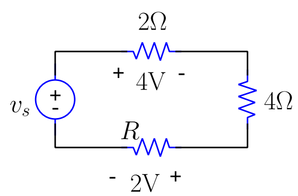
Errata:

1. Chapter 1, Problem 1.1 (Polarity of the voltage across resistor R should be the opposite of what is shown in the text, in order to obtain $R > 0$).

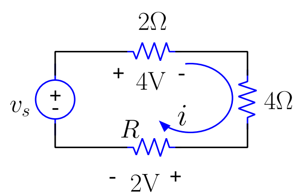
2. Chapter 3, Problem 3.7a (The correct specification of part (a) is given in the solution manual, and is different from what is printed in the text).

Chapter 1

1. In the following circuit determine R and v_s :



Erratum: In the text book the polarization of resistor R was inverted.
Solution:



Ohm's law at the 2Ω resistor states that

$$i = \frac{4\text{V}}{2\Omega} = 2\text{A}.$$

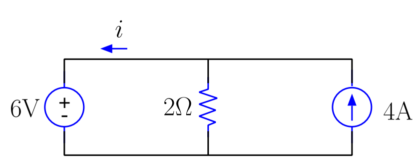
Likewise, using Ohm's law for the unknown resistor yields to

$$R = \frac{2\text{V}}{i} = \frac{2\text{V}}{2\text{A}} = 1\Omega.$$

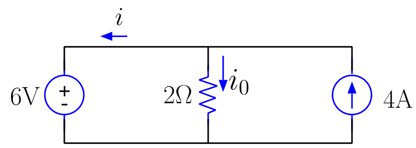
Finally applying KVL for the loop gives

$$v_s = 2\text{V} + (4\Omega)(2\text{A}) + 4\text{V} = 14\text{V}.$$

2. In the following circuit determine i:



Solution:



Applying Ohm's law at the resistor gives

$$i_0 = \frac{6\text{V}}{2\Omega} = 3\text{A}.$$

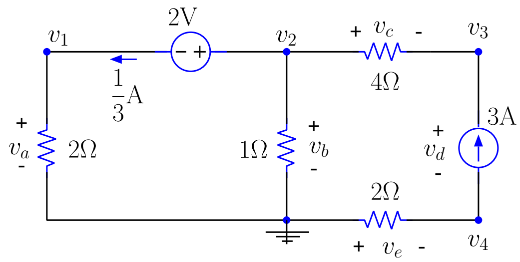
Then the KCL equation at the top node can be written as

$$4\text{A} = i + i_0.$$

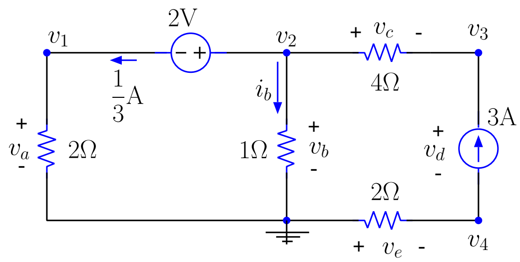
Substituting $i_0 = 3\text{A}$ into the KCL equation, we obtain

$$i = 4\text{A} - i_0 = 4\text{A} - 3\text{A} = 1\text{A}.$$

3. a) In the following circuit determine all the unknown element and node voltages.



b) What is the voltage drop in the circuit above from the reference to node 4?
c) What is the voltage rise from node 2 to node 3?
d) What is the voltage drop from node 1 to the reference?
Solution:



The KCL at node v_2 can be written as

$$\frac{1}{3}\text{A} + i_b = 3\text{A},$$

which yields to

$$i_b = \frac{8}{3}\text{A}.$$

a) Node Voltages:
Voltage drop in 2Ω resistor: $v_1 - 0 = \frac{1}{3}\text{A} \times 2\Omega \Longrightarrow v_1 = \frac{2}{3}\text{V}$
Voltage drop in 1Ω resistor: $v_2 - 0 = i_b \times 1\Omega \Longrightarrow v_2 = \frac{8}{3}\text{V}$
Voltage drop in 4Ω resistor: $v_3 - v_2 = 3\text{A} \times 4\Omega \Longrightarrow v_3 = \frac{44}{3}\text{V}$
Voltage drop in 2Ω resistor: $v_4 - 0 = (-3\text{A}) \times 2\Omega \Longrightarrow v_4 = -6\text{V}$
Element voltages:
$v_a = v_1 - 0 = \frac{2}{3}\text{V}$
$v_b = v_2 - 0 = \frac{8}{3}\text{V}$
$v_c = v_2 - v_3 = \frac{8}{3}\text{V} - \frac{44}{3}\text{V} = -12\text{V}$
$v_d = v_3 - v_4 = \frac{44}{3}\text{V} - (-6\text{V}) = \frac{62}{3}\text{V}$
$v_e = 0 - v_4 = 6\text{V}$.

