CHAPTER 1

1.1 You are given the following differential equation with the initial condition, $v(t = 0) = 0$,

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

Multiply both sides by m/c_d

$$\frac{m}{c_d}\frac{dv}{dt} = \frac{m}{c_d} g - v^2$$

Define $a = \sqrt{mg / c_d}$

$$\frac{m}{c_d}\frac{dv}{dt} = a^2 - v^2$$

Integrate by separation of variables,

$$\int \frac{dv}{a^2 - v^2} = \int \frac{c_d}{m} dt$$

A table of integrals can be consulted to find that

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \tanh^{-1} \frac{x}{a}$$

Therefore, the integration yields

$$\frac{1}{a} \tanh^{-1} \frac{v}{a} = \frac{c_d}{m} t + C$$

If $v = 0$ at $t = 0$, then because $\tanh^{-1}(0) = 0$, the constant of integration $C = 0$ and the solution is

$$\frac{1}{a} \tanh^{-1} \frac{v}{a} = \frac{c_d}{m} t$$

This result can then be rearranged to yield

$$v = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}} t\right)$$

1.2 (a) For the case where the initial velocity is positive (downward), Eq. (1.21) is

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

Multiply both sides by m/c_d

$$\frac{m}{c_d}\frac{dv}{dt} = \frac{m}{c_d} g - v^2$$

Define $a = \sqrt{mg / c_d}$

$$\frac{m}{c_d}\frac{dv}{dt} = a^2 - v^2$$

Integrate by separation of variables,

$$\int \frac{dv}{a^2 - v^2} = \int \frac{c_d}{m} dt$$

A table of integrals can be consulted to find that

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \tanh^{-1} \frac{x}{a}$$

Therefore, the integration yields

$$\frac{1}{a} \tanh^{-1} \frac{v}{a} = \frac{c_d}{m} t + C$$

If $v = +v_0$ at $t = 0$, then

$$C = \frac{1}{a} \tanh^{-1} \frac{v_0}{a}$$

Substitute back into the solution

$$\frac{1}{a} \tanh^{-1} \frac{v}{a} = \frac{c_d}{m} t + \frac{1}{a} \tanh^{-1} \frac{v_0}{a}$$

Multiply both sides by a, taking the hyperbolic tangent of each side and substituting a gives,

$$v = \sqrt{\frac{mg}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}} t + \tanh^{-1} \sqrt{\frac{c_d}{mg}} v_0 \right) \qquad (1)$$

(b) For the case where the initial velocity is negative (upward), Eq. (1.21) is

$$\frac{dv}{dt} = g + \frac{c_d}{m} v^2$$

Multiplying both sides of Eq. (1.8) by m/c_d and defining $a = \sqrt{mg / c_d}$ yields

$$\frac{m}{c_d}\frac{dv}{dt} = a^2 + v^2$$

Integrate by separation of variables,

$$\int \frac{dv}{a^2 + v^2} = \int \frac{c_d}{m} dt$$

A table of integrals can be consulted to find that

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

Therefore, the integration yields

$$\frac{1}{a} \tan^{-1} \frac{v}{a} = \frac{c_d}{m} t + C$$

The initial condition, $v(0) = v_0$ gives

$$C = \frac{1}{a} \tan^{-1} \frac{v_0}{a}$$

Substituting this result back into the solution yields

$$\frac{1}{a} \tan^{-1} \frac{v}{a} = \frac{c_d}{m} t + \frac{1}{a} \tan^{-1} \frac{v_0}{a}$$

Multiplying both sides by a and taking the tangent gives

$$v = a \tan\left(a \frac{c_d}{m} t + \tan^{-1} \frac{v_0}{a} \right)$$

or substituting the values for a and simplifying gives

$$v = \sqrt{\frac{mg}{c_d}} \tan\left(\sqrt{\frac{gc_d}{m}} t + \tan^{-1} \sqrt{\frac{c_d}{mg}} v_0 \right) \qquad (2)$$

(c) We use Eq. (2) until the velocity reaches zero. Inspection of Eq. (2) indicates that this occurs when the argument of the tangent is zero. That is, when

$$\sqrt{\frac{gc_d}{m}} t_{zero} + \tan^{-1} \sqrt{\frac{c_d}{mg}} v_0 = 0$$

The time of zero velocity can then be computed as

$$t_{zero} = -\sqrt{\frac{m}{gc_d}} \tan^{-1} \sqrt{\frac{c_d}{mg}} v_0$$

Thereafter, the velocities can then be computed with Eq. (1.9),

$$v = \sqrt{\frac{mg}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}}(t - t_{zero})\right) \qquad (3)$$

Here are the results for the parameters from Example 1.2, with an initial velocity of –40 m/s.

$$t_{zero} = -\sqrt{\frac{68.1}{9.81(0.25)}} \tan^{-1}\left(\sqrt{\frac{0.25}{68.1(9.81)}}(-40)\right) = 3.470239 \text{ s}$$

Therefore, for $t = 2$, we can use Eq. (2) to compute

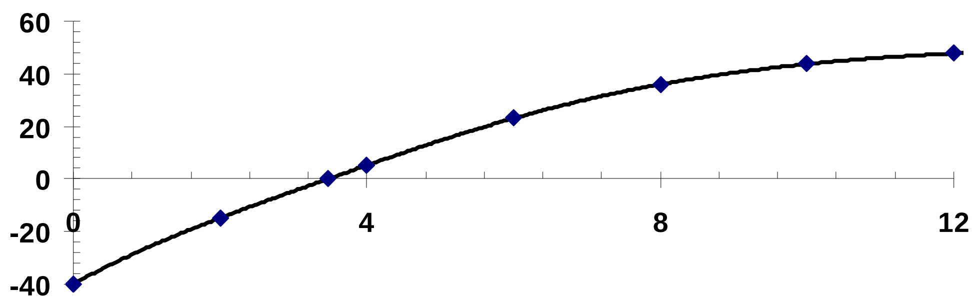
$$v = \sqrt{\frac{68.1(9.81)}{0.25}} \tan\left(\sqrt{\frac{9.81(0.25)}{68.1}}(2) + \tan^{-1}\sqrt{\frac{0.25}{68.1(9.81)}}(-40)\right) = -14.8093 \frac{\text{m}}{\text{s}}$$

