) By synthetic division:

$$\begin{array}{r|rrrrr} 1 & 1 & 3 & -5 & -21 & 22 \\ & & 1 & 4 & -1 & -22 \\ \hline & 1 & 4 & -1 & -22 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -1 & -22 \\ & & 2 & 12 & 22 \\ \hline & 1 & 6 & 11 & 0 \end{array}$$

The complex roots of $x^2 + 6x + 11$ are

$$x = \frac{-6 \pm \sqrt{6^2 - 4(11)}}{2} = -3 \pm \sqrt{2}i.$$

69. $h(x) = 8x^3 - 14x^2 + 18x - 9$

(a) The *root* feature yields the real root 0.75, and the complex roots $0.5 \pm 1.118i$.

(b) By synthetic division:

$$\begin{array}{r|rrrr} \frac{3}{4} & 8 & -14 & 18 & -9 \\ & & 6 & -6 & 9 \\ \hline & 8 & -8 & 12 & 0 \end{array}$$

The complex roots of $8x^2 - 8x + 12$ are

$$x = \frac{8 \pm \sqrt{64 - 4(8)(12)}}{2(8)} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}i.$$

71. To determine if the football reaches a height of 50 feet, set $h(t) = 50$ and solve for t.

$$-16t^2 + 48t = 50$$
$$-16t^2 + 48t - 50 = 0$$

Using the Quadratic formula:

$$t = \frac{-(48) \pm \sqrt{48^2 - 4(-16)(-50)}}{2(-16)}$$

$$t = \frac{-(48) \pm \sqrt{-896}}{-32}$$

Because the discriminant is negative, the solutions are not real, therefore the football does not reach a height of 50 feet.

73. False, a third-degree polynomial must have at least one real zero.

75. Answers will vary.

77. $f(x) = x^2 - 7x - 8$

$$= \left(x - \frac{7}{2}\right)^2 - \frac{81}{4}$$

A parabola opening upward with vertex

$$\left(\frac{7}{2},\ -\frac{81}{4}\right)$$

79. $f(x) = 6x^2 + 5x - 6$

$$= 6\left(x + \frac{5}{12}\right)^2 - \frac{169}{24}$$

A parabola opening upward with vertex

$$\left(-\frac{5}{12},\ -\frac{169}{24}\right)$$

Section 2.6

1. rational functions

3. To determine the vertical asymptote(s) of the graph of $y = \frac{9}{x-3}$, find the real zeros of the denominator of the equation. (Assuming no common factors in the numerator and denominator)

5. $f(x) = \frac{1}{x-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	−2
0.9	−10
0.99	−100
0.999	−1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	$0.\overline{01}$
1000	$0.\overline{001}$

x	$f(x)$
−5	$-0.1\overline{6}$
−10	$-0.\overline{09}$
−100	$-0.\overline{0099}$
−1000	$-0.\overline{00099}$

(c) f approaches $-\infty$ from the left of 1 and ∞ from the right of 1.

7. $f(x) = \frac{3x}{|x-1|}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	3
0.9	27
0.99	297
0.999	2997

x	$f(x)$
1.5	9
1.1	33
1.01	303
1.001	3003

x	$f(x)$
5	3.75
10	$3.\overline{33}$
100	$3.\overline{03}$
1000	$3.\overline{003}$

x	$f(x)$
−5	−2.5
−10	−2.727
−100	−2.970
−1000	−2.997

(c) f approaches ∞ from both the left and the right of 1.

9. $f(x) = \frac{3x^2}{x^2-1}$

(a) Domain: all $x \neq 1$

(b)

x	$f(x)$
0.5	−1
0.9	−12.79
0.99	−147.8
0.999	−1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502.3

x	$f(x)$
5	3.125
10	$3.\overline{03}$
100	$3.\overline{0003}$
1000	3

x	$f(x)$
−5	3.125
−10	$3.\overline{03}$
−100	$3.\overline{0003}$
−1000	3

(c) f approaches $-\infty$ from the left of 1, and ∞ from the right of 1. f approaches ∞ from the left of −1, and $-\infty$ from the right of −1.

11. $f(x) = \frac{2}{x+2}$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

Matches graph (a).

13. $f(x) = \frac{4x+1}{x}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 4$

Matches graph (c).

15. $f(x) = \frac{x-2}{x-4}$

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 1$

Matches graph (b).

17. $f(x) = \frac{1}{x^2}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$ or x-axis

19. $f(x) = \frac{2x^2}{x^2+x-6} = \frac{2x^2}{(x+3)(x-2)}$

Vertical asymptotes: $x = -3$, $x = 2$

Horizontal asymptote: $y = 2$

21. $f(x) = \frac{x(2+x)}{2x-x^2} = \frac{\cancel{x}(x+2)}{-\cancel{x}(x-2)} = -\frac{x+2}{x-2}, \ x \neq 0$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = -1$

Hole at $x = 0$

23. $f(x)=\dfrac{x^2-25}{x^2+5x}=\dfrac{\cancel{(x+5)}(x-5)}{x\cancel{(x+5)}}=\dfrac{x-5}{x},\ x\neq -5$

Vertical asymptote: $x=0$

Horizontal asymptote: $y=1$

Hole at $x=-5$

25. $f(x)=\dfrac{3x^2+x-5}{x^2+1}$

(a) Domain: all real numbers

(b) Continuous

(c) Vertical asymptote: none

Horizontal asymptote: $y=3$

27. $f(x)=\dfrac{x^2+3x-4}{-x^3+27}=\dfrac{(x+4)(x-1)}{-(x-3)(x^2+3x+9)}$

(a) Domain: all real numbers x except $x=3$

(b) Not continuous at $x=3$

(c) Vertical asymptote: $x=3$

Horizontal asymptote: $y=0$ or x-axis

29. $f(x)=\dfrac{x^2-16}{x-4}=\dfrac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}=x+4,\ x\neq 4$

$g(x)=x+4$

(a) Domain of f: all real x except $x=4$

Domain of g: all real x

(b) Vertical asymptote:

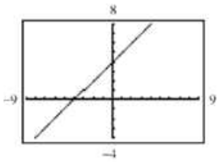
f has none. g has none.

Hole: f has a hole at $x=4$; g has none.

(c)

x	1	2	3	4	5	6	7
$f(x)$	5	6	7	Undef.	9	10	11
$g(x)$	5	6	7	8	9	10	11

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

31. $f(x)=\dfrac{x^2-1}{x^2-2x-3}=\dfrac{\cancel{(x+1)}(x-1)}{(x-3)\cancel{(x+1)}}=\dfrac{x-1}{x-3},\ x\neq -1$

$g(x)=\dfrac{x-1}{x-3}$

(a) Domain of f: all real x except $x=3$ and $x=-1$

Domain of g: all real x except $x=3$

(b) Vertical asymptote: f has a vertical asymptote at $x=3$.

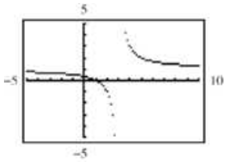
g has a vertical asymptote at $x=3$.

Hole: f has a hole at $x=-1$; g has none.

(c)

x	-2	-1	0	1	2	3	4
$f(x)$	$\frac{3}{5}$	Undef.	$\frac{1}{3}$	0	-1	Undef.	3
$g(x)$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{3}$	0	-1	Undef.	3

(d)



(e) Graphing utilities are limited in their resolution and therefore may not show a hole in a graph.

33. $f(x)=4-\dfrac{1}{x}$

As $x\to\pm\infty$, $f(x)\to 4$.

As $x\to\infty$, $f(x)\to 4$ but is less than 4.

As $x\to-\infty$, $f(x)\to 4$ but is greater than 4.

35. $f(x)=\dfrac{2x-1}{x-3}$

As $x\to\pm\infty$, $f(x)\to 2$.

As $x\to\infty$, $f(x)\to 2$ but is greater than 2.

As $x\to-\infty$, $f(x)\to 2$ but is less than 2.

37. $g(x)=\dfrac{x^2-4}{x+3}=\dfrac{(x-2)(x+2)}{x+3}$

The zeros of g correspond to the zeros of the numerator and are $x=\pm 2$.

39. $f(x)=1-\dfrac{2}{x-5}=\dfrac{x-7}{x-5}$

The zero of f corresponds to the zero of the numerator and is $x=7$.

41. $g(x)=\dfrac{x^2-2x-3}{x^2+1}=\dfrac{(x-3)(x+1)}{x^2+1}=0$

Zeros: $x=-1,\ 3$

43. $f(x)=\dfrac{2x^2-5x+2}{2x^2-7x+3}=\dfrac{(2x-1)(x-2)}{(2x-1)(x-3)}=\dfrac{x-2}{x-3},\ x\neq\dfrac{1}{2}$

Zero: $x=2$ ($x=\dfrac{1}{2}$ is not in the domain.)

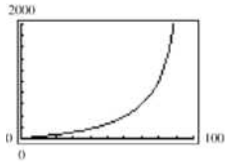
45. $C = \dfrac{255p}{100 - p},\ 0 \le p < 100$

(a) find $C(10) = \dfrac{255(10)}{100 - 10} = \$28.3$ million

$C(40) = \dfrac{255(40)}{100 - 40} = \$170$ million

$C(75) = \dfrac{255(75)}{100 - 75} = \$765$ million

(b)



(c) No. The function is undefined at $p = 100\,\%$.

47.

(a)

M	200	400	600	800	1000	1200	1400	1600	1800	2000
t	0.472	0.596	0.710	0.817	0.916	1.009	1.096	1.178	1.255	1.328

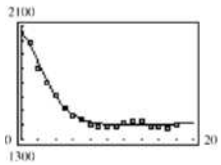
The greater the mass, the more time required per oscillation. The model is a good fit to the actual data.

(b) You can find M corresponding to $t = 1.056$ by finding the point of intersection of

$$t = \frac{38M + 16{,}965}{10(M + 500)} \text{ and } t = 1.056.$$

If you do this, you obtain $M \approx 1306$ grams.

49. (a) The model fits the data well.