b) Voltage drop from the reference to node 4 $= 0 - v_4 = v_e = 6\text{V}$.

c) Voltage rise from node 2 to node 3 $= v_3 - v_2 = -v_c = 12\text{V}$.

d) Voltage drop from node 1 to the reference $= v_1 - 0 = v_a = \frac{2}{3}\text{V}$.

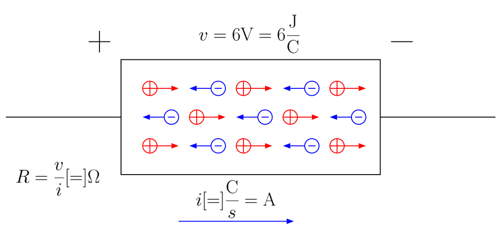
4. a) A volume of ionized gas filled with free electrons and protons can be modeled as a resistor. Consider such a resistor model supporting a 6 V potential difference between its terminals. We are told that in 1 s 6.2422×10^{18} protons move through the resistor in the direction of the 6 V drop (say, from left to right) and 1.24844×10^{19} electrons move in the opposite direction. What is the net amount of electrical charge that transits the element in 1 s in the direction of 6 V drop and what is the corresponding resistance R? Note that electrical charge q is 1.602×10^{-19} C for a proton and -1.602×10^{-19} C for an electron.

b) Does a proton gain or lose energy as it transits the resistor? How many joules? Explain.

c) Does an electron gain or lose energy as it transits the resistor? How many joules? Explain

d) A second resistor with 6 V potential difference conducts 1.87266×10^{19} electrons every second but no proton is allowed to move through it. Compare the current, resistance, and absorbed power of the two resistors.

Solution:
Consider the following figure:



a) Since negative charge(electrons) moving to the left, can be seen as positive charge moving to the right, the net amount of positive charge that transits the element in 1s in the direction of 6V drop (from left to right in the figure) is

$$\begin{aligned} i &= 6.2422 \times 10^{18} \frac{\text{proton}}{\text{s}} \times 1.602 \times 10^{-19} \frac{\text{C}}{\text{proton}} \\ &\quad -1.24844 \times 10^{19} \frac{\text{electrons}}{\text{s}} \times -1.602 \times 10^{-19} \frac{\text{C}}{\text{electrons}} \\ &= 3.00000132\text{A} \approx 3\text{A}. \end{aligned}$$

Consequently the corresponding resistance, according to Ohm's law is

$$R = \frac{6\text{V}}{3\text{A}} = 2\Omega.$$

b) From definition of voltage, we have

$$v = 6\text{V} = 6\frac{\text{J}}{\text{C}},$$

which is the **energy loss** per unit charge transported from terminal + to terminal -. So a proton lose the following energy

$$\Delta\omega = 6\frac{\text{J}}{\text{C}} \times 1.602 \times 10^{-19} \frac{\text{C}}{\text{proton}} = 9.612 \times 10^{-19}\text{J}.$$

c) Voltage can also be defined as the **energy gain** per unit charge transported from terminal - to terminal +. So an electron gain the following energy

$$\Delta\omega = 6\frac{\text{J}}{\text{C}} \times -1.602 \times 10^{-19} \frac{\text{C}}{\text{electrons}} = -9.612 \times 10^{-19}\text{J}.$$

Since the gain is negative, then we say that the electron is actually losing energy, and in the same amount as the proton (9.612×10^{-19}J).

d) In this case we can obtain the current as follows:

$$i = -1.87266 \times 10^{19} \frac{\text{electrons}}{\text{s}} \times -1.602 \times 10^{-19} \frac{\text{C}}{\text{electrons}} = 3.00000132\text{A} \approx 3\text{A}.$$