For $t = 4$, the jumper is now heading downward and Eq. (3) applies

$$v = \sqrt{\frac{68.1(9.81)}{0.25}} \tanh\left(\sqrt{\frac{9.81(0.25)}{68.1}}(4 - 3.470239)\right) = 5.17952 \frac{\text{m}}{\text{s}}$$

The same equation is then used to compute the remaining values. The results for the entire calculation are summarized in the following table and plot:

***t* (s)**	***v* (m/s)**
0	-40
2	-14.8093
3.470239	0
4	5.17952
6	23.07118
8	35.98203
10	43.69242
12	47.78758



1.3 (*a*) This is a transient computation. For the period ending June 1:

Balance = Previous Balance + Deposits – Withdrawals + Interest

Balance = 1512.33 + 220.13 – 327.26 + 0.01(1512.33) = 1420.32

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Interest	Balance
1-May				$1,512.33
	$220.13	$327.26	$15.12	
1-Jun				$1,420.32
	$216.80	$378.61	$14.20	
1-Jul				$1,272.72
	$450.25	$106.80	$12.73	
1-Aug				$1,628.89
	$127.31	$350.61	$16.29	
1-Sep				**$1,421.88**

(*b*) $\dfrac{dB}{dt} = D(t) - W(t) + iB$

(*c*) for $t = 0$ to 0.5:

$$\frac{dB}{dt} = 220.13 - 327.26 + 0.01(1512.33) = -92.01$$
$$B(0.5) = 1512.33 - 92.01(0.5) = 1466.33$$

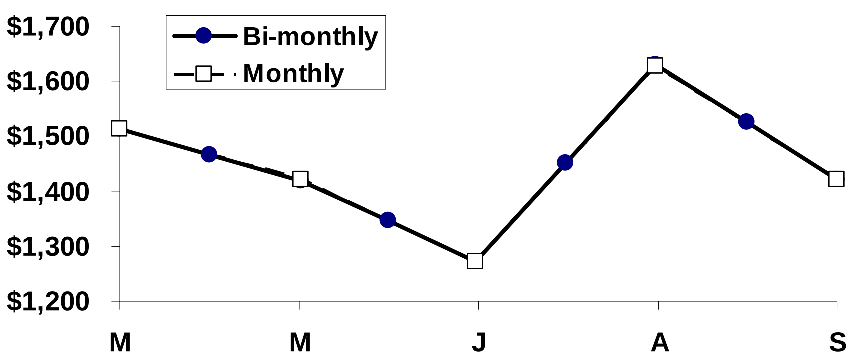
for $t = 0.5$ to 1:

$$\frac{dB}{dt} = 220.13 - 327.260 + 0.01(1466.33) = -92.47$$
$$B(0.5) = 1466.33 - 92.47(0.5) = 1420.09$$

The balances for the remainder of the periods can be computed in a similar fashion as tabulated below:

Date	Deposit	Withdrawal	Interest	*dB/dt*	Balance
1-May	$220.13	$327.26	$15.12	-$92.01	$1,512.33
16-May	$220.13	$327.26	$14.66	-$92.47	$1,466.33
1-Jun	$216.80	$378.61	$14.20	-$147.61	$1,420.09
16-Jun	$216.80	$378.61	$13.46	-$148.35	$1,346.29
1-Jul	$450.25	$106.80	$12.72	$356.17	$1,272.12
16-Jul	$450.25	$106.80	$14.50	$357.95	$1,450.20
1-Aug	$127.31	$350.61	$16.29	-$207.01	$1,629.18
16-Aug	$127.31	$350.61	$15.26	-$208.04	$1,525.67
1-Sep					**$1,421.65**

(*d*) As in the plot below, the results of the two approaches are very close.



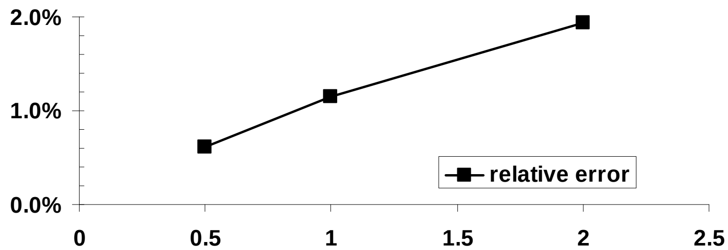
1.4 At $t = 12$ s, the analytical solution is 50.6175 (Example 1.1). The numerical results are:

step	v(12)	absolute relative error
2	51.6008	1.94%
1	51.2008	1.15%
0.5	50.9259	0.61%

where the relative error is calculated with

$$\text{absolute relative error} = \left|\frac{\text{analytical} - \text{numerical}}{\text{analytical}}\right| \times 100\%$$

The error versus step size can be plotted as



Thus, halving the step size approximately halves the error.

