

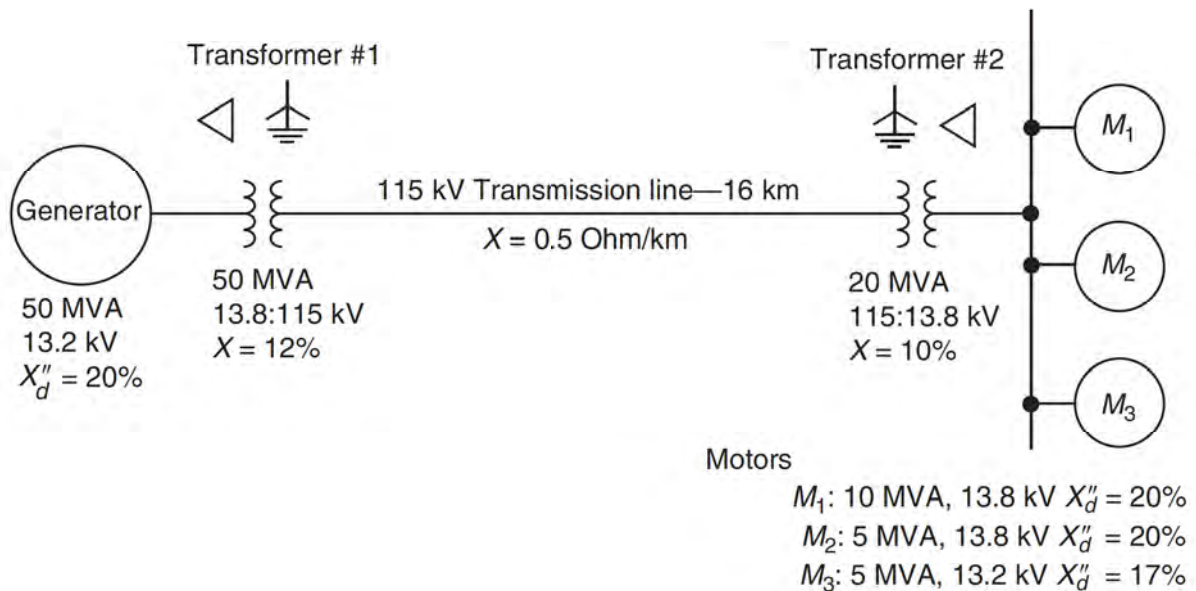
CHAPTER 2

- 2.1 A wye-connected generator has a nameplate rating of 200 MVA, 20 kV, and its subtransient reactance (X_d'') is 1.2 pu. Determine its reactance in ohms.
- 2.2 The generator of Problem 2.1 is connected in a power system where the base is specified as 100 MVA, 13.8 kV. What is the generator reactance (X_d'') in per unit on this system base?
- 2.3 Convert the per-unit answer calculated in Problem 2.2 to ohms. Does this match the value determined in Problem 2.1?
- 2.4 Three 5 MVA single-phase transformers, each rated 8:1.39 kV, have a leakage impedance of 6%. These can be connected in a number of different ways to supply three identical 5 Ω resistive loads. Various transformer and load connections are outlined in Table P2.4. Complete the table columns. Use a three-phase base of 15 MVA.

TABLE P2.4

Case No.	Transformer connection			Line-to-line base kV	Load R in per unit	Total Z as viewed from the high side	
	Pri	Sec	Load connection to secondary HV			Per unit	Ω
1	Wye	Wye	Wye				
2	Wye	Wye	Delta				
3	Wye	Delta	Wye				
4	Wye	Delta	Wye				
5	Delta	Wye	Wye				
6	Delta	Wye	Delta				
7	Delta	Delta	Wye				
8	Delta	Delta	Delta				

- 2.5 A three-phase generator feeds three large synchronous motors over a 16 km, 115 kV transmission line, through a transformer bank, as shown in Figure P2.5. Draw an equivalent single-line reactance diagram with all reactances indicated in per unit of a 100 MVA, 13.8 or 115 kV base.



2.6 In the system of Problem 2.5, it is desired to maintain the voltage at the motor bus of $1.0 \angle 0^\circ$ per unit. The three motors are operating at full rating and 90% pf.