The corresponding resistance, according to Ohm's law, is

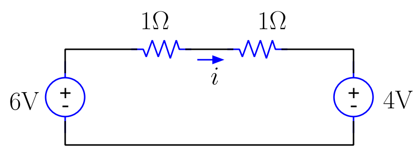
$$R = \frac{6\text{V}}{3\text{A}} = 2\Omega.$$

And the absorbed power is

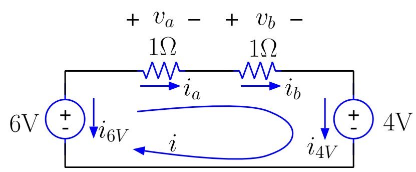
$$p = vi = (6\text{V})(3\text{A}) = 18\frac{\text{J}}{\text{C}}\frac{\text{C}}{\text{s}} = 18\frac{\text{J}}{\text{s}} = 18\text{W}.$$

The same as the first case.

5. In the circuit pictured here, one of the independent voltage sources is injecting energy into the circuit, while the other one is absorbing energy. Identify the source that is injecting the energy absorbed in the circuit and confirm that the sum of all absorbed powers equals zero.



Solution:



First we write the KVL equation as

$$6\text{V} - i \times (1\Omega) - i \times (1\Omega) - 4\text{V} \quad = \quad 0,$$

which yields to

$$i = 1\text{A}.$$

Applying Ohm's law at the resistors gives

$$v_a \quad = \quad v_b = (1\text{A})(1\Omega) = 1\text{V}.$$

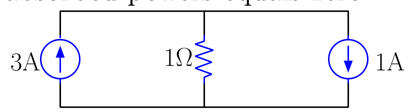
Then the absorbed powers can be calculated using $p = v_x i_x$, which gives

$$\begin{aligned} p_a &= p_b = v_a i_a = (1\text{V})(1\text{A}) = 1\text{W} \\ p_{6\text{V}} &= (6\text{V})i_{6V} = (6\text{V})(-1\text{A}) = -6\text{W} \\ p_{4\text{V}} &= (4\text{V})i_{4V} = (4\text{V})(1\text{A}) = 4\text{W}. \end{aligned}$$

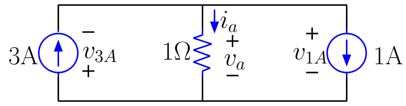
Since the absorbed power of the 6V voltage source is negative, the this is the source that is injecting energy to the circuit. Verifying that the sum of all absorbed powers equals zero:

$$p_{\text{absorved}} = p_a + p_b + p_{6\text{V}} + p_{4\text{V}} = 1\text{W} + 1\text{W} - 6\text{W} + 4\text{W} = 0\text{W}.$$

6. In the circuit pictured here, one of the independent current sources is injecting energy into the circuit, while the other one is absorbing energy. Identify the source that is injecting the energy absorbed in the circuit and confirm that the sum of all absorbed powers equals zero.



Solution:



The KCL in the top node can be written as

$$3\text{A} = i_a + 1\text{A},$$

yielding to

$$i_a = 2\text{A}.$$

Applying Ohm's law in the resistor gives

$$v_a = (1\Omega) \times i_a = 2\text{V},$$

which implies that

$$v_{1\text{A}} = -v_{3\text{A}} = v_a = 2\text{V}.$$

The absorbed powers can be obtained using $p = v_x i_x$,

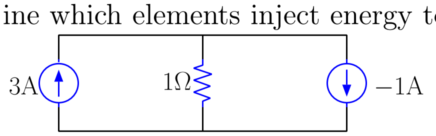
$$\begin{aligned} p_{1\Omega} &= v_a \times i_a = (2\text{V})(2\text{A}) = 4\text{W} \\ p_{3\text{A}} &= v_{3\text{A}} \times (3A) = (-2\text{V})(3\text{A}) = -6\text{W} \\ p_{1\text{A}} &= v_{1\text{A}} \times (1A) = (2\text{V})(1\text{A}) = 2\text{W}. \end{aligned}$$

Sum of all absorbed powers:

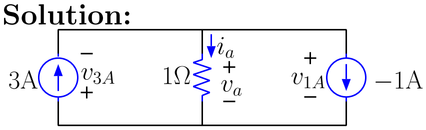
$$p_{\text{absorved}} = p_{1\Omega} + p_{3\text{A}} + p_{1\text{A}} = 4\text{W} - 6\text{W} + 2\text{W} = 0\text{W}.$$

The source injecting the energy is the 3A source.