1.5 (*a*) The force balance is

$$\frac{dv}{dt} = g - \frac{c'}{m} v$$

Applying Laplace transforms,

$$sV - v(0) = \frac{g}{s} - \frac{c'}{m} V$$

Solve for

$$V = \frac{g}{s(s + c'/m)} + \frac{v(0)}{s + c'/m} \qquad (1)$$

The first term to the right of the equal sign can be evaluated by a partial fraction expansion,

$$\frac{g}{s(s + c'/m)} = \frac{A}{s} + \frac{B}{s + c'/m} \qquad (2)$$

$$\frac{g}{s(s + c'/m)} = \frac{A(s + c'/m) + Bs}{s(s + c'/m)}$$

Equating like terms in the numerators yields

$$A + B = 0$$
$$g = \frac{c'}{m} A$$

Therefore,

$$A = \frac{mg}{c'} \qquad B = -\frac{mg}{c'}$$

These results can be substituted into Eq. (2), and the result can be substituted back into Eq. (1) to give

$$V = \frac{mg / c'}{s} - \frac{mg / c'}{s + c'/ m} + \frac{v(0)}{s + c'/ m}$$

Applying inverse Laplace transforms yields

$$v = \frac{mg}{c'} - \frac{mg}{c'} e^{-(c'/m)t} + v(0)e^{-(c'/m)t}$$

or

$$v = v(0)e^{-(c'/m)t} + \frac{mg}{c'}\left(1 - e^{-(c'/m)t}\right)$$

where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, $v(0) = 0$, so the final solution is

$$\boxed{v = \frac{mg}{c'}\left(1 - e^{-(c'/m)t}\right)}$$

Alternative solution: Another way to obtain solutions is to use separation of variables,

$$\int \frac{1}{g - \frac{c'}{m} v} dv = \int dt$$

The integrals can be evaluated as

$$-\frac{\ln\left(g - \frac{c'}{m} v\right)}{c'/ m} = t + C$$

where C = a constant of integration, which can be evaluated by applying the initial condition

$$C = -\frac{\ln\left(g - \frac{c'}{m} v(0)\right)}{c'/ m}$$

which can be substituted back into the solution

$$-\frac{\ln\left(g-\frac{c'}{m}v\right)}{c'/m}=t-\frac{\ln\left(g-\frac{c'}{m}v(0)\right)}{c'/m}$$

This result can be rearranged algebraically to solve for *v*,

$$v=v(0)e^{-(c'/m)t}+\frac{mg}{c'}\left(1-e^{-(c'/m)t}\right)$$

where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, $v(0) = 0$, so the final solution is

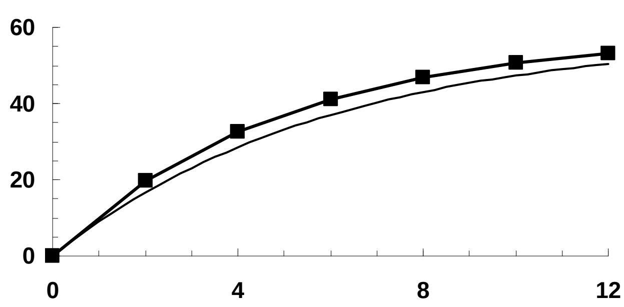
$$\boxed{v=\frac{mg}{c'}\left(1-e^{-(c'/m)t}\right)}$$

(b) The numerical solution can be implemented as

$$v(2)=0+\left[9.81-\frac{12.5}{68.1}(0)\right]2=19.62$$
$$v(4)=19.62+\left[9.81-\frac{12.5}{68.1}(19.62)\right]2=32.0374$$

The computation can be continued and the results summarized and plotted as:

t	*v*	*dv/dt*
0	0	9.81
2	19.6200	6.4968
4	32.6136	4.3026
6	41.2187	2.8494
8	46.9176	1.8871
10	50.6917	1.2497
12	53.1911	0.8276
∞	58.0923	



Note that the analytical solution is included on the plot for comparison.

1.6 $v(t)=\frac{gm}{c'}(1-e^{-(c'/m)\,t})$

jumper #1: $v(t)=\frac{9.81(70)}{12}(1-e^{-(12/70)\,9})=44.99204\,\frac{\text{m}}{\text{s}}$

jumper #2: $44.99204 = \frac{9.81(80)}{15}(1 - e^{-(15/80)\,t})$

$$44.99204 = 52.32 - 52.32e^{-0.1875\,t}$$
$$0.14006 = e^{-0.1875\,t}$$
$$t = \frac{\ln 0.14006}{-0.1875} = 10.4836 \text{ s}$$

1.7 Note that the differential equation should be formulated as

$$\frac{dv}{dt} = g - \frac{c_d}{m} v|v|$$

This ensures that the sign of the drag is correct when the parachutist has a negative upward velocity. Before the chute opens ($t < 10$), Euler's method can be implemented as

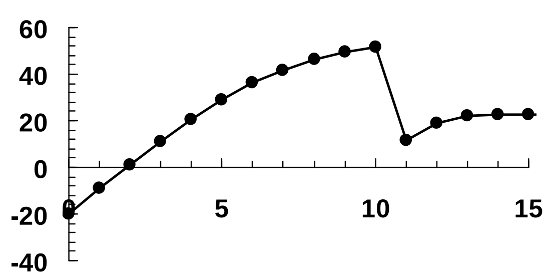
$$v(t + \Delta t) = v(t) + \left[9.81 - \frac{0.25}{80} v|v|\right]\Delta t$$

After the chute opens ($t \geq 10$), the drag coefficient is changed and the implementation becomes

$$v(t + \Delta t) = v(t) + \left[9.81 - \frac{1.5}{80} v|v|\right]\Delta t$$

Here is a summary of the results along with a plot:

Chute closed			Chute opened		
t	*v*	*dv/dt*	*t*	*v*	*dv/dt*
0	-20.0000	11.0600	10	51.5260	-39.9698
1	-8.9400	10.0598	11	11.5561	7.3060
2	1.1198	9.8061	12	18.8622	3.1391
3	10.9258	9.4370	13	22.0013	0.7340
4	20.3628	8.5142	14	22.7352	0.1183
5	28.8770	7.2041	15	22.8535	0.0172
6	36.0812	5.7417	16	22.8707	0.0025
7	41.8229	4.3439	17	22.8732	0.0003
8	46.1668	3.1495	18	22.8735	0.0000
9	49.3162	2.2097	19	22.8736	0.0000
			20	22.8736	0.0000



1.8 (a) The first two steps are

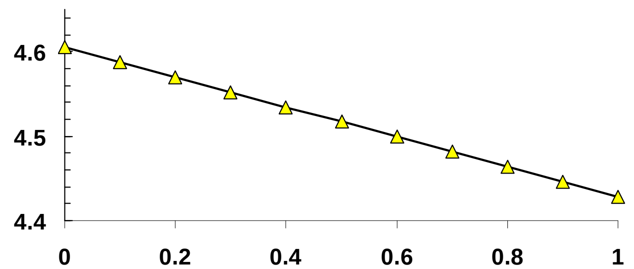
$$c(0.1) = 100 - 0.175(10)0.1 = 98.25 \text{ Bq/L}$$

$c(0.2) = 98.25 - 0.175(98.25)0.1 = 96.5306$ Bq/L

The process can be continued to yield

t	*c*	*dc/dt*
0	100.0000	-17.5000
0.1	98.2500	-17.1938
0.2	96.5306	-16.8929
0.3	94.8413	-16.5972
0.4	93.1816	-16.3068
0.5	91.5509	-16.0214
0.6	89.9488	-15.7410
0.7	88.3747	-15.4656
0.8	86.8281	-15.1949
0.9	85.3086	-14.9290
1	83.8157	-14.6678

(b) The results when plotted on a semi-log plot yields a straight line



The slope of this line can be estimated as

$$\frac{\ln(83.8157) - \ln(100)}{1} = -0.17655$$

Thus, the slope is approximately equal to the negative of the decay rate. If we had used a smaller step size, the result would be more exact.

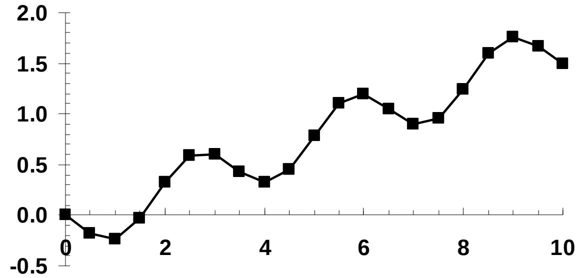
1.9 The first two steps yield

$$y(0.5) = 0 + \left[3\frac{450}{1250}\sin^2(0) - \frac{450}{1250}\right]0.5 = \ 0 + (-0.36)\ 0.5 = -0.18$$
$$y(1) = -0.18 + \left[3\frac{450}{1250}\sin^2(0.5) - \frac{450}{1250}\right]0.5 = \ -0.18 + (-0.11176)\ 0.5 = -0.23508$$

The process can be continued to give the following table and plot:

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.36000	5.5	1.10271	0.17761
0.5	-0.18000	-0.11176	6	1.19152	-0.27568
1	-0.23588	0.40472	6.5	1.05368	-0.31002
1.5	-0.03352	0.71460	7	0.89866	0.10616
2	0.32378	0.53297	7.5	0.95175	0.59023
2.5	0.59026	0.02682	8	1.24686	0.69714
3	0.60367	-0.33849	8.5	1.59543	0.32859
3.5	0.43443	-0.22711	9	1.75972	-0.17657

4	0.32087	0.25857	9.5	1.67144	-0.35390
4.5	0.45016	0.67201	10	1.49449	-0.04036
5	0.78616	0.63310			



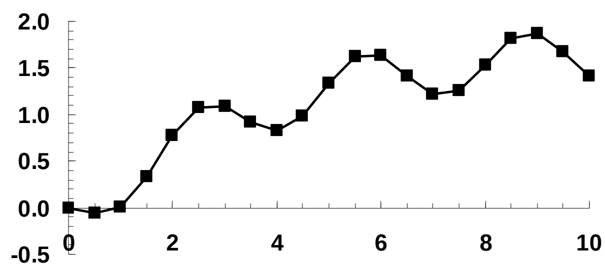
1.10 The first two steps yield

$$y(0.5) = 0 + \left[3\frac{450}{1250}\sin^2(0) - \frac{150(1+0)^{1.5}}{1250}\right]0.5 = 0 - 0.12(0.5) = \ -0.06$$

$$y(1) = -0.06 + \left[3\frac{450}{1250}\sin^2(0.5) - \frac{150(1-0.06)^{1.5}}{1250}\right]0.5 = -0.06 + 0.13887(0.5) = 0.00944$$

The process can be continued to give

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.12000	5.5	1.61981	0.02876
0.5	-0.06000	0.13887	6	1.63419	-0.42872
1	0.00944	0.64302	6.5	1.41983	-0.40173
1.5	0.33094	0.89034	7	1.21897	0.06951
2	0.77611	0.60892	7.5	1.25372	0.54423
2.5	1.08058	0.02669	8	1.52584	0.57542
3	1.09392	-0.34209	8.5	1.81355	0.12227
3.5	0.92288	-0.18708	9	1.87468	-0.40145
4	0.82934	0.32166	9.5	1.67396	-0.51860
4.5	0.99017	0.69510	10	1.41465	-0.13062
5	1.33772	0.56419			