(a) $N = \dfrac{77.095t^2 - 216.04t + 2050}{0.052t^2 - 0.8t + 1}$

$2009: N(19) \approx 1{,}412{,}000$

$2010: N(20) \approx 1{,}414{,}000$

$2011: N(11) \approx 1{,}416{,}000$

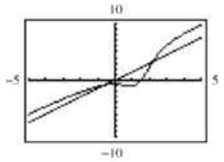
Answers will vary.

(b) Horizontal asymptote: $N = 1482.6$ (approximately) Answers will vary.

51. False. For example, $f(x) = \dfrac{1}{x^2 + 1}$ has no vertical asymptote.

53. No. If $x = c$ is also a zero of the denominator of f, then f is undefined at $x = c$, and the graph of f may have a hole of vertical asymptote at $x = c$.

55.



The graphs of $y_1 = \dfrac{3x^3 - 5x^2 + 4x - 5}{2x^2 - 6x + 7}$ and $y_2 = \dfrac{3x^3}{2x^2}$ are approximately the same graph as $x \to \infty$ and $x \to -\infty$.

Therefore as $x \to \pm\infty$, the graph of a rational function $y = \dfrac{a_n x^n + \ldots a_1 x + a_0}{b_m x^m + \ldots b_1 x + b_0}$ appears to be very close to the graph of $y = \dfrac{a_n x^n}{b_m x^m}$.

57. $y - 2 = \dfrac{-1 - 2}{0 - 3}(x - 3) = 1(x - 3)$

$y = x - 1$

$y - x + 1 = 0$

59. $y - 7 = \dfrac{10 - 7}{3 - 2}(x - 2) = 3(x - 2)$

$y = 3x + 1$

$3x - y + 1 = 0$

61.

$$\begin{array}{r|l} & x + 9 \\ \hline x - 4 & x^2 + 5x + 6 \\ & \underline{x^2 - 4x} \\ & \quad 9x + 6 \\ & \quad \underline{9x - 36} \\ & \qquad 42 \end{array}$$

$$\frac{x^2 + 5x + 6}{x - 4} = x + 9 + \frac{42}{x - 4}$$

63.

$$\begin{array}{r|l} & 2x^2 - 9 \\ \hline x^2 + 5 & 2x^4 + 0x^3 + x^2 + 0x - 11 \\ & \underline{2x^4 + \quad 10x^2} \\ & \quad -9x^2 - \quad 11 \\ & \quad \underline{-9x^2 - \quad 45} \\ & \qquad\qquad 34 \end{array}$$

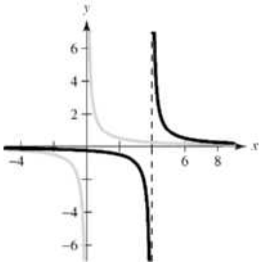
$$\frac{2x^4 + x^2 - 11}{x^2 + 5} = 2x^2 - 9 + \frac{34}{x^2 + 5}$$

Section 2.7

1. slant, asymptote

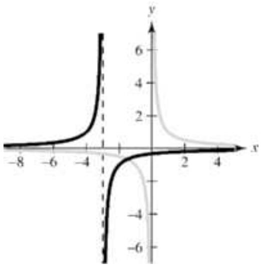
3. Yes. Because the numerator's degree is exactly 1 greater than that of the denominator, the graph of f has a slant asymptote.

5.



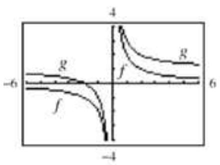
The graph of $g = \dfrac{1}{x-4}$ is a horizontal shift four units to the right of the graph of $f(x) = \dfrac{1}{x}$.

7.



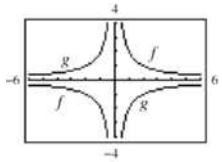
The graph of $g(x) = \dfrac{-1}{x+3}$ is a reflection in the x-axis and a horizontal shift three units to the left of the graph of $f(x) = \dfrac{1}{x}$.

9. $g(x) = \dfrac{2}{x} + 1$



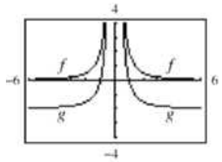
Vertical shift one unit upward

11. $g(x) = -\dfrac{2}{x}$



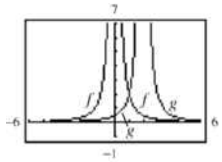
Reflection in the x-axis

13. $g(x) = \dfrac{2}{x^2} - 2$



Vertical shift two units downward

15. $g(x) = \dfrac{2}{(x-2)^2}$

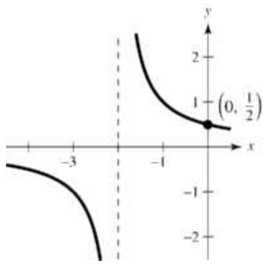


Horizontal shift two units to the right

17. $f(x) = \dfrac{1}{x+2}$

y-intercept: $\left(0, \dfrac{1}{2}\right)$

Vertical asymptote: $x = -2$
Horizontal asymptote: $y = 0$



x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$

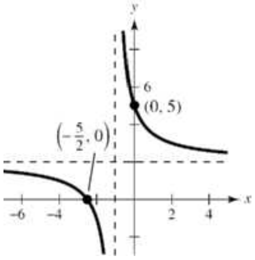
19. $C(x) = \dfrac{5+2x}{1+x} = \dfrac{2x+5}{x+1}$

x-intercept: $\left(-\dfrac{5}{2},\ 0\right)$

y-intercept: $(0,\ 5)$

Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 2$



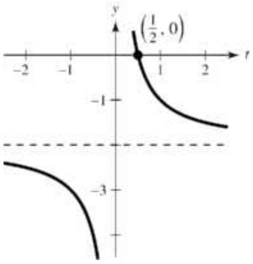
x	-4	-3	-2	0	1	2
$C(x)$	1	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3

21. $f(t) = \dfrac{1-2t}{t} = -\dfrac{2t-1}{t}$

t-intercept: $\left(\dfrac{1}{2},\ 0\right)$

Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$



x	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$

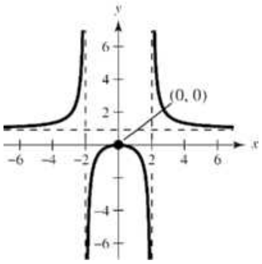
23. $f(x) = \dfrac{x^2}{x^2-4}$

Intercept: $(0,\ 0)$

Vertical asymptotes: $x = 2,\ x = -2$

Horizontal asymptote: $y = 1$

y-axis symmetry



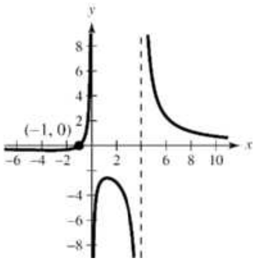
x	-4	-1	0	-1	4
y	$\frac{4}{3}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{4}{3}$

25. $g(x) = \dfrac{4(x+1)}{x(x-4)}$

Intercept: $(-1,\ 0)$

Vertical asymptotes: $x = 0$ and $x = 4$

Horizontal asymptote: $y = 0$



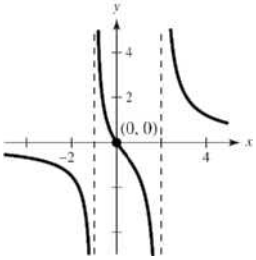
x	-2	-1	1	2	3	5	6
y	$-\frac{1}{3}$	0	$-\frac{8}{3}$	-3	$-\frac{16}{3}$	$\frac{24}{5}$	$\frac{7}{3}$

27. $f(x) = \dfrac{3x}{x^2 - x - 2} = \dfrac{3x}{(x+1)(x-2)}$

Intercept: $(0,\ 0)$

Vertical asymptotes: $x = -1,\ 2$

Horizontal asymptote: $y = 0$



x	-3	0	1	3	4
y	$-\dfrac{9}{10}$	0	$-\dfrac{3}{2}$	$\dfrac{9}{4}$	$\dfrac{6}{5}$

29. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x-2)(x+3)} = \dfrac{x}{x-2},$

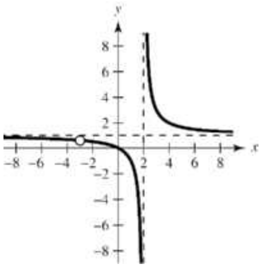
$x \neq -3$

Intercept: $(0,\ 0)$

Vertical asymptote: $x = 2$

(There is a hole at $x = -3$.)

Horizontal asymptote: $y = 1$

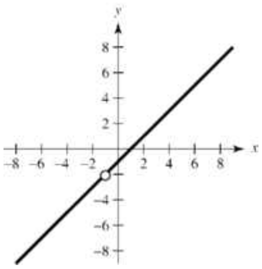


x	-2	-1	0	1	2	3
y	$\dfrac{1}{2}$	$\dfrac{1}{3}$	0	-1	Undef.	3

31. $f(x) = \dfrac{x^2 - 1}{x + 1} = \dfrac{(x+1)(x-1)}{x+1} = x - 1,$

$x \neq -1$

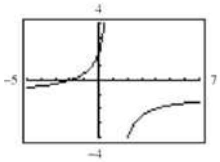
The graph is a line, with a hole at $x = -1$.



33. $f(x) = \dfrac{2 + x}{1 - x} = -\dfrac{x+2}{x-1}$

Vertical asymptote: $x = 1$

Horizontal asymptote: $y = -1$

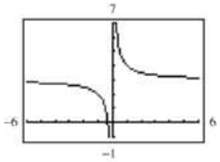


Domain: $x \neq 1$ or $(-\infty,\ 1) \cup (1, \infty)$

35. $f(t) = \dfrac{3t + 1}{t}$

Vertical asymptote: $t = 0$

Horizontal asymptote: $y = 3$

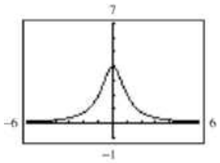


Domain: $t \neq 0$ or $(-\infty,\ 0) \cup (0,\ \infty)$

37. $h(t) = \dfrac{4}{t^2 + 1}$

Domain: all real numbers or $(-\infty,\ \infty)$

Horizontal asymptote: $y = 0$

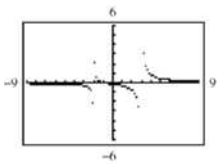


39. $f(x) = \dfrac{x + 1}{x^2 - x - 6} = \dfrac{x+1}{(x-3)(x+2)}$

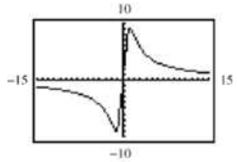
Domain: all real numbers except $x = 3,\ -2$

Vertical asymptotes: $x = 3, x = -2$

Horizontal asymptote: $y = 0$

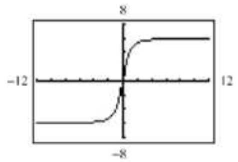


41. $f(x) = \dfrac{20x}{x^2+1} - \dfrac{1}{x} = \dfrac{19x^2-1}{x(x^2+1)}$



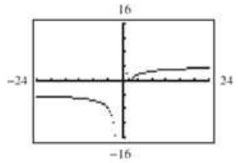
Domain: all real numbers except 0, or $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 0$
Horizontal asymptote: $y = 0$

43. $h(x) = \dfrac{6x}{\sqrt{x^2+1}}$



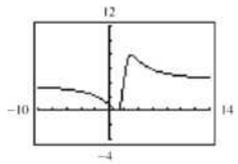
There are two horizontal asymptotes, $y = \pm 6$.