7. Calculate the absorbed power for each element in the following circuit and determine which elements inject energy to the circuit.



Solution:



Applying KCL at the top node,

$$3\text{A} = i_a + (-1\text{A}) \Rightarrow i_a = 4\text{A}.$$

From Ohm's law in resistor we have

$$v_a = (1\Omega) \times i_a = 4\text{V}.$$

Therefore,

$$v_{-1\text{A}} = -v_{3\text{A}} = v_a = 2\text{V}.$$

The Absorbed powers for the different elements are

$$p_{1\Omega} = v_a \times i_a = (4\text{V})(4\text{A}) = 16\text{W}.$$

$$p_{3\text{A}} = v_{3\text{A}} \times 3\text{A} = (-4\text{V})(3\text{A}) = -12\text{W}$$

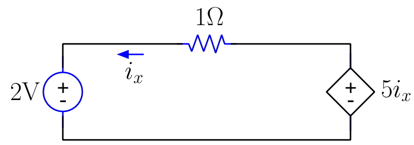
$$p_{-1\text{A}} = v_{-1\text{A}} \times (-1\text{A}) = (4\text{V})(-1\text{A}) = -4\text{W}$$

Sum of all absorbed powers

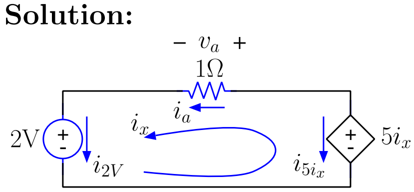
$$p_{\text{absorved}} = p_{1\Omega} + p_{3\text{A}} + p_{-1\text{A}} = 16\text{W} - 12\text{W} - 4\text{W} = 0\text{W}.$$

Both sources inject energy to the circuit.

8. In the circuit given, determine i_x and calculate the absorbed power for each circuit element. Which element is injecting the energy absorbed in the circuit?



Solution:

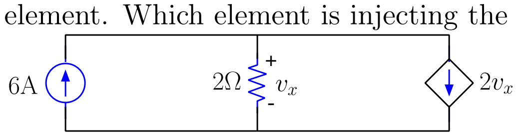


Applying KVL :

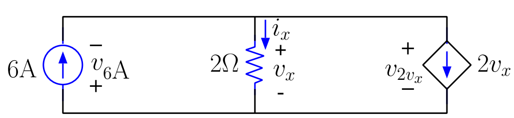
$$2 + (1) \times i - 5i_x = 0 \Rightarrow i_x = \frac{1}{2}\text{A}.$$

Absorbed powers:
$p_{2\text{V}} = v_{2\text{V}} \times i_{2\text{V}} = 2\text{V} \times \frac{1}{2}\text{A} = 1\text{W}$ (absorbs power)
$p_a = v_a \times i_a = 1\Omega \times i_a^2 = \frac{1}{4}\text{W}$ (absorbs power)
$p_{5i_x} = 5i_x \times i_{5i_x} = 5 \times \frac{1}{2}\text{V} \times (-\frac{1}{2}\text{A}) = -\frac{5}{4}\text{W}$ (injects power)
The dependent source is injecting the energy absorbed in the circuit.

9. In the circuit given, determine v_x and calculate the absorbed power for each circuit element. Which element is injecting the energy absorbed in the circuit?



Solution:



The KCL equation at the top node can be written as

$$6 = i_x + 2v_x,$$

but from Ohm's law in the resistor,

$$v_x = 2i_x.$$

Hence,

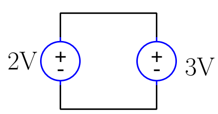
$$6 = i_x + 2 \times 2i_x \Rightarrow i_x = \frac{6}{5}\mathrm{A} = 1.2\,\mathrm{A},$$

and

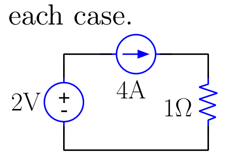
$$v_x = 2.4\,\mathrm{V}.$$

Then the absorbed powers are:
$p_{2\Omega} = v_x \times i_x = 2.4\mathrm{V} \times 1.2\,\mathrm{A} = 2.88\,\mathrm{W}$ (absorbs energy)
$p_{6\mathrm{A}} = v_{6\mathrm{A}} \times 6\mathrm{A} = -2.4\,\mathrm{V} \times 6\,\mathrm{A} = -14.4\,\mathrm{W}$ (injects energy)
$p_{2v_x} = v_{2v_x} \times (2v_x) = 2.4\,\mathrm{V} \times (4.8\,\mathrm{A}) = 11.52\,\mathrm{W}$ (absorbs energy)
The independent 6A current source is injecting the energy.