1.11 When the water level is above the outlet pipe, the volume balance can be written as

$$\frac{dV}{dt} = 3\sin^2(t) - 3(y - y_{\text{out}})^{1.5}$$

In order to solve this equation, we must relate the volume to the level. To do this, we recognize that the volume of a cone is given by $V = \pi r^2 y/3$. Defining the side slope as $s = y_{top}/r_{top}$, the radius can be related to the level ($r = y/s$) and the volume can be reexpressed as

$$V = \frac{\pi}{3s^2} y^3$$

which can be solved for

$$y = \sqrt[3]{\frac{3s^2 V}{\pi}} \qquad (1)$$

and substituted into the volume balance

$$\frac{dV}{dt} = 3\sin^2(t) - 3\left(\sqrt[3]{\frac{3s^2 V}{\pi}} - y_{\text{out}}\right)^{1.5} \qquad (2)$$

For the case where the level is below the outlet pipe, outflow is zero and the volume balance simplifies to

$$\frac{dV}{dt} = 3\sin^2(t) \qquad (3)$$

These equations can then be used to solve the problem. Using the side slope of $s = 4/2.5 = 1.6$, the initial volume can be computed as

$$V(0) = \frac{\pi}{3(1.6)^2} 0.8^3 = 0.20944 \text{ m}^3$$

For the first step, $y < y_{out}$ and Eq. (3) gives

$$\frac{dV}{dt}(0) = 3\sin^2(0) = 0$$

and Euler's method yields

$$V(0.5) = V(0) + \frac{dV}{dt}(0)\Delta t = 0.20944 + 0(0.5) = 0.20944$$

For the second step, Eq. (3) still holds and

$$\frac{dV}{dt}(0.5) = 3\sin^2(0.5) = 0.689547$$

$$V(1) = V(0.5) + \frac{dV}{dt}(0.5)\Delta t = 0.20944 + 0.689547(0.5) = 0.554213$$

Equation (1) can then be used to compute the new level,

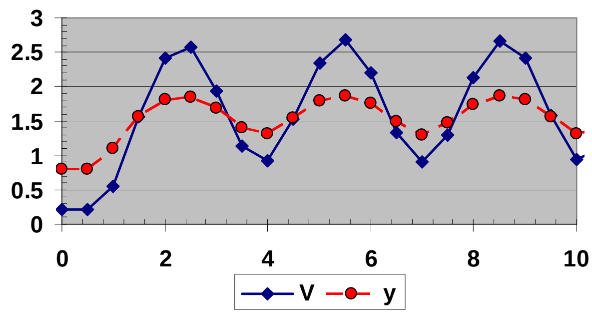
$$y = \sqrt[3]{\frac{3(1.6)^2(0.554213)}{\pi}} = 1.106529 \text{ m}$$

Because this level is now higher than the outlet pipe, Eq. (2) holds for the next step

$$\frac{dV}{dt}(1) = 2.12422 - 3(1.106529 - 1)^{1.5} = 2.019912$$
$$V(1.5) = 0.554213 + 2.019912(0.5) = 2.984989$$

The remainder of the calculation is summarized in the following table and figure.

t	Q_{in}	***V***	***y***	Q_{out}	***dV/dt***
0	0	0.20944	0.8	0	0
0.5	0.689547	0.20944	0.8	0	0.689547
1	2.12422	0.554213	1.106529	0.104309	2.019912
1.5	2.984989	1.564169	1.563742	1.269817	1.715171
2	2.480465	2.421754	1.809036	2.183096	0.29737
2.5	1.074507	2.570439	1.845325	2.331615	-1.25711
3	0.059745	1.941885	1.680654	1.684654	-1.62491
3.5	0.369147	1.12943	1.40289	0.767186	-0.39804
4	1.71825	0.93041	1.31511	0.530657	1.187593
4.5	2.866695	1.524207	1.55031	1.224706	1.641989
5	2.758607	2.345202	1.78977	2.105581	0.653026
5.5	1.493361	2.671715	1.869249	2.431294	-0.93793
6	0.234219	2.202748	1.752772	1.95937	-1.72515
6.5	0.13883	1.340173	1.48522	1.013979	-0.87515
7	1.294894	0.902598	1.301873	0.497574	0.79732
7.5	2.639532	1.301258	1.470703	0.968817	1.670715
8	2.936489	2.136616	1.735052	1.890596	1.045893
8.5	1.912745	2.659563	1.866411	2.419396	-0.50665
9	0.509525	2.406237	1.805164	2.167442	-1.65792
9.5	0.016943	1.577279	1.568098	1.284566	-1.26762
10	0.887877	0.943467	1.321233	0.5462	0.341677



1.12

$$Q_{\text{students}} = 35 \text{ ind} \times 80 \frac{\text{J}}{\text{ind s}} \times 20 \text{ min} \times 60 \frac{\text{s}}{\text{min}} \times \frac{\text{kJ}}{1000 \text{ J}} = 3{,}360 \text{ kJ}$$

$$m = \frac{PV\text{Mwt}}{RT} = \frac{(101.325 \text{ kPa})(11\text{m} \times 8\text{m} \times 3\text{m} - 35 \times 0.075 \text{ m}^3)(28.97 \text{ kg/kmol})}{(8.314 \text{ kPa m}^3/(\text{kmol K})((20+273.15)\text{K})} = 314.796 \text{ kg}$$

$$\Delta T = \frac{Q_{\text{students}}}{mC_v} = \frac{3{,}360 \text{ kJ}}{(314.796 \text{ kg})(0.718 \text{ kJ/(kg K)})} = 14.86571 \text{ K}$$

Therefore, the final temperature is 20 + 14.86571 = 34.86571°C.