45. $g(x) = \dfrac{4|x-2|}{x+1}$



There are two horizontal asymptotes, $y = \pm 4$ and one vertical asymptote, $x = -1$.

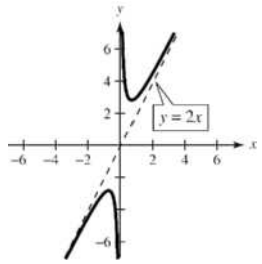
47. $f(x) = \dfrac{4(x-1)^2}{x^2-4x+5}$



The graph crosses its horizontal asymptote, $y = 4$.

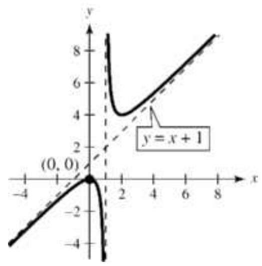
49. $f(x) = \dfrac{2x^2+1}{x} = 2x + \dfrac{1}{x}$

Vertical asymptote: $x = 0$
Slant asymptote: $y = 2x$
Origin symmetry



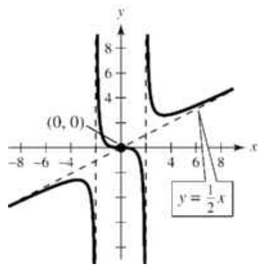
51. $h(x) = \dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$

Intercept: $(0, 0)$
Vertical asymptote: $x = 1$
Slant asymptote: $y = x + 1$



53. $g(x) = \dfrac{x^3}{2x^2-8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2-8}$

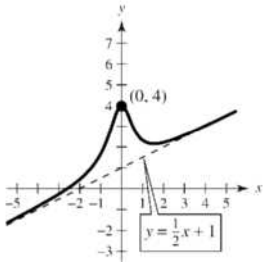
Intercept: $(0, 0)$
Vertical asymptotes: $x = \pm 2$
Slant asymptote: $y = \dfrac{1}{2}x$
Origin symmetry



55. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1} = \dfrac{x}{2} + 1 + \dfrac{3 - \frac{x}{2}}{2x^2 + 1}$

Intercepts: $(-2.594, 0)$, $(0, 4)$

Slant asymptote: $y = \dfrac{x}{2} + 1$



57. $y = \dfrac{x+1}{x-3}$

x-intercept: $(-1, 0)$

$0 = \dfrac{x+1}{x-3}$

$0 = x + 1$

$-1 = x$

59. $y = \dfrac{1}{x} - x$

x-intercepts: $(\pm 1, 0)$

$0 = \dfrac{1}{x} - x$

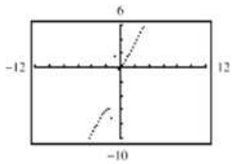
$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$

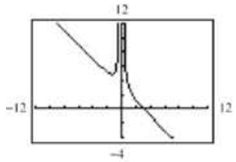
61. $y = \dfrac{2x^2 + x}{x+1} = 2x - 1 + \dfrac{1}{x+1}$

Domain: all real numbers except $x = -1$
Vertical asymptote: $x = -1$
Slant asymptote: $y = 2x - 1$



63. $y = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers except 0
or $(-\infty, 0) \cup (0, \infty)$
Vertical asymptote: $x = 0$
Slant asymptote: $y = -x + 3$



65. $f(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x-4)(x-1)}{(x-2)(x+2)}$

Vertical asymptotes: $x = 2$, $x = -2$
Horizontal asymptote: $y = 1$
No slant asymptotes, no holes

67. $f(x) = \dfrac{2x^2 - 5x + 2}{2x^2 - x - 6} = \dfrac{(2x-1)(x-2)}{(2x+3)(x-2)} = \dfrac{2x-1}{2x+3}$,

$x \neq 2$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = 1$
No slant asymptotes
Hole at $x = 2$, $\left(2, \dfrac{3}{7}\right)$

69. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$= \dfrac{(x-1)(x+1)(2x-1)}{(x+1)(x+2)}$

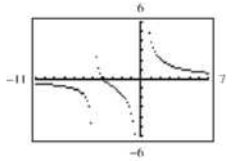
$= \dfrac{(x-1)(2x-1)}{x+2}$, $x \neq -1$

Long division gives

$f(x) = \dfrac{2x^2 - 3x + 1}{x+2} = 2x - 7 + \dfrac{15}{x+2}$.

Vertical asymptote: $x = -2$
No horizontal asymptote
Slant asymptote: $y = 2x - 7$
Hole at $x = -1$, $(-1, 6)$

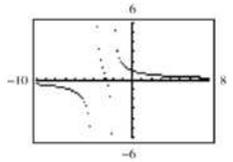
71. $y = \dfrac{1}{x+5} + \dfrac{4}{x}$



x-intercept: $(-4, 0)$

$$\begin{aligned} 0 &= \frac{1}{x+5} + \frac{4}{x} \\ -\frac{4}{x} &= \frac{1}{x+5} \\ -4(x+5) &= x \\ -4x - 20 &= x \\ -5x &= 20 \\ x &= -4 \end{aligned}$$

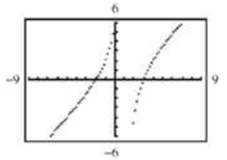
73. $y = \dfrac{1}{x+2} + \dfrac{2}{x+4}$



x-intercept: $\left(-\dfrac{8}{3}, 0\right)$

$$\begin{aligned} \frac{1}{x+2} + \frac{2}{x+4} &= 0 \\ \frac{1}{x+2} &= \frac{-2}{x+4} \\ x + 4 &= -2x - 4 \\ 3x &= -8 \\ x &= -\frac{8}{3} \end{aligned}$$

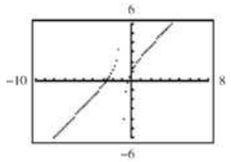
75. $y = x - \dfrac{6}{x-1}$



x-intercepts: $(-2, 0)$, $(3, 0)$

$$\begin{aligned} 0 &= x - \frac{6}{x-1} \\ \frac{6}{x-1} &= x \\ 6 &= x(x-1) \\ 0 &= x^2 - x - 6 \\ 0 &= (x+2)(x-3) \\ x &= -2,\ x = 3 \end{aligned}$$

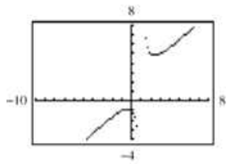
77. $y = x + 2 - \dfrac{1}{x+1}$



x-intercepts: $(-2.618, 0)$, $(-0.382, 0)$

$$\begin{aligned} x + 2 &= \frac{1}{x+2} \\ x^2 + 3x + 2 &= 1 \\ x^2 + 3x + 1 &= 0 \\ x &= \frac{-3 \pm \sqrt{9-4}}{2} \\ &= -\frac{3}{2} \pm \frac{\sqrt{5}}{2} \\ &\approx -2.618, -0.382 \end{aligned}$$

79. $y = x + 1 + \dfrac{2}{x-1}$

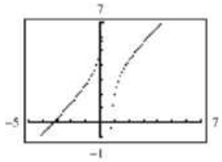


No x-intercepts

$$\begin{aligned} x + 1 + \frac{2}{x-1} &= 0 \\ \frac{2}{x-1} &= -x - 1 \\ 2 &= -x^2 + 1 \\ x^2 + 1 &= 0 \end{aligned}$$

No real zeros

81. $y = x + 3 - \dfrac{2}{2x-1}$



x-intercepts: $(0.766, 0)$, $(-3.266, 0)$

$$x + 3 - \frac{2}{2x-1} = 0$$
$$x + 3 = \frac{2}{2x-1}$$
$$2x^2 + 5x - 3 = 2$$
$$2x^2 + 5x - 5 = 0$$
$$x = \frac{-5 \pm \sqrt{25 - 4(2)(-5)}}{4}$$
$$= \frac{-5 \pm \sqrt{65}}{4}$$
$$\approx 0.766, -3.266$$

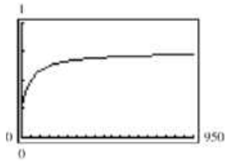
83.

(a) $0.25(50) + 0.75(x) = C(50 + x)$

$$\frac{12.5 + 0.75x}{50 + x} = C$$
$$\frac{50 + 3x}{200 + 4x} = C$$
$$C = \frac{3x + 50}{4(x + 50)}$$

(b) Domain: $x \ge 0$ and $x \le 1000 - 50 = 950$
Thus, $0 \le x \le 950$.

(c)



As the tank fills, the rate that the concentration is increasing slows down. It approaches the horizontal asymptote $C = \dfrac{3}{4} = 0.75$. When the tank is full $(x = 950)$, the concentration is $C = 0.725$.

85.

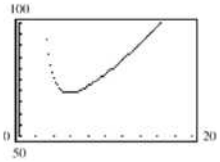
(a) $A = xy$ and

$$(x-2)(y-4) = 30$$
$$y - 4 = \frac{30}{x-2}$$
$$y = 4 + \frac{30}{x-2} = \frac{4x+22}{x-2}$$

Thus, $A = xy = x\left(\dfrac{4x+22}{x-2}\right) = \dfrac{2x(2x+11)}{x-2}$.