- a.** Determine the voltage required at the generator terminals assuming that there is no voltage regulating taps or similar equipment in this system.
- b.** What is the voltage required behind the subtransient reactance?

2.7 The percent impedance of a transformer is typically determined by a short circuit test. In such a test, the secondary of the transformer is shorted and the voltage on the primary is increased until rated current flows in the transformer windings. The applied voltage that produces rated current divided by the rated voltage of the transformer is equal to the per-unit impedance of the transformer.

A short circuit test on a 150 KVA, 7200–240 V transformer provides the following results:
Primary voltage at 20.8 primary amperes = 208.8 V

- a.** Determine the %Z of the transformer.
- b.** Calculate the ohmic impedance of the transformer in primary and secondary terms.
- c.** How much current would flow in the transformer if its secondary would become shorted during normal operating conditions? (Consider source impedance to be zero.)

■ 2.1

Find impedance in ohms from per-unit.

$$\text{MVA}_B = 200, \quad \text{kV}_B = 20, \quad X''_d = Z_{\text{pu}} = 1.2$$

$$\text{Per Eq. (2.17)} \quad Z_{\Omega} = \frac{\text{kV}_B^2 \times Z_{\text{pu}}}{\text{MVA}_B}$$

$$\frac{20^2 \times 1.2}{200}$$

$$2.4$$

Impedance $X''_d = 2.4$ ohms

■ 2.2 - Method 1

Convert per-unit impedance from one base to another.

$$\text{MVA}_1 = 200, \text{ MVA}_2 = 100, \text{ kV}_1 = 20, \text{ kV}_2 = 13.8, X''_d = Z_{1\text{PU}} = 1.2$$

$$\text{Per Eq. (2.33)} \quad Z_{2\text{PU}} = Z_{1\text{PU}} \times \frac{\text{MVA}_2}{\text{MVA}_1} \times \frac{\text{kV}_1^2}{\text{kV}_2^2}$$

$$1.2 \times \frac{100}{200} \times \frac{20^2}{13.8^2}$$

$$1.26024$$

Per-unit impedance in new base $Z_{2\text{PU}} = 1.26024$

■ 2.2 - Method 2

Convert impedance from ohms (Result 2.1) to per-unit.

$$\text{MVA}_B = 100, \text{ kV}_B = 13.8, X''_d = Z_{\Omega} = 2.4$$

$$\text{Per Eq. (2.15)} \quad Z_{\text{PU}} = \frac{Z_{\Omega}}{Z_B} = \frac{\text{MVA}_B \times Z_{\Omega}}{\text{kV}_B^2}$$

$$\frac{100 \times 2.4}{13.8^2}$$

$$1.26024$$

Per-unit impedance $Z_{\text{PU}} = 1.26024$

■ 2.3

Convert impedance from per-unit (Result 2.2) back to ohms.

$$\text{MVA}_B = 100, \quad \text{kV}_B = 13.8, \quad X''_d = Z_{\text{pu}} = 1.26024$$

$$\begin{aligned} \text{Per Eq. (2.17)} \quad Z_{\Omega} &= \frac{\text{kV}_B^2 \times Z_{\text{pu}}}{\text{MVA}_B} \\ &= \frac{13.8^2 \times 1.26024}{100} \end{aligned}$$

$$2.4$$

Impedance $X''_d = 2.4$ ohms. This matches Result 2.1.

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The transformer per unit impedance can be referred to either side, so when either winding is connected in wye, the transformer impedance will be $6\% I^2$ equivalent impedance in the line referred to the wye-connected side.

When both windings are connected in delta, the equivalent line impedance will be $1/3$ of the winding impedance, or $2\% I^2$.

Similarly, the load resistors, when connected in delta, have an equivalent wye resistance of $1/3$ the ohmic value of resistors used to make up the delta.

Transformer impedance is $6\% I^2$ on transformer 1-phase
Base of 5 MVA per phase, equivalent to 15 MVA 3-phase.

The equivalent wye-connected load resistance can be expressed on per unit basis by dividing by the low-side base impedance. The high-side per unit impedance is the same as the low-side per unit impedance; the high-side equivalent impedance in ohms can be found by multiplying the per unit impedance by the high-side base impedance.