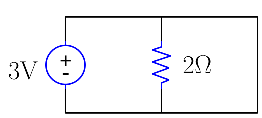
10. Some of the following circuits violate KVL/KCL and/or basic definitions of two-terminal elements given in Section 1.3. Identify these ill-specified circuits and explain the problem in each case.



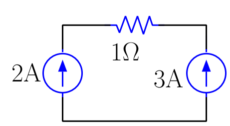
(a)



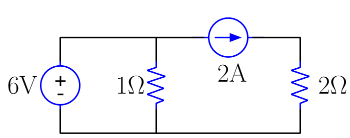
(b)



(c)



(d)



(e)

Solution:

a) This circuit violates KVL, top node cannot be both 2V and 3V!

b) Applying KVL

$$2\mathrm{V} - v_{4\mathrm{A}} - (4\mathrm{A})(1\Omega) = 0 \Rightarrow v_{4\mathrm{A}} = -2\mathrm{V}.$$

Circuit is correct.

c) Violates KVL : outer loop with source and short doesn't sum to zero!

d) Violates KCL : currents entering don't sum zero!

e) KVL on outer loop: $6\text{V} - v_{2\text{A}} - (2\text{A})(2\Omega) = 0 \Rightarrow v_{2\text{A}} = 2\text{V}$.
KCL at the top left node: $i_{6\text{V}} + \frac{6\text{V}}{1\Omega} + 2\text{A} = 0 \Rightarrow i_{6\text{V}} = -8\text{A}$. Circuit is correct.

11. a) Let $A = 3 - j3$. Express A in exponential form.

b) Let $B = -1 - j1$. Express B in exponential form.

c) Determine the magnitudes of $A + B$ and $A - B$.

d) Express AB and A/B in rectangular form.

Solution:

a) $A = 3 - j3$, $|A| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$, $\angle A = \arctan(\frac{-3}{3}) = -\frac{\pi}{4}$
$\therefore A = 3\sqrt{2}e^{-j\frac{\pi}{4}}$

b) $B = -1 - j1$, $|B| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$, $\angle B = \pi + \arctan(\frac{-1}{-1}) = \frac{5\pi}{4}$
recall π must be added because$\text{Re}\{B\} < 0$
$\therefore B = \sqrt{2}e^{j\frac{5\pi}{4}}$

c) $|A + B| = |3 - 3j - 1 - j1| = |2 - j4| = \sqrt{2^2 + (-4)^2} = 2\sqrt{5}$
$|A - B| = |3 - 3j + 1 + j1| = |4 - j2| = \sqrt{4^2 + (-2)^2} = 2\sqrt{5}$

d) $AB = (3\sqrt{2}e^{-j\frac{\pi}{4}})(\sqrt{2}e^{j\frac{5\pi}{4}}) = 6e^{j\pi} = -6$
$A/B = (3\sqrt{2}e^{-j\frac{\pi}{4}})/(\sqrt{2}e^{j\frac{5\pi}{4}}) = 3e^{-j\frac{3\pi}{2}} = j3$

12. Do Exercise 1.11(a) through (d), but with $A = -3 - j3$ and $B = 1 + j2$.
Solution:

a) $A = -3 - j3$, $|A| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$, $\angle A = \pi + arctan(\frac{-3}{-3}) = \frac{5\pi}{4}$
$\therefore A = 3\sqrt{2}e^{j\frac{5\pi}{4}}$

b) $B = 1 + j2$, $|B| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$, $\angle B = arctan(\frac{2}{1})$
$\therefore B = \sqrt{5}e^{j \cdot arctan(2)}$

c) $|A + B| = |-3 - j3 + 1 + j2| = |-2 - j1| = \sqrt{(-2)^2 + (-1)^2} = \sqrt{5}$
$|A - B| = |-3 - j3 - 1 - j2| = |-4 - j5| = \sqrt{(-4)^2 + (-5)^2} = \sqrt{41}$

d) $AB = (3\sqrt{2}e^{j\frac{5\pi}{4}})(\sqrt{5}e^{j \cdot arctan(2)}) = 3\sqrt{10}e^{j(\frac{5\pi}{4} + arctan(2))}$
$A/B = (3\sqrt{2}e^{j\frac{5\pi}{4}})/(\sqrt{5}e^{j \cdot arctan(2)}) = \frac{3}{5}\sqrt{10}e^{j(\frac{5\pi}{4} - arctan(2))}$