1.13 $\sum \mathrm{M_{in}} - \sum \mathrm{M_{out}} = 0$

$$\text{Food} + \text{Drink} + \text{Air In} + \text{Metabolism} = \text{Urine} + \text{Skin} + \text{Feces} + \text{Air Out} + \text{Sweat}$$
$$\text{Drink} = \text{Urine} + \text{Skin} + \text{Feces} + \text{Air Out} + \text{Sweat} - \text{Food} - \text{Air In} - \text{Metabolism}$$
$$\text{Drink} = 1.4 + 0.35 + 0.2 + 0.4 + 0.3 - 1 - 0.05 - 0.3 = 1.3 \text{ L}$$

1.14 (a) The force balance can be written as:

$$m\frac{dv}{dt} = -mg(0)\frac{R^2}{(R+x)^2} + c_d v|v|$$

Dividing by mass gives

$$\frac{dv}{dt} = -g(0)\frac{R^2}{(R+x)^2} + \frac{c_d}{m} v|v|$$

(b) Recognizing that $dx/dt = v$, the chain rule is

$$\frac{dv}{dt} = v\frac{dv}{dx}$$

Setting drag to zero and substituting this relationship into the force balance gives

$$\frac{dv}{dx} = -\frac{g(0)}{v}\frac{R^2}{(R+x)^2}$$

(c) Using separation of variables

$$v\ dv = -g(0)\frac{R^2}{(R+x)^2}dx$$

Integrating gives

$$\frac{v^2}{2} = g(0)\frac{R^2}{R+x} + C$$

Applying the initial condition yields

$$\frac{v_0^2}{2} = g(0)\frac{R^2}{R+0} + C$$

which can be solved for $C = v_0^2/2 - g(0)R$, which can be substituted back into the solution to give

$$\frac{v^2}{2} = g(0)\frac{R^2}{R+x} + \frac{v_0^2}{2} - g(0)R$$

or

$$v = \pm\sqrt{v_0^2 + 2g(0)\frac{R^2}{R+x} - 2g(0)R}$$

Note that the plus sign holds when the object is moving upwards and the minus sign holds when it is falling.

(d) Euler's method can be developed as

$$v(x_{i+1}) = v(x_i) + \left[-\frac{g(0)}{v(x_i)}\frac{R^2}{(R+x_i)^2}\right](x_{i+1} - x_i)$$

The first step can be computed as

$$v(10,000) = 1,500 + \left[-\frac{9.81}{1,500}\frac{(6.37\times10^6)^2}{(6.37\times10^6+0)^2}\right](10,000-0) = 1,500 + (-0.00654)10,000 = 1434.600$$

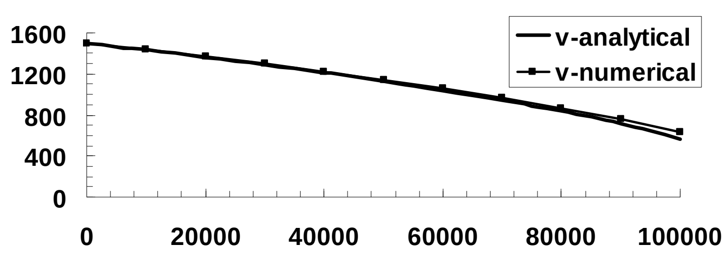
The remainder of the calculations can be implemented in a similar fashion as in the following table

x	*v*	*dv/dx*	*v*-analytical
0	1500.000	-0.00654	1500.000
10000	1434.600	-0.00682	1433.216
20000	1366.433	-0.00713	1363.388
30000	1295.089	-0.00750	1290.023
40000	1220.049	-0.00794	1212.475
50000	1140.643	-0.00847	1129.884
60000	1055.973	-0.00912	1041.049
70000	964.798	-0.00995	944.206
80000	865.317	-0.01106	836.579
90000	754.742	-0.01264	713.299
100000	628.359	-0.01513	564.197

For the analytical solution, the value at 10,000 m can be computed as

$$v = \sqrt{1,500^2 + 2(9.81)\frac{(6.37\times10^6)^2}{(6.37\times10^6+10,000)} - 2(9.81)(6.37\times10^6)} = 1433.216$$

The remainder of the analytical values can be implemented in a similar fashion as in the last column of the above table. The numerical and analytical solutions can be displayed graphically.



1.15 The volume of the droplet is related to the radius as

$$V = \frac{4\pi r^3}{3} \tag{1}$$

This equation can be solved for radius as

$$r = \sqrt[3]{\frac{3V}{4\pi}} \tag{2}$$

The surface area is

$$A = 4\pi r^2 \tag{3}$$

Equation (2) can be substituted into Eq. (3) to express area as a function of volume

$$A = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$$

This result can then be substituted into the original differential equation,

$$\frac{dV}{dt} = -k4\pi\left(\frac{3V}{4\pi}\right)^{2/3} \tag{4}$$

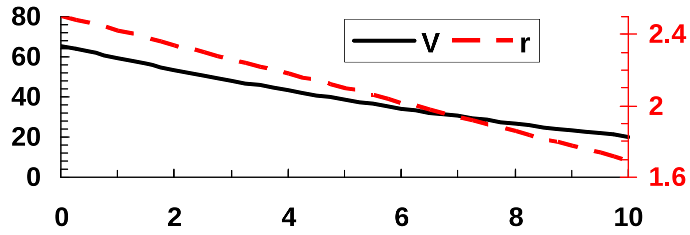
The initial volume can be computed with Eq. (1),

$$V = \frac{4\pi r^3}{3} = \frac{4\pi(2.5)^3}{3} = 65.44985 \text{ mm}^3$$

Euler's method can be used to integrate Eq. (4). Here are the beginning and last steps

t	***V***	***dV/dt***
0	65.44985	-6.28319
0.25	63.87905	-6.18225
0.5	62.33349	-6.08212
0.75	60.81296	-5.98281
1	59.31726	-5.8843
•		
•		
•		
9	23.35079	-3.16064
9.25	22.56063	-3.08893
9.5	21.7884	-3.01804
9.75	21.03389	-2.94795
10	20.2969	-2.87868