2.4 cont

	HV	LV	LOAD Connection	V _B HV	V _B LV	Leakage Reactance	EAIV Y Reactance	PU EA Y	HV EA Y
1	Y	Y	Y	13.856	2.408	j0.06	5	31.15+j0.06	
2	Y	Y	Δ	13.856	2.408	j0.06	1.667	10.38+j0.06	
3	Y	Δ	Y	13.856	1.390	j0.06	5	38.82+j0.06	
4	Y	Δ	Δ	13.856	1.390	j0.06	1.667	12.94+j0.06	
5	Δ	Y	Y	8	2.408	j0.06	5	31.15+j0.06	
6	Δ	Y	Δ	8	2.408	j0.06	1.667	10.38+j0.06	
7	Δ	Δ	Y	8	1.390	j0.02	5	38.82+j0.02	
8	Δ	Δ	Δ	8	1.390	j0.02	1.667	12.94+j0.02	

Wye connected HV $8 \text{ kV} \times \sqrt{3} = 13.856 \text{ kV}$, $Z_B = \frac{(13.856 \text{ kV})^2}{15 \text{ MVA}} = 12.94 \Omega$

Δ connected HV 8 kV , $Z_B = \frac{(8 \text{ kV})^2}{15 \text{ MVA}} = 4.267 \Omega$

Wye connected LV $1.39 \text{ kV} \times \sqrt{3} = 2.408 \text{ kV}$, $Z_B = \frac{(2.408 \text{ kV})^2}{15 \text{ MVA}} = 0.3864 \Omega$

Δ connected LV 1.39 kV , $Z_B = \frac{(1.39 \text{ kV})^2}{15 \text{ MVA}} = 0.1288 \Omega$

$$\frac{5}{0.3864} = 12.94$$

$$\frac{5/3}{0.1605} = 4.31$$

$$\frac{5}{0.1288} = 38.82$$

$$\frac{5/3}{0.1288} = 12.94$$

See spreadsheet.

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Problem 2.4

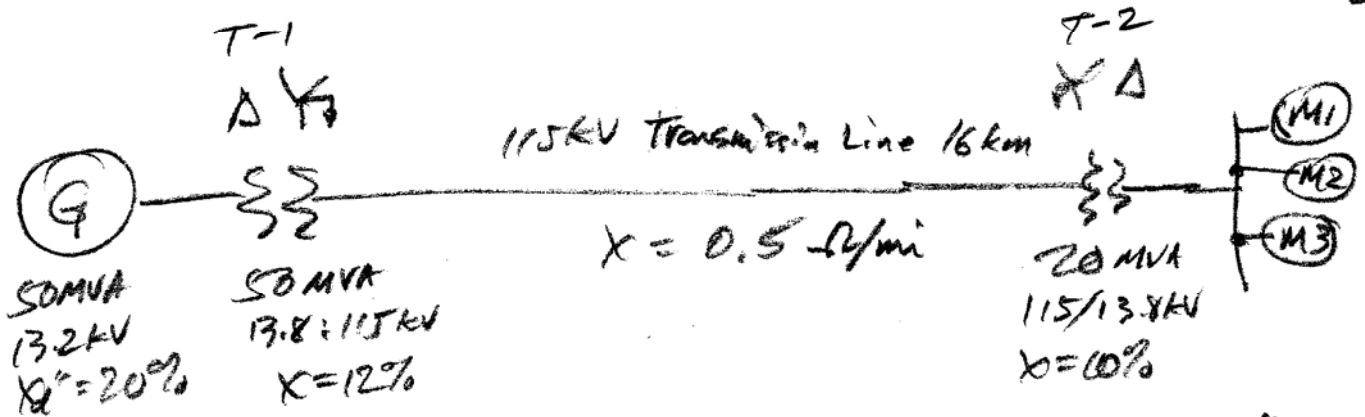
						Intermediate calculations						
Col A	Col B	Col C	Col D	Col E	Col F	Col G	Col H	Col I	Col J	Col K	Col L	Col M
	Winding connection			LL Voltage Base (kV)		Base Impedance (ohms)		PU leakage reactance	equivalent Y-connected resistive load (ohms on LV side)	PU equivalent wye-connected load resistance	Total Z viewed from HV side	
Case	HV	LV	Load connection	HV	LV	HV	LV	equivalent in line		(Col J / Col H)	per unit (Col K + j*Col i)	ohms (Col L * Col G)
1	wye	wye	wye	13.856	2.407	12.799	0.386	0.060	5.000	12.940	12.940 + j0.060	165.623 + j0.768
2	wye	wye	delta	13.856	2.407	12.799	0.386	0.060	1.667	4.313	4.313 + j0.060	55.208 + j0.768
3	wye	delta	wye	13.856	1.390	12.799	0.129	0.060	5.000	38.818	38.818 + j0.060	496.840 + j0.768
4	wye	delta	delta	13.856	1.390	12.799	0.129	0.060	1.667	12.939	12.939 + j0.060	165.613 + j0.768
5	delta	wye	wye	8.000	2.407	4.267	0.386	0.060	5.000	12.940	12.940 + j0.060	55.211 + j0.256
6	delta	wye	delta	8.000	2.407	4.267	0.386	0.060	1.667	4.313	4.313 + j0.060	18.404 + j0.256
7	delta	delta	wye	8.000	1.390	4.267	0.129	0.020	5.000	38.818	38.818 + j0.020	165.623 + j0.085
8	delta	delta	delta	8.000	1.390	4.267	0.129	0.020	1.667	12.939	12.939 + j0.020	55.208 + j0.085