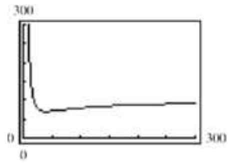
(b) Domain: Since the margins on the left and right are each 1 inch, $x > 2$, or $(2, \infty)$.

(c)



The area is minimum when $x \approx 5.87$ in. and $y \approx 11.75$ in.

87. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right)$, $1 \le x$



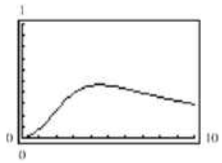
The minimum occurs when $x = 40.4 \approx 40$.

89.

$C = \dfrac{3t^2 + t}{t^3 + 50}$, $0 \le t$

(a) The horizontal asymptote is the t-axis, or $C = 0$. This indicates that the chemical eventually dissipates.

(b)

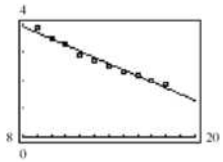


The maximum occurs when $t \approx 4.5$.

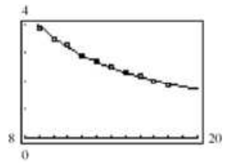
(c) Graph C together with $y = 0.345$. The graphs intersect at $t \approx 2.65$ and $t \approx 8.32$. $C < 0.345$ when $0 \le t \le 2.65$ hours and when $t > 8.32$ hours.

91.

(a) $A = -0.2182t + 5.665$



(b) $A = \dfrac{1}{0.0302t - 0.020}$



(c)

Year	1999	2000	2001	2002
Original data, A	3.9	3.5	3.3	2.9
Model from (a), A	3.7	3.5	3.3	3.0
Model from (b), A	4.0	3.5	3.2	2.9

Year	2003	2004	2005	2006
Original data, A	2.7	2.5	2.3	2.2
Model from (a), A	2.8	2.6	2.4	2.2
Model from (b), A	2.7	2.5	2.3	2.2

Year	2007	2008
Original data, A	2.0	1.9
Model from (a), A	2.0	1.7
Model from (b), A	2.0	1.9

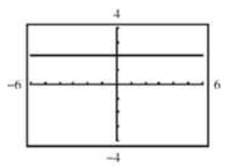
Answers will vary.

93. False. The graph of a rational function is continuous when the polynomial in the denominator has no real zeros.

95. $h(x) = \dfrac{6-2x}{3-x} = \dfrac{2(3-x)}{3-x} = 2,\ x \neq 3$

Since *h(x)* is not reduced and $(3-x)$ is a factor of both the numerator and the denominator, $x = 3$ is not a horizontal asymptote.

There is a hole in the graph at $x = 3$.



97. *Horizontal asymptotes*:

If the degree of the numerator is greater than the degree of the denominator, then there is no horizontal asymptote. If the degree of the numerator is less than the degree of the denominator, then there is a horizontal asymptote at $y = 0$.

If the degree of the numerator is equal to the degree of the denominator, then there is a horizontal asymptote at the line given by the ratio of the leading coefficients.

Vertical asymptotes:

Set the denominator equal to zero and solve.

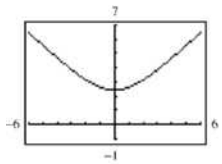
Slant asymptotes:

If there is no horizontal asymptote and the degree of the numerator is exactly one greater than the degree of the denominator, then divide the numerator by the denominator. The slant asymptote is the result, not including the remainder.

99. $\left(\dfrac{x}{8}\right)^{-3} = \left(\dfrac{8}{x}\right)^{3} = \dfrac{512}{x^3}$

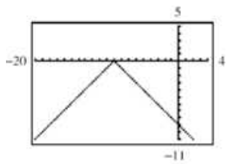
101. $\dfrac{3^{7/6}}{3^{1/6}} = 3^{6/6} = 3$

103.



Domain: all x
Range: $y \geq \sqrt{6}$

105.



Domain: all x
Range: $y \leq 0$

107. Answers will vary.

Section 2.8

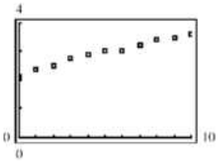
1. quadratic

3. Quadratic

5. Linear

7. Neither

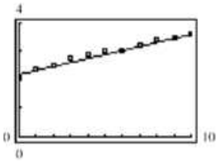
9. (a)



(b) Linear model is better.

(c) $y = 0.14x + 2.2$, linear

(d)

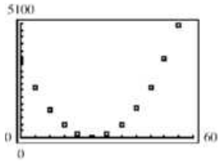


(e)

x	0	1	2	3	4
y	2.1	2.4	2.5	2.8	2.9
Model	2.2	2.4	2.5	2.6	2.8

x	5	6	7	8	9	10
y	3.0	3.0	3.2	3.4	3.5	3.6
Model	2.9	3.0	3.2	3.4	3.5	3.6

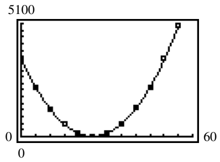
11. (a)



(b) Quadratic model is better.

(c) $y = 5.55x^2 - 277.5x + 3478$

(d)

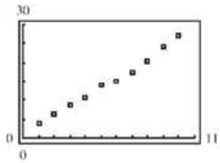


(e)

x	0	5	10	15	20	25
y	3480	2235	1250	565	150	12
Model	3478	2229	1258	564	148	9

x	30	35	40	45	50	55
y	145	575	1275	2225	3500	5010
Model	148	564	1258	2229	3478	5004

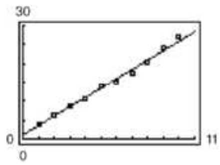
13. (a)



(b) Linear model is better.

(c) $y = 2.48x + 1.1$

(d)

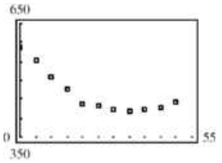


(e)

x	1	2	3	4	5
Actual, y	4.0	6.5	8.8	10.6	13.9
Model, y	3.6	6.1	8.5	11.0	13.5

x	6	7	8	9	10
Actual, y	15.0	17.5	20.1	24.0	27.1
Model, y	16.0	18.5	20.9	23.4	25.9

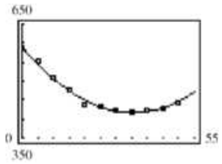
15. (a)



(b) Quadratic is better.

(c) $y = 0.14x^2 - 9.9x + 591$

(d)

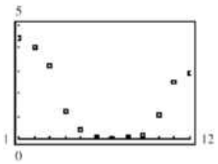


(e)

x	0	5	10	15	20	25
Actual, y	587	551	512	478	436	430
Model, y	591	545	506	474	449	431

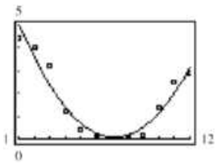
x	30	35	40	45	50
Actual, y	424	420	423	429	444
Model, y	420	416	419	429	446

17. (a)



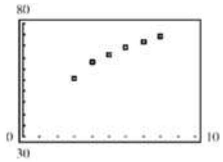
(b) $P = 0.1323t^2 - 1.893t + 6.85$

(c)



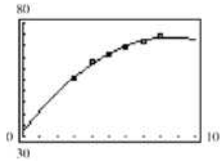
(d) The model's minimum is $H \approx 0.1$ at $t = 7.4$. This corresponds to July.

19. (a)



(b) $P = -0.5638t^2 + 9.690t + 32.17$

(c)



(d) To determine when the percent P of the U.S. population who used the Internet falls below 60%, set P = 60. Using the Quadratic Formula, solve for t.

$$-0.5638t^2 + 9.690t + 32.17 = 60$$

$$-0.5638t^2 + 9.690t - 27.83 = 0$$

$$t = \frac{-(9.690) \pm \sqrt{(9.690)^2 - 4(-0.5638)(-27.83)}}{2(-0.5638)}$$

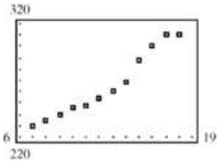
$$t = \frac{-9.690 \pm \sqrt{156.658316}}{-1.1276}$$

$$t \approx 13.54 \text{ or } 2014$$

Therefore after 2014, the percent of the U.S. population who use the Internet will fall below 60%.

This is not a good model for predicting future years. In fact, by 2021, the model gives negative values for the percentage.

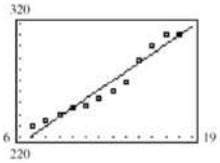
21. (a)



(b) $T = 7.97t + 166.1$

$r^2 \approx 0.9469$

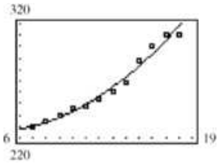
(c)



(d) $T = 0.459t^2 - 3.51t + 232.4$

$r^2 \approx 0.9763$

(e)



(f) The quadratic model is a better fit. Answers will vary.

(g) To determine when the number of televisions, T, in homes reaches 350 million, set T for the model equal to 350 and solve for t.

Linear model:

$$7.97t + 166.1 = 350$$
$$7.97t = 183.9$$
$$t \approx 23.1 \text{ or } 2014$$

Quadratic model:

$$0.459t^2 - 3.51t + 232.4 = 350$$
$$0.459t^2 - 3.51t - 117.6 = 0$$

Using the Quadratic Formula,

$$t = \frac{-(-3.51) \pm \sqrt{(-3.51)^2 - 4(0.459)(-117.6)}}{2(0.459)}$$

$$t = \frac{3.51 \pm \sqrt{228.2337}}{0.918}$$

$$t \approx 20.3 \text{ or } 2011$$

23. True. A quadratic model with a positive leading coefficient opens upward. So, its vertex is at its lowest point.

25. The model is above all data points.

27. (a) $(f \circ g)(x) = f(x^2 + 3) = 2(x^2 + 3) - 1 = 2x^2 + 5$

(b) $(g \circ f)(x) = g(2x - 1) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4$

29. (a) $(f \circ g)(x) = f(\sqrt[3]{x + 1}) = x + 1 - 1 = x$

(b) $(g \circ f)(x) = g(x^3 - 1) = \sqrt[3]{x^3 - 1 + 1} = x$

31. f is one-to-one.

$$y = 2x + 5$$
$$x = 2y + 5$$
$$2y = x - 5$$
$$y = \frac{(x - 5)}{2} \Rightarrow f^{-1}(x) = \frac{x - 5}{2}$$

33. f is one-to-one on $[0, \infty)$.

$$y = x^2 + 5,\ x \ge 0$$
$$x = y^2 + 5,\ y \ge 0$$
$$y^2 = x - 5$$
$$y = \sqrt{x - 5} \Rightarrow f^{-1}(x) = \sqrt{x - 5},\ x \ge 5$$

35. For $1 - 3i$, the complex conjugate is $1 + 3i$.

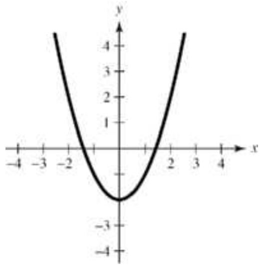
$$(1 - 3i)(1 + 3i) = 1 - 9i^2$$
$$= 1 + 9$$
$$= 10$$

37. For $-5i$, the complex conjugate is $5i$.

$$(-5i)(5i) = -25i^2$$
$$= 25$$

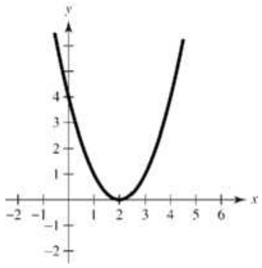
Chapter 2 Review Exercises

1.