A plot of the results is shown below. We have included the radius on this plot (dashed line and right scale):



Eq. (2) can be used to compute the final radius as

$$r = \sqrt[3]{\frac{3(20.2969)}{4\pi}} = 1.692182$$

Therefore, the average evaporation rate can be computed as

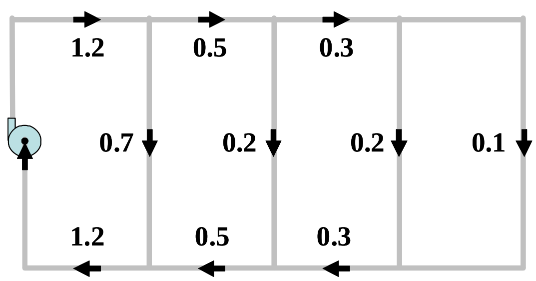
$$k = \frac{(2.5 - 1.692182)\text{ mm}}{10\text{ min}} = 0.080782\,\frac{\text{mm}}{\text{min}}$$

which is approximately equal to the given evaporation rate of 0.08 mm/min.

1.16 Continuity at the nodes can be used to determine the flows as follows:

$$Q_1 = Q_2 + Q_3 = 0.7 + 0.5 = 1.2\ \text{m}^3/\text{s}$$
$$Q_{10} = Q_1 = 1.2\ \text{m}^3/\text{s}$$
$$Q_9 = Q_{10} - Q_2 = 1.2 - 0.7 = 0.5\ \text{m}^3/\text{s}$$
$$Q_4 = Q_9 - Q_8 = 0.5 - 0.3 = 0.2\ \text{m}^3/\text{s}$$
$$Q_5 = Q_3 - Q_4 = 0.5 - 0.2 = 0.3\ \text{m}^3/\text{s}$$
$$Q_6 = Q_5 - Q_7 = 0.3 - 0.1 = 0.2\ \text{m}^3/\text{s}$$

Therefore, the final results are

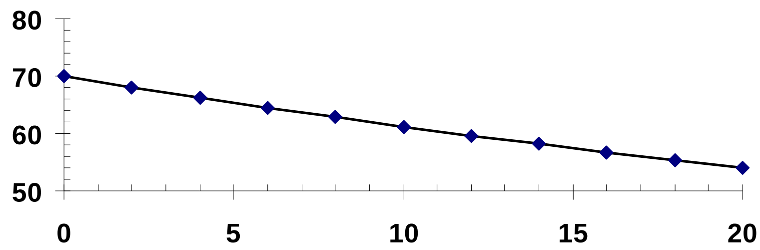


1.17 The first two steps can be computed as

$$T(1) = 70 + \left[-0.019(70 - 20)\right]\ 2 = 68 + (-0.95)2 = 68.1$$
$$T(2) = 68.1 + \left[-0.019(68.1 - 20)\right]\ 2 = 68.1 + (-0.9139)2 = 66.2722$$

The remaining results are displayed below along with a plot of the results.

t	T	dT/dt	t	T	dT/dt
0	70.00000	-0.95000	12.00000	59.62967	-0.75296
2	68.10000	-0.91390	14.00000	58.12374	-0.72435
4	66.27220	-0.87917	16.00000	56.67504	-0.69683
6	64.51386	-0.84576	18.00000	55.28139	-0.67035
8	62.82233	-0.81362	20.00000	53.94069	-0.64487
10	61.19508	-0.78271			



1.18 (a) For the constant temperature case, Newton's law of cooling is written as

$$\frac{dT}{dt} = -0.135(T - 10)$$

The first two steps of Euler's methods are

$$T(0.5) = T(0) - \frac{dT}{dt}(0) \times \Delta t = 37 + 0.12(10 - 37)(0.5) = 37 - 3.2400 \times 0.50 = 35.3800$$
$$T(1) = 35.3800 + 0.12(10 - 35.3800)(0.5) = 35.3800 - 3.0456 \times 0.50 = 33.8572$$

The remaining calculations are summarized in the following table:

t	T_a	T	dT/dt
0:00	10	37.0000	-3.2400
0:30	10	35.3800	-3.0456
1:00	10	33.8572	-2.8629
1:30	10	32.4258	-2.6911
2:00	10	31.0802	-2.5296
2:30	10	29.8154	-2.3778
3:00	10	28.6265	-2.2352
3:30	10	27.5089	-2.1011
4:00	10	26.4584	-1.9750
4:30	10	25.4709	-1.8565
5:00	10	24.5426	-1.7451

(b) For this case, the room temperature can be represented as

$$T_a = 20 - 2t$$

where t = time (hrs). Newton's law of cooling is written as

$$\frac{dT}{dt} = -0.12(T - 20 + 2t)$$

The first two steps of Euler's methods are

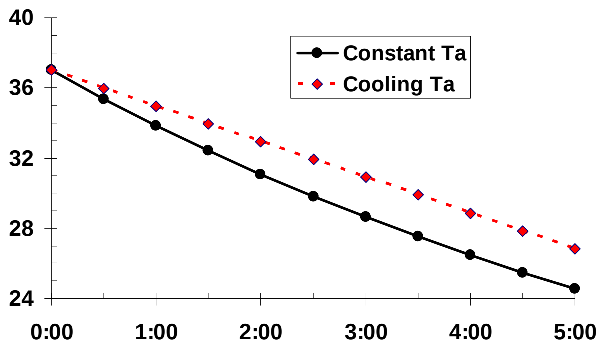
$$T(0.5) = 37 + 0.12(20 - 37)(0.5) = 37 - 2.040 \times 0.50 = 35.9800$$
$$T(1) = 35.9800 + 0.12(19 - 35.9800)(0.5) = 35.9800 - 2.0376 \times 0.50 = 34.9612$$