Base power (3-phase MVA)	HV winding voltage (kV)	HV delta LL voltage (kV)	HV wye LL voltage (kV)	Zbase for HV delta	Zbase for HV wye
15.000	8.000	8.000	13.856	4.267	12.799

Base power (3-phase MVA)	LV winding voltage (kV)	LV delta LL voltage (kV)	LV wye LL voltage (kV)	Zbase for LV delta	Zbase for LV wye
15.000	1.390	1.390	2.407	0.129	0.386

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M₁: 10 MVA, 13.8 kV, $X_d'' = 20\%$

M₂: 5 MVA, 13.8 kV, $X_d'' = 20\%$

M₃: 5 MVA, 13.2 kV, $X_d'' = 17\%$

Changing Base:

$$Z_{pu, New} = Z_{pu, Old} \left(\frac{MVA_{New}}{MVA_{Old}} \right) \left(\frac{KV_{Old}}{KV_{New}} \right)^2$$

Generator $X_d'' = 20\% \left(\frac{100}{50} \right) \left(\frac{13.2}{13.8} \right)^2 = 36.6\%, 0.366 pu$

T-1 $X = 12\% \left(\frac{100}{50} \right) = 24\%, 0.24 pu$

T-2 $X = 10\% \left(\frac{100}{20} \right) = 50\%, 0.50 pu$

M1 $X_d'' = 20\% \left(\frac{100}{10} \right) = 200\%, 2.0 pu$

M2 $X_d'' = 20\% \left(\frac{100}{5} \right) = 400\%, 4.0 pu$

M3 $X_d'' = 17\% \left(\frac{100}{5} \right) \left(\frac{13.2}{13.8} \right)^2 = 311\%, 3.11 pu$

Line impedance: $Z_{pu} = \frac{Z}{Z_{base}}$

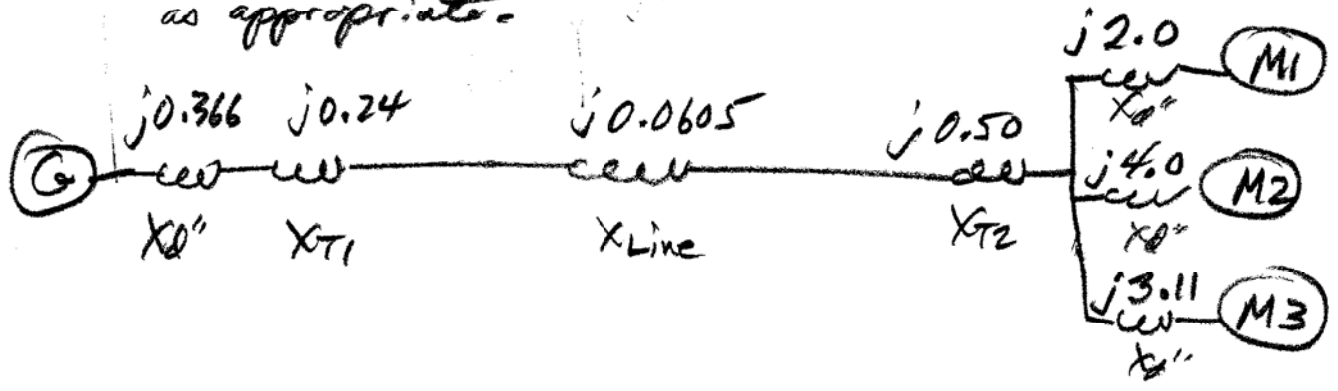
$Z_{base 115KV} = \frac{(115KV)^2}{100 MVA} = 132.25 \Omega$