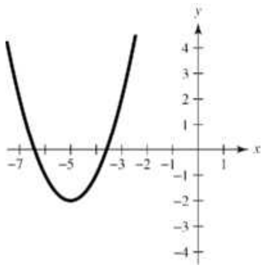
The graph of $y = x^2 - 2$ is a vertical shift two units downward of $y = x^2$.

3.



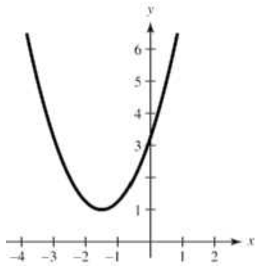
The graph of $y = (x-2)^2$ is a horizontal shift two units to the right of $y = x^2$.

5.



The graph of $y = (x+5)^2 - 2$ is a horizontal shift five units to the left and a vertical shift two units downward of $y = x^2$.

7. The graph of $f(x) = \left(x + 3/2\right)^2 + 1$ is a parabola opening upward with vertex $\left(-\frac{3}{2},\ 1\right)$, and no x-intercepts.



9.
$$\begin{aligned} f(x) &= \frac{1}{3}(x^2 + 5x - 4) \\ &= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) - \frac{4}{3} \\ &= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12} \end{aligned}$$

The graph of f is a parabola opening upward with vertex $\left(-\frac{5}{2},\ -\frac{41}{12}\right)$.

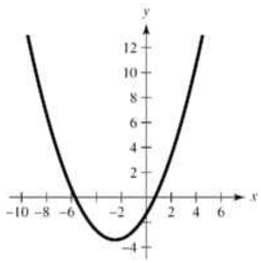
x-intercepts: $\frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12} = 0$.

or

$$\frac{1}{3}\left(x^2 + 5x - 4\right) = 0$$
$$x^2 + 5x - 4 = 0$$

Use Quadratic formula.

$$x = \frac{-5 \pm \sqrt{41}}{2}$$
$$\left(\frac{-5 + \sqrt{41}}{2},\ 0\right), \left(\frac{-5 - \sqrt{41}}{2},\ 0\right)$$



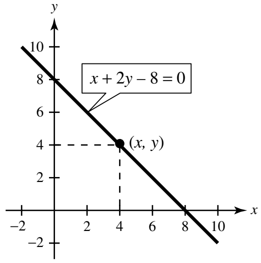
11. Vertex : $(1,-4) \Rightarrow f(x) = a(x-1)^2 - 4$

Point: $(2,-3) \Rightarrow -3 = a(2-1)^2 - 4$

$$1 = a$$

Thus, $f(x) = (x-1)^2 - 4$.

13.



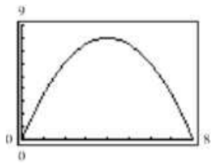
(a) $A = xy$

If $x + 2y - 8 = 0$, then $y = \dfrac{8-x}{2}$.

$$A = x\left(\frac{8-x}{2}\right)$$

$$A = 4x - \frac{1}{2}x^2,\ 0 < x < 8$$

(b)



Using the graph, when $x = 4$, the area is a maximum.

When $x = 4$, $y = \dfrac{8-4}{2} = 2$.

(c) $A = -\dfrac{1}{2}x^2 + 4x$

$$= -\frac{1}{2}(x^2 - 8x)$$

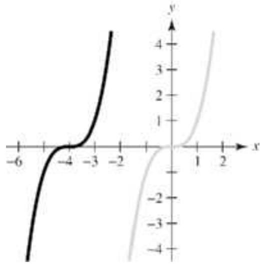
$$= -\frac{1}{2}(x^2 - 8x + 16 - 16)$$

$$= -\frac{1}{2}(x-4)^2 + 8$$

The graph of A is a parabola opening downward, with vertex $(4,\ 8)$. Therefore, when $x = 4$, the maximum area is 8 square units.

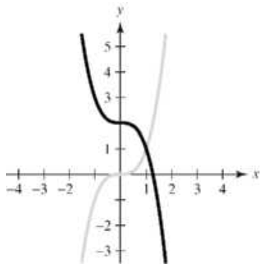
Yes, graphically and algebraically the same dimensions result.

15.



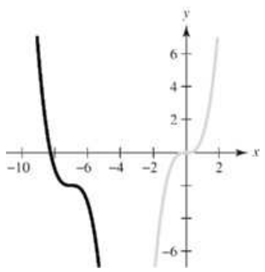
The graph of $f(x) = (x+4)^3$ is a horizontal shift four units to the left of $y = x^3$.

17.



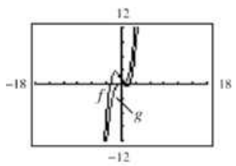
The graph if $f(x) = -x^3 + 2$ is a reflection in the x-axis and a vertical shift two units upward of $y = x^3$.

19.



The graph of $f(x) = -\left(x+7\right)^3 - 2$ is a horizontal shift seven units to the left, a reflection in the x-axis, and a vertical shift two units downward of $y = x^3$.

21. $f(x) = \dfrac{1}{2}x^3 - 2x + 1$; $g(x) = \dfrac{1}{2}x^3$



The graphs have the same end behavior. Both functions are of the same degree and have positive leading coefficients.

23. $f(x) = -x^2 + 6x + 9$

The degree is even and the leading coefficient is negative. The graph falls to the left and right.

25. $f(x) = \frac{3}{4}\left(x^4 + 3x^2 + 2\right)$

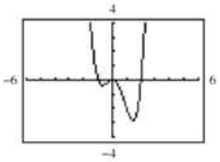
The degree is even and the leading coefficient is positive. The graph rises to the left and right.

27. (a) $x^4 - x^3 - 2x^2 = x^2\left(x^2 - x - 2\right)$

$$= x^2\left(x-2\right)\left(x+1\right) = 0$$

Zeros: $x = -1,\ 0,\ 0,\ 2$

(b)

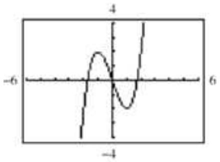


(c) Zeros: $x = -1,\ 0,\ 0,\ 2$; the same

29. (a) $t^3 - 3t = t\left(t^2 - 3\right) = t\left(t + \sqrt{3}\right)\left(t - \sqrt{3}\right) = 0$

Zeros: $t = 0, \pm\sqrt{3}$

(b)

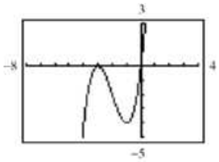


(c) Zeros: $t = 0, \pm 1.732$; the same

31. (a) $x\left(x+3\right)^2 = 0$

Zeros: $x = 0, -3, -3$

(b)



(c) Zeros: $x = -3,\ -3,\ 0$; the same

33. $f(x) = \left(x+2\right)\left(x-1\right)^2\left(x-5\right)$

$$= x^4 - 5x^3 - 3x^2 + 17x - 10$$

35. $f(x) = \left(x-3\right)\left(x-2+\sqrt{3}\right)\left(x-2-\sqrt{3}\right)$

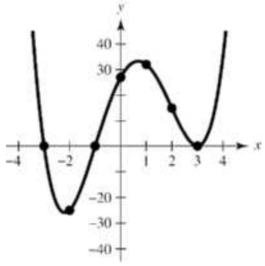
$$= x^3 - 7x^2 + 13x - 3$$

37. (a) The degree of *f* is even and the leading coefficient is 1. The graph rises to the left and rises to the right.

(b)
$$\begin{aligned} f(x) &= x^4 - 2x^3 - 12x^2 + 18x + 27 \\ &= (x^4 - 12x^2 + 27) - (2x^3 - 18x) \\ &= (x^2 - 9)(x^2 - 3) - 2x(x^2 - 9) \\ &= (x^2 - 9)(x^2 - 3 - 2x) \\ &= (x+3)(x-3)(x^2 - 2x - 3) \\ &= (x+3)(x-3)(x-3)(x+1) \end{aligned}$$

Zeros: $-3,\ 3,\ 3,\ -1$

(c) and (d)

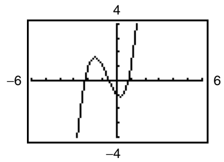


39. $f(x) = x^3 + 2x^2 - x - 1$

(a) $f(-3) < 0, f(-2) > 0 \Rightarrow$ zero in $(-3,\ -2)$

$f(-1) > 0, f(0) < 0 \Rightarrow$ zero in $(-1,\ 0)$

$f(0) < 0, f(1) > 0 \Rightarrow$ zero in $(0,\ 1)$

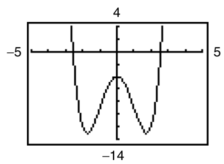


(b) Zeros: $-2.247,\ -0.555,\ 0.802$

41. $f(x) = x^4 - 6x^2 - 4$

(a) $f(-3) > 0, f(-2) < 0 \Rightarrow$ zero in $(-3,\ -2)$

$f(2) < 0, f(3) > 0 \Rightarrow$ zero in $(2,\ 3)$



(b) Zeros: ± 2.570

43.

$$\begin{array}{r} 8x + 5 \\ 3x - 2\,\overline{)\,24x^2 - x - 8} \\ \underline{24x^2 - 16x} \\ 15x - 8 \\ \underline{15x - 10} \\ 2 \end{array}$$

Thus, $\dfrac{24x^2 - x - 8}{3x - 2} = 8x + 5 + \dfrac{2}{3x - 2}$.

