PART I

ORGANIZING A MODELING COURSE

ISSUES

Organizing a modeling course is a significant, educational challenge. A number of resource materials must be gathered: an appropriate text, supplemental references both for students as well as the instructor, sources and scenarios for student projects, and, possibly, computer software. Furthermore, there are crucial pedagogical issues that must be resolved in designing the course, including the following:

1. Course objectives
2. Course prerequisites
3. Course content
4. Number and type of student projects
5. Individual versus group projects
6. The role of computation
7. Grading considerations
8. Opportunities for follow-on modeling courses.

We believe that a textbook can only serve as a base for a modeling course, which must then be tailored to meet the specific needs of students, as well as overall objectives in the curriculum. Moreover, a modeling course needs to be flexible and dynamic to allow for each individual instructor to take advantage of his or her particular mathematical expertise, experiences, and modeling preferences.

OBJECTIVES

The overall goal of our course is to provide a thorough introduction to the entire modeling process while affording students the opportunity to practice:

1. **Creative and Empirical Model Construction:** Given a real-world scenario, the student must identify a problem, make assumptions and collect data, propose a model, test the assumptions, refine the model as necessary, fit the model to data if appropriate, and analyze the underlying mathematical structure of the model in order to appraise the sensitivity of the conclusions in relation to the assumptions. Furthermore, the student should be able to generalize the construction to related scenarios.

2. **Model Analysis:** Given a model, the student must work backward to uncover the implicit underlying assumptions, assess critically how well the assumptions reflect the scenario at hand, and estimate the sensitivity of the conclusions when the assumptions are not precisely met.

3. **Model Research:** The student investigates an area of interest to gain knowledge, understanding, and an ability to use what already has been created or discovered. Model research provides for determining the 'state of the art' in a subject area.

To accomplish our goals, we provide students with a diversity of scenarios for practicing all three of these facets of modeling. In addition, normal and routine exercises are assigned to test the student's understanding of the instructional material we present. Thus, our textbook provides expository material and a framework around which a diversity of additional materials can be organized in support of a *modeling course.*

COURSE PREREQUISITES

There are strong arguments to require such courses as advanced calculus, linear algebra, differential equations, probability, numerical analysis, and optimization as prerequisites to an introductory modeling course. Certainly the level and sophistication of the mathematics that students are capable of using increases significantly as more advanced courses are added to their programs. However, our desire is to gain the modeling experience as early as possible in the student's career. Although some unfamiliar mathematical ideas are taught as part of the modeling process, the emphasis is on using mathematics already known to the student after completing high school. As outlined in the preface to the text, a course can be constructed requiring only high school mathematics as a prerequisite. Some sections do require an introductory calculus course as a prerequisite or corequisite, as detailed in the preface to the text. In our modeling courses, we emphasize teaching students how to use mathematics they already know in a context of significant applications with which they can readily identify. This approach stimulates student interest in mathematics and motivates them to study more advanced topics such as those mentioned above. Moreover, our students are eager to see meaningful applications of the mathematics they have learned.

COURSE CONTENT

Many modeling courses select from an inventory of specific model types which can be adapted to a variety of situations. Certainly model selection is a valid step in the problem-solving process and it is important that students learn to use what already has been created. However, our experience is that undergraduate students seldom comprehend the assumptions inherent in type models. We want our students to realize the necessity of making assumptions, the need to determine the appropriateness of the assumptions, and the importance of investigating how sensitive the conclusions are to the assumptions. Consequently, while we do discuss how to fit type models in the text, we have chosen to emphasize model construction, leaving the study of type models for more advanced courses.

We feel that model construction promotes student creativity, demonstrates the artistic nature of model building, and develops an appreciation for how mathematics can be used effectively in various settings. Since the student needs practice in the first several steps of the problem-solving process–identifying the problem, making assumptions, determining interrelationships between the variables and submodels–it is tempting to compose an entire course of creative model construction. However, there are serious difficulties with such a course. Typically, students are very anxious, at least initially, because they don't know how to begin the modeling process. After all, they have probably never before attacked an open-ended problem. When they are successful in constructing a model, they usually find the procedure enjoyable and exhilarating. Nevertheless, they tend to "burn out" if an entire quarter or semester is dedicated to creative model construction. Moreover, a course consisting entirely of creative model construction cannot address other important aspects of modeling, like experimentation and simulation. Furthermore, there are difficulties with such a course for the instructor. Preparation of the course requires enormous effort in researching and generating scenarios to be modeled. Grading is difficult and tends to be subjective because each student approaches each project in a different way, yet students need constant feedback on their work. These difficulties then contribute to the anxieties of the student who is overly concerned about being graded in class under a time constraint in an area perceived to be relatively subjective. Under these conditions, success of the course becomes highly instructor dependent and circumstantial.

For the above reasons, we have chosen to design a course consisting of a mixture of both *creative* and *empirical* modeling projects, along with projects in model analysis and model research. We begin by interactively constructing graphical models in class to engage students immediately in model analysis, which is relatively familiar to them. The transition to creative model construction commences with the students learning to make assumptions about a real-world behavior and by providing them with data to check their assumptions (initially with simple proportionality arguments). We then apply the modeling process to construct interactively in class relatively simple submodels and models in a variety of settings covering many disciplines. Students begin to see that situations arise where it may be very difficult or impossible to construct an analytic model, yet predictive capabilities are highly desirable. This perception motivates the study of empirical modeling. Students find empirical model construction more procedural and reproducible than creative model construction, and we find they welcome the mixture. In the text, some scenarios are modeled several ways creatively, and several ways empirically, so students can experience the alternatives that may be available.

STUDENT PROJECTS

We have developed our modeling courses to promote progressive development in the student's modeling capabilities. To achieve our objectives, we require each student to complete at least 6 **significant** problem assignments or projects to be handed-in for a grade. Each student is assigned a mixture of problems/projects in creative and empirical model construction, model analysis, and model research. We purposely select problems/projects which address scenarios for which there are no unique solutions. Several of the projects include **real** data that the student is either given or can **readily** collect.

If the course is taught early in the student's program we recommend a combination of individual and group projects. Individual work is essential if the student is to develop adequate modeling skills. By way of contrast, group projects are exhilarating and allow for the experience of the synergistic effect that takes place in a 'brainstorming' session. For example, during the instruction of Chapter 2 on proportionality, we give a choice of 5 or 6 different scenarios for which students individually create a model. During the instruction of the next chapter on model fitting, teams are organized to test, refine, and fit these models to data. The resulting group model is typically substantially superior to any of the individual models. A similar procedure is followed during the instruction of the simulation chapter where students are required to develop models individually followed by a team effort to refine the models and implement them on the computer.