13. a) Determine the rectangular forms of e^{j0}, $e^{j\frac{\pi}{2}}$, $e^{-j\frac{\pi}{2}}$, $e^{j\pi}$, $e^{-j\pi}$, and $e^{j2\pi}$.

b) Simplify $P = e^{j\pi} + e^{-j\pi}$, $Q = e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}}$, $R = 1 - e^{j\pi}$.

c) Show that $e^{j\frac{\pi}{2}m} = (-1)^{\frac{m}{2}}$.

Solution:

a) $e^{j0} = \cos(0) + j\sin(0) = 1$
$e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j$
$e^{-j\frac{\pi}{2}} = \cos(-\frac{\pi}{2}) + j\sin(-\frac{\pi}{2}) = -j$
$e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1$
$e^{-j\pi} = \cos(-\pi) + j\sin(-\pi) = -1$
$e^{j2\pi} = \cos(2\pi) + j\sin(2\pi) = 1$

b) $P = e^{j\pi} + e^{-j\pi} = -1 - 1 = -2$
$Q = e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}} = j - j = 0$
$R = 1 - e^{j\pi} = 1 - (-1) = 2$

c) $e^{j\frac{\pi}{2}m} = (e^{j\pi})^{\frac{m}{2}} = (-1)^{\frac{m}{2}}$

14. a) Determine the rectangular forms of $7e^{j\frac{\pi}{4}}$, $7e^{-j\frac{\pi}{4}}$, $5e^{j\frac{3\pi}{4}}$, and $5e^{-j\frac{3\pi}{4}}$.

b) Simplify $P = 2e^{j\frac{5\pi}{4}} - 2e^{-j\frac{5\pi}{4}}$, $Q = 8e^{-j\frac{\pi}{4}} - 8e^{j\frac{\pi}{4}}$, and $R = \frac{e^{j\frac{3\pi}{4}}}{e^{-j\frac{\pi}{4}}}$.

Solution:

a) $7e^{j\frac{\pi}{4}} = 7(\cos(\frac{\pi}{4}) + j\sin(\frac{\pi}{4})) = \frac{7\sqrt{2}}{2} + j\frac{7\sqrt{2}}{2}$
$7e^{-j\frac{\pi}{4}} = 7(\cos(-\frac{\pi}{4}) + j\sin(-\frac{\pi}{4})) = \frac{7\sqrt{2}}{2} - j\frac{7\sqrt{2}}{2}$
$5e^{j\frac{3\pi}{4}} = 5(\cos(\frac{3\pi}{4}) + j\sin(\frac{3\pi}{4})) = -\frac{5\sqrt{2}}{2} + j\frac{5\sqrt{2}}{2}$
$5e^{-j\frac{3\pi}{4}} = 5(\cos(-\frac{3\pi}{4}) + j\sin(-\frac{3\pi}{4})) = -\frac{5\sqrt{2}}{2} - j\frac{5\sqrt{2}}{2}$

b) $P = 2e^{j\frac{5\pi}{4}} - 2e^{-j\frac{5\pi}{4}} = j4\sin(\frac{5\pi}{4}) = -j2\sqrt{2}$
$Q = 8e^{-j\frac{\pi}{4}} - 8e^{j\frac{\pi}{4}} = -j16\sin(\frac{\pi}{4}) = -j8\sqrt{2}$
$R = \frac{e^{j\frac{3\pi}{4}}}{e^{-j\frac{\pi}{4}}} = e^{j\pi} = -1$

15. a) Prove that $CC^* = |C|^2$.

b) Prove that $(C_1C_2)^* = C_1^*C_2^*$.