45.
$$\begin{array}{r} x^2-2 \\ x^2-1\overline{)x^4-3x^2+2} \\ \underline{x^4-x^2} \\ -2x^2+2 \\ \underline{-2x^2+2} \\ 0 \end{array}$$

Thus, $\dfrac{x^4-3x^2+2}{x^2-1}=x^2-2,\ (x\pm 1).$

47.
$$\begin{array}{r} 5x+2 \\ x^2-3x+1\overline{)5x^3-13x^2-\ x+2} \\ \underline{5x^3-15x^2+5x} \\ 2x^2-6x+2 \\ \underline{2x^2-6x+2} \\ 0 \end{array}$$

Thus, $\dfrac{5x^3-13x^2-x+2}{x^2-3x+1}=5x+2,$

$\left(x\neq\dfrac{1}{2}(3\pm\sqrt{5})\right).$

49.
$$\begin{array}{r} 3x^2+5x+8 \\ 2x^2+0x-1\overline{)6x^4+10x^3+13x^2-5x+2} \\ \underline{6x^4+\ 0x^3\ -3x^2} \\ 10x^3+16x^2-5x \\ \underline{10x^3+\ 0x^2-5x} \\ 16x^2-0+2 \\ \underline{16x^2+0-8} \\ 10 \end{array}$$

Thus,

$\dfrac{6x^4+10x^3+13x^2-5x+2}{2x^2-1}=3x^2+5x+8+\dfrac{10}{2x^2-1}.$

51.
$$\begin{array}{r|rrrrr} -2 & 0.25 & -4 & 0 & 0 & 0 \\ & & -\frac{1}{2} & 9 & -18 & 36 \\ \hline & \frac{1}{4} & -\frac{9}{2} & 9 & -18 & 36 \end{array}$$

Thus, $\dfrac{0.25x^4-4x^3}{x+2}=\dfrac{1}{4}x^3-\dfrac{9}{2}x^2+9x-18+\dfrac{36}{x+2}.$

53.
$$\begin{array}{r|rrrrr} \frac{2}{3} & 6 & -4 & -27 & 18 & 0 \\ & & 4 & 0 & -18 & 0 \\ \hline & 6 & 0 & -27 & 0 & 0 \end{array}$$

Thus, $\dfrac{6x^4-4x^3-27x^2+18x}{x-(2/3)}=6x^3-27x,\ x\neq\dfrac{2}{3}.$

55.
$$\begin{array}{r|rrrr} 4 & 3 & -10 & 12 & -22 \\ & & 12 & 8 & 80 \\ \hline & 3 & 2 & 20 & 58 \end{array}$$

Thus, $\dfrac{3x^3-10x^2+12x-22}{x-4}=3x^2+2x+20+\dfrac{58}{x-4}.$

57. (a)
$$\begin{array}{r|rrrrl} -3 & 1 & 10 & -24 & 20 & 44 \\ & & -3 & -21 & 135 & -465 \\ \hline & 1 & 7 & -45 & 155 & -421=f(-3) \end{array}$$

(b)
$$\begin{array}{r|rrrrl} -2 & 1 & 10 & -24 & 20 & 44 \\ & & -2 & -16 & 80 & -200 \\ \hline & 1 & 8 & -40 & 100 & -156=f(-2) \end{array}$$

59. $f(x)=x^3+4x^2-25x-28$

(a)
$$\begin{array}{r|rrrr} 4 & 1 & 4 & -25 & -28 \\ & & 4 & 32 & 28 \\ \hline & 1 & 8 & 7 & 0 \end{array}$$

$(x-4)$ is a factor.

(b) $x^2+8x+7=(x+1)(x+7)$

Remaining factors: $(x+1),\ (x+7)$

(c) $f(x)=(x-4)(x+1)(x+7)$

(d) Zeros: $4,\ -1,\ -7$

61. $f(x)=x^4-4x^3-7x^2+22x+24$

(a)
$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$(x+2)$ is a factor.

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

$(x-3)$ is a factor.

(b) $x^2-3x-4=(x-4)(x+1)$

Remaining factors: $(x-4),\ (x+1)$

(c) $f(x)=(x+2)(x-3)(x-4)(x+1)$

(d) Zeros: $-2,\ 3,\ 4,\ -1$

63. $f(x)=4x^3-11x^2+10x-3$

Possible rational zeros: $\pm1,\ \pm3,\ \pm\frac{3}{2},\ \pm\frac{3}{4},\ \pm\frac{1}{2},\ \pm\frac{1}{4}$

Zeros: $1,\ 1,\ \dfrac{3}{4}$

65. $g(x)=5x^3-6x+9$ has two variation in sign $\Rightarrow 0$ or 2 positive real zeros.

$g(-x)=-5x^3+6x+9$ has one variation in sign $\Rightarrow 1$ negative real zero.

67. $\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$

All entries positive; $x = 1$ is upper bound.

$\begin{array}{r|rrrr} -\frac{1}{4} & 4 & -3 & 4 & -3 \\ & & -1 & 1 & -\frac{5}{4} \\ \hline & 4 & -4 & 5 & -\frac{17}{4} \end{array}$

Alternating signs; $x = -\frac{1}{4}$ is lower bound.

Real zero: $x = \dfrac{3}{4}$

69. $f(x) = 6x^3 + 31x^2 - 18x - 10$

Possible rational zeros:

$\pm 1,\ \pm 2,\ \pm 5,\ \pm 10,\ \pm \frac{1}{2},\ \pm \frac{5}{2},\ \pm \frac{1}{3},\ \pm \frac{2}{3},\ \pm \frac{5}{3},\ \pm \frac{10}{3},\ \pm \frac{1}{6},\ \pm \frac{5}{6}$

Using synthetic division and a graph check $x = \frac{5}{6}$.

$\begin{array}{r|rrrr} \frac{5}{6} & 6 & 31 & -18 & -10 \\ & & 5 & 30 & 10 \\ \hline & 6 & 36 & 12 & 0 \end{array}$

$x = \frac{5}{6}$ is a real zero.

Rewrite in polynomial form and use the Quadratic Formula:

$$6x^2 + 36x + 12 = 0$$
$$6(x^2 + 6x + 2) = 0$$
$$x^2 + 6x + 2 = 0$$
$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{-6 \pm \sqrt{28}}{2} = \frac{-6 \pm 2\sqrt{7}}{2} = -3 \pm \sqrt{7}$$

Real zeros: $x = \frac{5}{6},\ -3 \pm \sqrt{7}$

71. $f(x) = 6x^4 - 25x^3 + 14x^2 + 27x - 18$

Possible rational zeros:

$\pm 1,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 9,\ \pm 18,\ \pm \frac{1}{2},\ \pm \frac{3}{2},\ \pm \frac{9}{2},\ \pm \frac{1}{3},\ \pm \frac{2}{3},\ \pm \frac{1}{6}$

Use a graphing utility to see that $x = -1$ and $x = 3$ are probably zeros.

$\begin{array}{r|rrrrr} -1 & 6 & -25 & 14 & 27 & -18 \\ & & -6 & 31 & -45 & 18 \\ \hline & 6 & -31 & 45 & -18 & 0 \end{array}$

$\begin{array}{r|rrrr} 3 & 6 & -31 & 45 & -18 \\ & & 18 & -39 & 18 \\ \hline & 6 & -13 & 6 & 0 \end{array}$

$$\begin{aligned} 6x^4 - 25x^3 + 14x^2 + 27x - 18 &= (x+1)(x-3)(6x^2 - 13x + 6) \\ &= (x+1)(x-3)(3x-2)(2x-3) \end{aligned}$$

Zeros: $x = -1,\ 3,\ \dfrac{2}{3},\ \dfrac{3}{2}$

73. $6 + \sqrt{-25} = 6 + 5i$

75. $-2i^2 + 7i = 2 + 7i$

77. $$\begin{aligned} (7+5i) + (-4+2i) &= (7-4) + (5i+2i) \\ &= 3 + 7i \end{aligned}$$

79. $5i(13 - 8i) = 65i - 40i^2 = 40 + 65i$

81. $$\begin{aligned} (10-8i)(2-3i) &= 20 - 30i - 16i + 24i^2 \\ &= -4 - 46i \end{aligned}$$

83. $$\begin{aligned} (3+7i)^2 + (3-7i)^2 &= (9 + 42i - 49) + (9 - 42i - 49) \\ &= -80 \end{aligned}$$

85. $$\begin{aligned} \left(\sqrt{-16} + 3\right)\left(\sqrt{-25} - 2\right) &= (4i + 3)(5i - 2) \\ &= -20 - 8i + 15i - 6 \\ &= -26 + 7i \end{aligned}$$

87. $$\begin{aligned} \sqrt{-9} + 3 + \sqrt{-36} &= 3i + 3 + 6i \\ &= 3 + 9i \end{aligned}$$

89. $$\begin{aligned} \frac{6+i}{i} = \frac{6+i}{i} \cdot \frac{-i}{-i} &= \frac{-6i - i^2}{-i^2} \\ &= \frac{-6i + 1}{1} = 1 - 6i \end{aligned}$$

91. $$\begin{aligned} \frac{3+2i}{5+i} \cdot \frac{5-i}{5-i} &= \frac{15 + 10i - 3i + 2}{25 + 1} \\ &= \frac{17}{26} + \frac{7}{26}i \end{aligned}$$

93. $$\begin{aligned} x^2 + 16 &= 0 \\ x^2 &= -16 \\ x &= \pm\sqrt{-16} \\ x &= \pm 4i \end{aligned}$$

95. $x^2 + 3x + 6 = 0$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(6)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{-15}}{2}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}i$$

97. $3x^2 - 5x + 6 = 0$
$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(6)}}{2(3)}$$
$$x = \frac{5 \pm \sqrt{-47}}{6}$$
$$x = \frac{5}{6} \pm \frac{\sqrt{47}}{6}i$$