The remaining calculations are summarized in the following table:

t	T_a	***T***	***dT/dt***
0:00	20	37.0000	-2.0400
0:30	19	35.9800	-2.0376
1:00	18	34.9612	-2.0353
1:30	17	33.9435	-2.0332
2:00	16	32.9269	-2.0312
2:30	15	31.9113	-2.0294
3:00	14	30.8966	-2.0276
3:30	13	29.8828	-2.0259
4:00	12	28.8699	-2.0244
4:30	11	27.8577	-2.0229
5:00	10	26.8462	-2.0215

Comparison with (a) indicates that the effect of the room air temperature has a significant effect on the expected temperature at the end of the 5-hr period (difference = 26.8462 – 24.5426 = 2.3036°C).

(c) The solutions for (a) Constant T_a, and (b) Cooling T_a are plotted below:



1.19 The two equations to be solved are

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$
$$\frac{dx}{dt} = v$$

Euler's method can be applied for the first step as

$$v(2) = v(0) + \frac{dv}{dt}(0)\Delta t = 0 + \left(9.81 - \frac{0.25}{68.1}(0)^2\right)(2) = 19.6200$$
$$x(2) = x(0) + \frac{dx}{dt}(0)\Delta t = 0 + 0(2) = 0$$

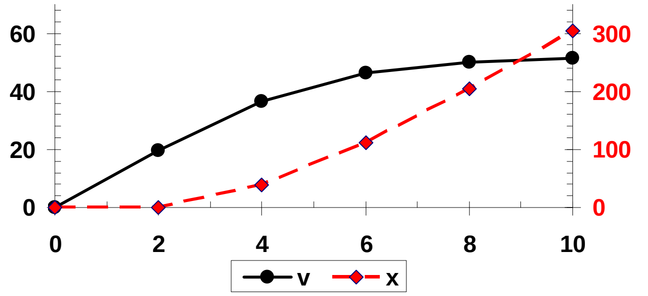
For the second step:

$$v(4) = v(2) + \frac{dv}{dt}(2)\Delta t = 19.6200 + \left(9.81 - \frac{0.25}{68.1}(19.6200)^2\right)(2) = 19.6200 + 8.3968(2) = 36.4137$$

$$x(4) = x(2) + \frac{dx}{dt}(2)\Delta t = 0 + 19.6200(2) = 39.2400$$

The remaining steps can be computed in a similar fashion as tabulated and plotted below:

t	*x*	*v*	*dx/dt*	*dv/dt*
0	0.0000	0.0000	0.0000	9.8100
2	0.0000	19.6200	19.6200	8.3968
4	39.2400	36.4137	36.4137	4.9423
6	112.0674	46.2983	46.2983	1.9409
8	204.6640	50.1802	50.1802	0.5661
10	305.0244	51.3123	51.3123	0.1442



1.20 (a) The force balance with buoyancy can be written as

$$m\frac{dv}{dt} = mg - \frac{1}{2}\rho v|v| AC_d - \rho Vg$$

Divide both sides by mass,

$$\frac{dv}{dt} = g\left(1 - \frac{\rho V}{m}\right) - \frac{\rho AC_d}{2m} v|v|$$

(b) For a sphere, the mass is related to the volume as in $m = \rho_s V$ where ρ_s = the sphere's density (kg/m^3). Substituting this relationship gives

$$\frac{dv}{dt} = g\left(1 - \frac{\rho}{\rho_s}\right) - \frac{\rho AC_d}{2\rho_s V} v|v|$$

The formulas for the volume and projected area can be substituted to give

$$\frac{dv}{dt} = g\left(1 - \frac{\rho}{\rho_s}\right) - \frac{3\rho C_d}{4\rho_s d} v|v|$$

(c) At steady state ($dv/dt = 0$),

$$g\left(\frac{\rho_s - \rho}{\rho_s}\right) = \frac{3\rho C_d}{4\rho_s d} v^2$$

which can be solved for the terminal velocity

$$v = \sqrt{\frac{4gd}{3C_d}\left(\frac{\rho_s - \rho}{\rho}\right)}$$

(d) Before implementing Euler's method, the parameters can be substituted into the differential equation to give

$$\frac{dv}{dt} = 9.81\left(1 - \frac{1000}{2700}\right) - \frac{3(1000)0.47}{4(2700)(0.01)}v^2 = 6.176667 - 13.055556v^2$$

The first two steps for Euler's method are

$$v(0.03125) = 0 + (6.176667 - 13.055556(0)^2)0.03125 = 0.193021$$
$$v(0.0625) = 0.193021 + (6.176667 - 13.055556(0.193021)^2)0.03125 = 0.370841$$

The remaining steps can be computed in a similar fashion as tabulated and plotted below:

t	*v*	*dv/dt*
0	0.000000	6.176667
0.03125	0.193021	5.690255
0.0625	0.370841	4.381224
0.09375	0.507755	2.810753
0.125	0.595591	1.545494
0.15625	0.643887	0.763953
0.1875	0.667761	0.355136
0.21875	0.678859	0.160023
0.25	0.683860	0.071055