$Z_{line pu} = \frac{(0.5 \frac{\Omega}{mi})(16 mi)}{132.25 \Omega} = 0.0605 pu$

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Reactance diagram

All impedances in PU on 100 MVA base
and either 13.8 kV base or 115 kV base
as appropriate.



Problem 2.6

$$\text{PU} \equiv 1$$

Prob 2.6.a – Voltage at generator bus to keep 1 PU voltage at motor bus

$$V_{\text{motor}} := 1.0 \cdot \text{PU}$$

Load current - motors at rated power and 0.9 PF. Problem does not say explicitly, but we assume that the PF is lagging as this is the most probable.

Total motor bus power in per unit

$$S_{\text{motor}} := \frac{10 \cdot \text{MVA} + 5 \cdot \text{MVA} + 5 \cdot \text{MVA}}{100 \cdot \text{MVA}} \cdot e^{j \cdot \arccos(0.9)} = (0.18 + 0.087j) \text{ PU}$$

$$|S_{\text{motor}}| = 0.2 \text{ PU} \quad \arg(S_{\text{motor}}) = 25.842 \text{ deg}$$

Current is found from the relation that complex power is equal to voltage times complex conjugate of current.

$$I_{\text{motor}} := \left(\frac{\overline{S_{\text{motor}}}}{V_{\text{motor}}} \right) = (0.18 - 0.087j) \text{ PU} \quad |I_{\text{motor}}| = 0.2 \text{ PU} \quad \arg(I_{\text{motor}}) = -25.842 \text{ deg}$$

$$\text{Line reactance in PU (from prob. 2.5)} \quad Z_{\text{Line}} := j \cdot 0.0605 \text{ PU}$$

$$\text{T1 reactance in PU (from prob. 2.5)} \quad Z_{\text{T1}} := j \cdot 0.24 \text{ PU}$$

$$\text{T2 reactance in PU (from prob. 2.5)} \quad Z_{\text{T2}} := j \cdot 0.50 \text{ PU}$$

Generator bus voltage is motor bus voltage plus voltage drop in T1, line, and T2.

$$V_{\text{Gen.bus}} := V_{\text{motor}} + I_{\text{motor}} \cdot (Z_{\text{T1}} + Z_{\text{Line}} + Z_{\text{T2}}) = (1.07 + 0.144j) \text{ PU}$$

$$|V_{\text{Gen.bus}}| = 1.079 \text{ PU} \quad \arg(V_{\text{Gen.bus}}) = 7.671 \text{ deg}$$

Prob 2.6 b - Generator internal voltage behind subtransient reactance: add internal voltage drop to the generator terminal voltage

$$\text{Generator subtransient reactance in PU (from prob. 2.5)} \quad Z_{\text{Gen}} := j \cdot 0.366$$

$$V_{\text{Gen.internal}} := V_{\text{Gen.bus}} + I_{\text{motor}} \cdot Z_{\text{Gen}} = (1.102 + 0.21j) \text{ PU}$$

$$|V_{\text{Gen.internal}}| = 1.122 \text{ PU} \quad \arg(V_{\text{Gen.internal}}) = 10.791 \text{ deg}$$

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$$\text{kVA} \equiv 1000 \cdot \text{volt} \cdot \text{amp}$$

Problem 2.7

$$\text{MVA} \equiv 10^6 \cdot \text{volt} \cdot \text{amp}$$

$$\text{PU} \equiv 1$$

Transformer rated power and rated voltages

$$S_{\text{rated}} := 150 \cdot \text{kVA} \quad V_{\text{rated.primary}} := 7200 \cdot \text{volt} \quad V_{\text{rated.secondary}} := 240 \cdot \text{volt}$$

Transformer rated current

$$I_{\text{rated.primary}} := \frac{150 \cdot \text{kVA}}{7.2 \cdot \text{kV}} = 20.833 \text{ A}$$

$$I_{\text{rated.secondary}} := \frac{150000 \cdot \text{volt} \cdot \text{amp}}{240 \cdot \text{volt}} = 625 \text{ A}$$

This confirms that the short circuit test was in fact performed at rated current. The impedance voltage is therefore equal to the voltage measured during the test, or 208.8 volts.