99. $x^2+6x+9=0$

$(x+3)^2=0$

$x+3=0$

$x=-3$

101. $x^3+16x=0$

$x(x^2+16)=0$

$x=0 \quad x^2+16=0$

$x^2=-16$

$x=\pm 4i$

103. $f(x)=3x(x-2)^2$

Zeros: 0, 2, 2

105. $h(x)=x^3-7x^2+18x-24$

$$\begin{array}{r|rrrr} 4 & 1 & -7 & 18 & -24 \\ & & 4 & -12 & 24 \\ \hline & 1 & -3 & 6 & 0 \end{array}$$

$x=4$ is a zero. Applying the Quadratic Formula on x^2-3x+6,

$$x=\frac{3\pm\sqrt{9-4(6)}}{2}=\frac{3}{2}\pm\frac{\sqrt{15}}{2}i.$$

Zeros: $4, \frac{3}{2}+\frac{\sqrt{15}}{2}i, \ \frac{3}{2}-\frac{\sqrt{15}}{2}i$

$$h(x)=(x-4)\left(x-\frac{3+\sqrt{15}i}{2}\right)\left(x-\frac{3-\sqrt{15}i}{2}\right)$$

107. $f(x)=2x^4-5x^3+10x-12$

$$\begin{array}{r|rrrrr} 2 & 2 & -5 & 0 & 10 & -12 \\ & & 4 & -2 & -4 & 12 \\ \hline & 2 & -1 & -2 & 6 & 0 \end{array}$$

$x=2$ is a zero.

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & -2 & 6 \\ & & -3 & 6 & -6 \\ \hline & 2 & -4 & 4 & 0 \end{array}$$

$x=-\frac{3}{2}$ is a zero.

$$f(x)=(x-2)\left(x+\frac{3}{2}\right)(2x^2-4x+4)$$

$$=(x-2)(2x+3)(x^2-2x+2)$$

By the Quadratic Formula, applied to x^2-2x+2,

$$x=\frac{2\pm\sqrt{4-4(2)}}{2}=1\pm i.$$

Zeros: $2, -\frac{3}{2}, \ 1\pm i$

$$f(x)=(x-2)(2x+3)(x-1+i)(x-1-i)$$

109. $f(x)=x^5+x^4+5x^3+5x^2$

$=x^2(x^3+x^2+5x+5)$

$=x^2\left[x^2(x+1)+5(x+1)\right]$

$=x^2(x+1)(x^2+5)$

$=x^2(x+1)(x+\sqrt{5}i)(x-\sqrt{5}i)$

Zeros: $0, 0, -1, \pm\sqrt{5}i$

111. $f(x)=x^3-4x^2+6x-4$

(a) $x^3-4x^2+6x-4=(x-2)(x^2-2x+2)$

By the Quadratic Formula for x^2-2x+2,

$$x=\frac{2\pm\sqrt{(-2)^2-4(2)}}{2}=1\pm i$$

Zeros: $2, 1+i, 1-i$

(b) $f(x)=(x-2)(x-1-i)(x-1+i)$

(c) x-intercept: $(2,0)$

113. (a) $f(x)=-3x^3-19x^2-4x+12$

$=-(x+1)(3x^2+16x-12)$

$$\begin{array}{r|rrrr} -1 & -3 & -19 & -4 & 12 \\ & & 3 & 16 & -12 \\ \hline & -3 & -16 & 12 & 0 \end{array}$$

$3x^2+16x-12=0$

$(3x-2)(x+6)=0$

$3x-2=0 \Rightarrow x=\frac{2}{3}$

$x+6=0 \Rightarrow x=-6$

Zeros: $-1, \frac{2}{3}, -6$

(b) $f(x)=-(x+1)(3x-2)(x+6)$

(c) x-intercepts: $(-1, 0), (-6, 0), \left(\frac{2}{3}, 0\right)$

115. $f(x)=x^4+34x^2+225$

(a) $x^4+34x^2+225=(x^2+9)(x^2+25)$

Zeros: $\pm 3i, \pm 5i$

(b) $(x+3i)(x-3i)(x+5i)(x-5i)$

(c) No x-intercepts

117. Since $5i$ is a zero, so is $-5i$.

$f(x)=(x-4)(x+2)(x-5i)(x+5i)$

$=(x^2-2x-8)(x^2+25)$

$=x^4-2x^3+17x^2-50x-200$

119. Since $-3+5i$ is a zero, so is $-3-5i$.

$$\begin{aligned} f(x) &= (x-1)(x+4)(x+3-5i)(x+3+5i) \\ &= (x^2+3x-4)((x+3)^2+25) \\ &= (x^2+3x-4)(x^2+6x+34) \\ &= x^4+9x^3+48x^2+78x-136 \end{aligned}$$

121. $f(x) = x^4-2x^3+8x^2-18x-9$

(a) $f(x) = (x^2+9)(x^2-2x-1)$

(b) For the quadratic

$x^2-2x-1, x = \dfrac{2 \pm \sqrt{(-2)^2-4(-1)}}{2} = 1 \pm \sqrt{2}.$

$f(x) = \left(x^2+9\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

(c) $f(x) = \left(x+3i\right)\left(x-3i\right)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

123. Zeros: $-2i$, 2i

$(x+2i)(x-2i) = x^2+4$ is a factor.

$f(x) = (x^2+4)(x+3)$

Zeros: $\pm 2i, -3$

125. $f(x) = \dfrac{2-x}{x+3}$

(a) Domain: all $x \neq -3$

(b) Not continuous

(c) Horizontal asymptote: $y = -1$

Vertical asymptote: $x = -3$

127. $f(x) = \dfrac{2}{x^2-3x-18} = \dfrac{2}{(x-6)(x+3)}$

(a) Domain: all $x \neq 6, -3$

(b) Not continuous

(c) Horizontal asymptote: $y = 0$

Vertical asymptotes: $x = 6$, $x = -3$

129. $f(x) = \dfrac{7+x}{7-x}$

(a) Domain: all $x \neq 7$

(b) Not continuous

(c) Horizontal asymptote: $y = -1$

Vertical asymptote: $x = 7$

131. $f(x) = \dfrac{4x^2}{2x^2-3}$

(a) Domain: all $x \neq \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

(b) Not continuous

(c) Horizontal asymptote: $y = 2$

Vertical asymptote: $x = \pm\sqrt{\dfrac{3}{2}} = \pm\dfrac{\sqrt{6}}{2}$

133. $f(x) = \dfrac{2x-10}{x^2-2x-15} = \dfrac{2(x-5)}{(x-5)(x+3)} = \dfrac{2}{x+3}, \ x \neq 5$

(a) Domain: all $x \neq 5, -3$

(b) Not continuous

(c) Vertical asymptote: $x = -3$

(There is a hole at $x = 5.$)

Horizontal asymptote: $y = 0$

135. $f(x) = \dfrac{x-2}{|x|+2}$

(a) Domain: all real numbers

(b) Continuous

(c) No vertical asymptotes

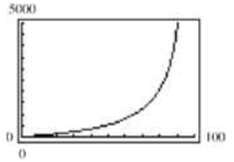
Horizontal asymptotes: $y = 1$, $y = -1$

137. $C = \dfrac{528p}{100-p}, \ 0 \leq p < 100$

(a) When $p = 25$, $C = \dfrac{528(25)}{100-25} = \$176$ million.

When $p = 50$, C$= \dfrac{528(50)}{100-50} = \$528$ million.

When $p = 75$, C$= \dfrac{528(75)}{100-75} = \$1584$ million.

(b) 

Answers will vary.

(c) No. As $p \to 100$, C approaches infinity.

139. $f(x) = \dfrac{x^2-5x+4}{x^2-1}$

$= \dfrac{(x-4)(x-1)}{(x-1)(x+1)}$

$= \dfrac{x-4}{x+1}, \ x \neq 1$

Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 1$

No slant asymptotes

Hole at $x = 1$

141. $f(x) = \dfrac{3x^2 + 5x - 2}{x+1}$

$= \dfrac{(3x-1)(x+2)}{x+1}$

Vertical asymptote: $x = -1$
Horizontal asymptote: none
Long division gives:

$$\begin{array}{r} 3x+2 \\ x+1 \overline{\smash{\big)}\,3x^2+5x-2} \\ \underline{3x^2+3x} \\ 2x-2 \\ \underline{2x+2} \\ -4 \end{array}$$

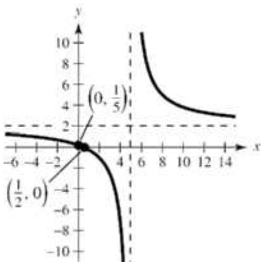
Slant asymptote: $y = 3x + 2$

143. $f(x) = \dfrac{2x-1}{x-5}$

Intercepts: $\left(0, \dfrac{1}{5}\right), \left(\dfrac{1}{2}, 0\right)$

Vertical asymptote: $x = 5$
Horizontal asymptote: $y = 2$

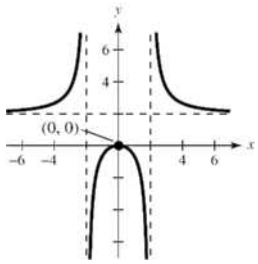
x	-4	-1	0	$\frac{1}{2}$	1	6	8
y	1	$\frac{1}{2}$	$\frac{1}{5}$	0	$-\frac{1}{4}$	11	5



145. $f(x) = \dfrac{2x^2}{x^2 - 4}$

Intercept: $(0, 0)$
y-axis symmetry
Vertical asymptotes: $x = \pm 2$
Horizontal asymptote: $y = 2$

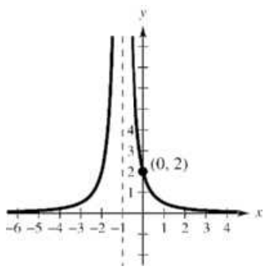
	-6	-4	-1	0	1	4	6
y	$\frac{9}{4}$	$\frac{8}{3}$	$-\frac{2}{3}$	0	$-\frac{2}{3}$	$\frac{8}{3}$	$\frac{9}{4}$