Solution:

a) Since $C = |C|e^{j\theta}$,
and $C^* = |C|e^{-j\theta}$,
then $CC^* = |C|^2$

b)

$$\begin{aligned}
(C_1C_2)^* &= (|C_1|e^{j\theta_1}|C_2|e^{j\theta_2})^* \\
&= (|C_1||C_2|e^{j(\theta_1+\theta_2)})^* \\
&= |C_1||C_2|e^{-j(\theta_1+\theta_2)} \\
&= |C_1|e^{-j\theta_1}|C_2|e^{-j\theta_2} \\
&= C_1^*C_2^*
\end{aligned}$$

16.

a) Prove that $|C_1C_2| = |C_1||C_2|$.

b) Prove that $|\frac{1}{C}| = \frac{1}{|C|}$.

c) Prove that $|\frac{C_1}{C_2}| = \frac{|C_1|}{|C_2|}$.

Solution:

a)

$$\begin{aligned} |C_1C_2| &= ||C_1|e^{j\theta_1}|C_2|e^{j\theta_2}| \\ &= |C_1||C_2||e^{j(\theta_1+\theta_2)}| \\ &= |C_1||C_2|. \end{aligned}$$

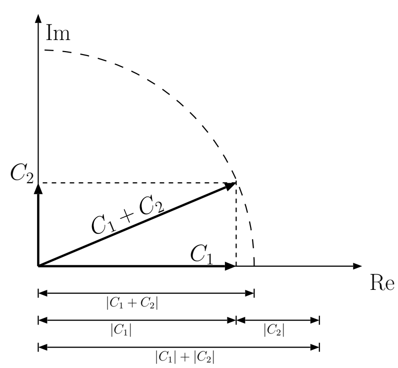
b)

$$\begin{aligned} |\frac{1}{C}| &= |\frac{1}{|C|e^{j\theta}}| \\ &= ||C|^{-1}e^{-j\theta}| \\ &= |C|^{-1}|e^{-j\theta}| \\ &= \frac{1}{|C|}. \end{aligned}$$

c)

$$\begin{aligned} |\frac{C_1}{C_2}| &= |\frac{|C_1|}{|C_2|}e^{j(\theta_1-\theta_2)}| \\ &= \frac{|C_1|}{|C_2|}|e^{j(\theta_1-\theta_2)}| \\ &= \frac{|C_1|}{|C_2|}. \end{aligned}$$

17. Show graphically on the complex plane that $|C_1 + C_2| \leq |C_1| + |C_2|$.
Solution:



Clearly we see that $|C_1| + |C_2|$ is greater than $|C_1 + C_2|$. This is just one example, but can be proven generally.

18.

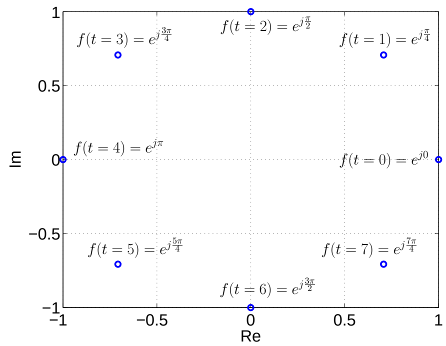
a) The function $f(t) = e^{j\frac{\pi}{4}t}$, for real-valued t, takes on complex values. Plot the values of $f(t)$ on the complex plane, for $t = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,$ and 7.

b) Repeat (a), but for the complex-valued function $g(t) = e^{(-\frac{1}{8}+j\frac{\pi}{4})t}$.

Solution:

a) Notice that $f(t)$ has a magnitude of $|f(t)| = 1$ for all t, then all the points will have the same distance from the origin, forming a circle.

t	$f(t)$
0	1
1	$0.7071 + j0.7071$
2	j
3	$-0.7071 + j0.7071$
4	-1
5	$-0.7071 - j0.7071$
6	$-j$
7	$0.7071 - j0.7071$



b) Notice that $g(t) = e^{(-\frac{1}{8}+j\frac{\pi}{4})t} = e^{-\frac{1}{8}t}f(t)$ decreases in magnitude as t increases, forming an spiral.

t	$g(t)$
0	1
1	$0.6240 + j0.6240$
2	$j0.7788$
3	$-0.4860 + j0.4860$
4	-0.6065
5	$-0.3785 - j0.3785$
6	$-j0.4724$
7	$0.2948 - j0.2948$