2.7.a The transformer per cent impedance voltage (%IZ or %Z) is equal to the measured impedance voltage expressed as per cent of the rated voltage or in this case:

$$V_{\text{test}} := 208.8 \cdot \text{volt} \quad X_T := \frac{V_{\text{test}}}{V_{\text{rated.primary}}} = 0.029 \text{ PU} \quad X_T = 2.9 \%$$

$$\frac{208.8 \cdot \text{volt}}{7200 \cdot \text{volt}} = 0.029 \text{ PU}$$

2.7.b The equivalent ohmic impedance referred to the primary or secondary is equal to the corresponding base impedance times the per unit impedance.

$$Z_{\text{base.primary}} := \frac{V_{\text{rated.primary}}^2}{S_{\text{rated}}} = 345.6 \Omega \quad Z_{\text{base.secondary}} := \frac{V_{\text{rated.secondary}}^2}{S_{\text{rated}}} = 0.384 \Omega$$

$$\frac{(7.2 \cdot \text{kV})^2}{0.15 \cdot \text{MVA}} = 345.6 \Omega$$

$$\frac{(0.24 \cdot \text{kV})^2}{0.150 \cdot \text{MVA}} = 0.384 \Omega$$

$$X_{T.\text{ohms.primary}} := X_T \cdot Z_{\text{base.primary}} = 10.022 \Omega$$

$$X_{T.\text{ohms.secondary}} := X_T \cdot Z_{\text{base.secondary}} = 0.0111 \Omega$$

2.7.c There are a few ways to arrive at the short circuit current.

One way is to divide the rated voltage by the equivalent ohmic impedance on either primary or secondary.

Alternatively, 1.0 PU voltage can be divided by the PU impedance to determine the short circuit current in PU of rated current. The PU short circuit current can then be multiplied by the base current (i.e., rated current) on either primary or secondary as desired. This method can be further simplified to "divide rated current by per unit impedance."

Both methods are illustrated here:

$$\frac{V_{\text{rated.primary}}}{X_{T.\text{ohms.primary}}} = 718.391 \text{ A} \qquad \frac{7200 \cdot \text{volt}}{10.022 \cdot \text{ohm}} = 718.419 \text{ A}$$

$$\frac{V_{\text{rated.secondary}}}{X_{T.\text{ohms.secondary}}} = 21551.724 \text{ A} \qquad \frac{240 \cdot \text{volt}}{.0111 \cdot \text{ohm}} = 21621.622 \text{ A}$$

(NOTE: this result can also be used to illustrate the concept of significant digits. The result is 21.6 kA, although the computer internally carries out calculations to greater precision than is significant.)

Another method to arrive at the same result:

$$I_{\text{SC.PU}} := \frac{1}{X_T} = 34.483 \text{ PU} \qquad I_{\text{SC.Primary}} := I_{\text{SC.PU}} \cdot I_{\text{rated.primary}} = 718.391 \text{ A}$$

$$34.5 \cdot 20.8 \cdot \text{amp} = 717.6 \text{ A}$$

$$I_{\text{SC.secondary}} := I_{\text{SC.PU}} \cdot I_{\text{rated.secondary}} = 21.552 \text{ kA}$$

$$34.5 \cdot 625 \cdot \text{amp} = 21562.5 \text{ A}$$

And finally, another method, often the simplest:

$$\frac{I_{\text{rated.primary}}}{X_T} = 718.391 \text{ A} \qquad \frac{20.8 \cdot \text{amp}}{.029} = 717.241 \text{ A}$$

$$\frac{I_{\text{rated.secondary}}}{X_T} = 21551.724 \text{ A} \qquad \frac{625 \cdot \text{amp}}{0.029} = 21551.724 \text{ A}$$