147. $f(x) = \dfrac{2}{(x+1)^2}$

Intercept: $(0, 2)$
Horizontal asymptote: $y = 0$
Vertical asymptote: $x = -1$

x	-4	-3	-2	0	1	2
y	$\frac{2}{9}$	$\frac{1}{2}$	2	2	$\frac{1}{2}$	$\frac{2}{9}$



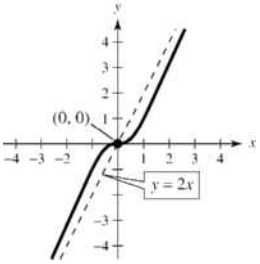
149. $f(x) = \dfrac{2x^3}{x^2+1} = 2x - \dfrac{2x}{x^2+1}$

Intercept: $(0, 0)$

Origin symmetry

Slant asymptote: $y = 2x$

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



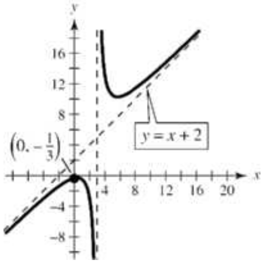
151. $f(x) = \dfrac{x^2 - x + 1}{x - 3} = x + 2 + \dfrac{7}{x-3}$

Intercept: $\left(0, -\dfrac{1}{3}\right)$

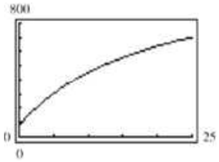
Vertical asymptote: $x = 3$

Slant asymptote: $y = x + 2$

x	-4	0	2	4	5
y	-3	$-\frac{1}{3}$	-3	13	10.5



153. (a)



5 years: $N(5) = \dfrac{20(4+3(5))}{1+0.05(5)} = 304$ thousand fish

10 years: $N(10) = \dfrac{20(4+3(10))}{1+0.05(10)} = 453.\overline{3}$ thousand fish

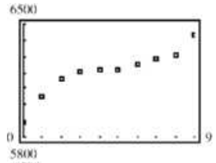
25 years: $N(25) = \dfrac{20(4+3(25))}{1+0.05(25)} = 702.\overline{2}$ thousand fish

(b) The maximum number of fish is $N = 1{,}200{,}000$. The graph of N has a horizontal asymptote at $N = 1200$ or 1,200,000 fish.

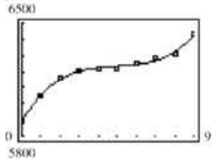
155. Quadratic model

157. Linear model

159. (a)

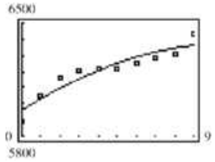


(b)



(c) $S = -3.49t^2 + 76.3t + 5958$; $R^2 \approx 0.8915$

(d)



(e) The cubic model is a better fit because it more closely follows the pattern of the data.

(f) For 2012, let $t = 12$.

$$S(12) = 2.520(12)^3 - 37.51(12)^2 + 192.4(12) + 5895$$
$$= 7157 \text{ stations}$$

161. False. The degree of the numerator is two more than the degree of the denominator.

163. False. $(1+i) + (1-i) = 2$, a real number

165. Not every rational function has a vertical asymptote. For example,

$$y = \frac{x}{x^2+1}.$$

167. The error is $\sqrt{-4} \neq 4i$. In fact,

$$-i\left(\sqrt{-4} - 1\right) = -i(2i - 1) = 2 + i.$$

Chapter 2 Test

1. $y = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1$

Vertex: $(-2, \ -1)$

$x = 0 \Rightarrow y = 3$

$y = 0 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow (x+3)(x+1) = 0 \Rightarrow x = -1, -3$

Intercepts: $(0, 3)$, $(-1, 0)$, $(-3, 0)$

2. Let $y = a(x-h)^2 + k$. The vertex $(3, \ -6)$ implies that $y = a(x-3)^2 - 6$. For $(0, 3)$ you obtain

$3 = a(0-3)^2 - 6 = 9a - 6 \Rightarrow a = 1.$

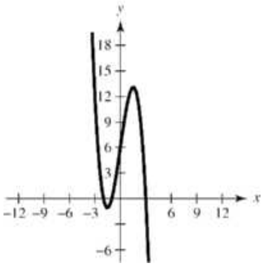
Thus, $y = (x-3)^2 - 6 = x^2 - 6x + 3.$

3. $f(x) = 4x^3 + 4x^2 + x = x(4x^2 + 4x + 1) = x(2x+1)^2$

Zeros: 0 (multiplicity 1)

$-\frac{1}{2}$ (multiplicity 2)

4. $f(x) = -x^3 + 7x + 6$



5.

$$\begin{array}{r}
3x \\
x^2+1 \overline{\smash{\big)}\, 3x^3 + 0x^2 + 4x - 1} \\
\underline{3x^3 + 3x } \\
x - 1
\end{array}$$

$3x + \frac{x-1}{x^2+1}$

6.

$$\begin{array}{r|rrrrr}
2 & 2 & 0 & -5 & 0 & -3 \\
 & & 4 & 8 & 6 & 12 \\
\hline
 & 2 & 4 & 3 & 6 & 9
\end{array}$$

$2x^3 + 4x^2 + 3x + 6 + \frac{9}{x-2}$

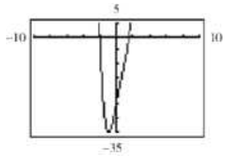
7.

$$\begin{array}{r|rrrrr}
-2 & 3 & 0 & -6 & 5 & -1 \\
 & & -6 & 12 & -12 & 14 \\
\hline
 & 3 & -6 & 6 & -7 & 13
\end{array}$$

$f(-2) = 13$

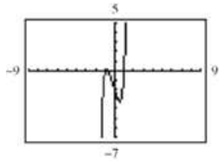
8. Possible rational zeros:

$\pm 24, \ \pm 12, \ \pm 8, \ \pm 6, \ \pm 4, \ \pm 3, \ \pm 2, \ \pm 1, \ \pm \frac{3}{2}, \ \pm \frac{1}{2}$



Rational zeros: $-2, \ \frac{3}{2}$

9. Possible rational zeros: $\pm 2, \ \pm 1, \ \pm \frac{2}{3}, \ \pm \frac{1}{3}$



Rational zeros: $\pm 1, \ -\frac{2}{3}$

10. $f(x) = x^3 - 7x^2 + 11x + 19$

$= (x+1)(x^2 - 8x + 19)$

For the quadratic,

$x = \frac{8 \pm \sqrt{64 - 4(19)}}{2} = 4 \pm \sqrt{3}i.$

Zeros: $-1, \ 4 \pm \sqrt{3}\, i$

$f(x) = (x+1)(x - 4 + \sqrt{3}i)(x - 4 - \sqrt{3}i)$

11. $(-8 - 3i) + (-1 - 15i) = -9 - 18i$

12. $\left(10 + \sqrt{-20}\right) - \left(4 - \sqrt{-14}\right) = 6 + 2\sqrt{5}i + \sqrt{14}i = 6 + \left(2\sqrt{5} + \sqrt{14}\right)i$

13. $(2+i)(6-i) = 12 + 6i - 2i + 1 = 13 + 4i$

14. $(4+3i)^2 - (5+i)^2 = (16 + 24i - 9) - (25 + 10i - 1) = -17 + 14i$

15. $\frac{8+5i}{6-i} \cdot \frac{6+i}{6+i} = \frac{48 + 30i + 8i - 5}{36+1} = \frac{43}{37} + \frac{38}{37}i$

16. $\frac{5i}{2+i} \cdot \frac{2-i}{2-i} = \frac{10i + 5}{4+1} = 1 + 2i$

17. $\frac{(2i-1)}{(3i+2)} \cdot \frac{2-3i}{2-3i} = \frac{6 - 2 + 4i + 3i}{4+9}$

$= \frac{4}{13} + \frac{7}{13}i$

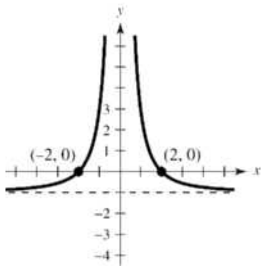
18. $x^2 + 75 = 0$

$$x^2 = -75$$
$$x = \pm\sqrt{-75}$$
$$= \pm 5\sqrt{3}i$$

19. $x^2 - 2x + 8 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(8)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{-28}}{2}$$
$$x = \frac{2 \pm 2\sqrt{7}i}{2}$$
$$x = 1 \pm \sqrt{7}i$$

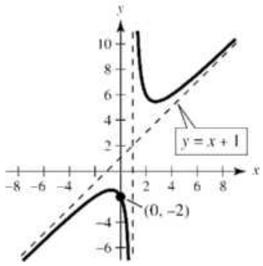
20.



Vertical asymptote: $x = 0$
Intercepts: $(2, 0)$, $(-2, 0)$
Symmetry: y-axis
Horizontal asymptote: $y = -1$

21. $g(x) = \dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$

Vertical asymptote: $x = 1$
Intercept: $(0, -2)$
Slant asymptote: $y = x + 1$

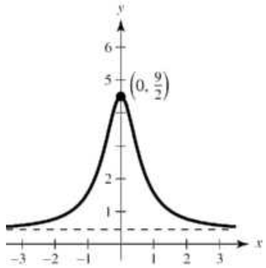


22. $f(x) = \dfrac{2x^2 + 9}{5x^2 + 2}$

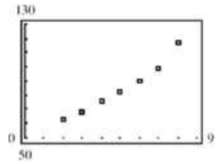
Horizontal asymptote: $y = \dfrac{2}{5}$

y-axis symmetry

Intercept: $\left(0, \dfrac{9}{2}\right)$

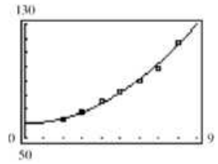


23. (a)



(b) $A = 0.861t^2 + 0.03t + 60.0$

(c)



The model fits the data well.

(d) For 2010, let $t = 10$.

$$A(10) = 0.861(10)^2 + 0.03(10) + 60.0$$
$$= \$146.4 \text{ million}$$

For 2012, let $t = 12$.

$$A(12) = 0.861(12)^2 + 0.03(12) + 60.0$$
$$\approx \$184.3 \text{ billion}$$

(e) Answers will vary.