Chapter 1

Analyzing Algorithms and Problems: Principles and Examples

Section 1.2: *Java as an Algorithm Language*

1.1

It is correct for instance fields whose type is an inner class to be declared before that inner class (as in Figure 1.2 in the text) or after (as here). Appendix A.7 gives an alternative to spelling out all the instance fields in the copy methods (functions).

```
class  Personal
    {
    public static class  Name
        {
        String  firstName;
        String  middleName;
        String  lastName;
        public static Name copy(Name n)
            {
            Name  n2;
            n2.firstName = n.firstName;
            n2.middleName = n.middleName;
            n2.lastName = n.lastName;
            return n2;
            }
        }

    public static class  Address
        {
        String  street;
        String  city;
        String  state;
        public static Address copy(Address a) {/* similar to Name.copy() */ }
        }

    public static class  PhoneNumber
        {
        int  areaCode;
        int  prefix;
        int  number;
        public static PhoneNumber copy(PhoneNumber n) {/* similar to Name.copy() */ }
        }

    Name  name;
    Address  address;
    PhoneNumber  phone;
    String  eMail;

    public static Personal copy(Personal p);
        {
        Personal  p2;
        p2.name = Name.copy(p.name);
        p2.address = Address.copy(p.address);
        p2.phone = PhoneNumber.copy(p.phone);
        p2.eMail = p.eMail;
        return p2;
        }
    }
```

Section 1.3: *Mathematical Background*

1.2

For $0 < k < n$, we have

$$\binom{n-1}{k} = \frac{(n-1)!}{k!(n-1-k)!} = \frac{(n-1)!(n-k)}{k!(n-k)!}$$

$$\binom{n-1}{k-1} = \frac{(n-1)!}{(k-1)!(n-k)!} = \frac{(n-1)!(k)}{k!(n-k)!}$$

Add them giving:

$$\frac{(n-1)!(n)}{k!(n-k)!} = \binom{n}{k}$$

For $0 < n \leq k$ we use the fact that $\binom{a}{b} = 0$ whenever $a < b$. (There is no way to choose more elements than there are in the whole set.) Thus $\binom{n-1}{k} = 0$ in all these cases. If $n < k$, $\binom{n-1}{k-1}$ and $\binom{n}{k}$ are both 0, confirming the equation. If $n = k$, $\binom{n-1}{k-1}$ and $\binom{n}{k}$ are both 1, again confirming the equation. (We need the fact that $0! = 1$ when $n = k = 1$.)

1.4

It suffices to show:

$$\log_c x \log_b c = \log_b x.$$

Consider b raised to each side.

$$b^{\text{left side}} = b^{\log_b c \log_c x} = (b^{\log_b c})^{\log_c x} = c^{\log_c x} = x$$

$$b^{\text{right side}} = b^{\log_b x} = x$$

So left side = right side.

1.6

Let $x = \lceil \lg(n+1) \rceil$. The solution is based on the fact that $2^{x-1} < n+1 \leq 2^x$.

```
x = 0;
twoToTheX = 1;
while (twoToTheX < n+1)
     x += 1;
     twoToTheX *= 2;
return x;
```

The values computed by this procedure for small n and the approximate values of $\lg(n+1)$ are:

n	x	$\lg(n+1)$
0	0	0.0
1	1	1.0
2	2	1.6
3	2	2.0
4	3	2.3
5	3	2.6
6	3	2.8
7	3	3.0
8	4	3.2
9	4	3.3

1.8

$$Pr(S \mid T) = \frac{Pr(S \text{ and } T)}{Pr(T)} = \frac{Pr(S)Pr(T)}{Pr(T)} = Pr(S)$$

The second equation is similar.

1.10

We know $A < B$ and $D < C$. By direct counting:

$$Pr(A < C \mid A < B \text{ and } D < C) = \frac{Pr(A < C \text{ and } A < B \text{ and } D < C)}{Pr(A < B \text{ and } D < C)} = \frac{5/24}{6/24} = \frac{5}{6}$$

$$Pr(A < D \mid A < B \text{ and } D < C) = \frac{Pr(A < D < C \text{ and } A < B)}{Pr(A < B \text{ and } D < C)} = \frac{3/24}{6/24} = \frac{3}{6} = \frac{1}{2}$$

$$Pr(B < C \mid A < B \text{ and } D < C) = \frac{Pr(A < B < C \text{ and } D < C)}{Pr(A < B \text{ and } D < C)} = \frac{3/24}{6/24} = \frac{3}{6} = \frac{1}{2}$$

$$Pr(B < D \mid A < B \text{ and } D < C) = \frac{Pr(A < B < D < C)}{Pr(A < B \text{ and } D < C)} = \frac{1/24}{6/24} = \frac{1}{6}$$

1.12

We assume that the probability of each coin being chosen is 1/3, that the probability that it shows “heads” after being flipped is 1/2 and that the probability that it shows “tails” after being flipped is 1/2. Call the coins A, B, and C. Define the elementary events, each having probability 1/6, as follows.

AH	A is chosen and flipped and comes out “heads”.
AT	A is chosen and flipped and comes out “tails”.
BH	B is chosen and flipped and comes out “heads”.
BT	B is chosen and flipped and comes out “tails”.
CH	C is chosen and flipped and comes out “heads”.
CT	C is chosen and flipped and comes out “tails”.

a) BH and CH cause a majority to be “heads”, so the probability is 1/3.

b) No event causes a majority to be “heads”, so the probability is 0.

c) AH, BH, CH and CT cause a majority to be “heads”, so the probability is 2/3.

1.13

The entry in row i, column j is the probability that D_i will beat D_j.

$$\begin{pmatrix} - & \frac{22}{36} & \frac{18}{36} & \frac{12}{36} \\ \frac{12}{36} & - & \frac{22}{36} & \frac{16}{36} \\ \frac{18}{36} & \frac{12}{36} & - & \frac{22}{36} \\ \frac{22}{36} & \frac{20}{36} & \frac{12}{36} & - \end{pmatrix}$$

Note that D_1 beats D_2, D_2 beats D_3, D_3 beats D_4, and D_4 beats D_1.

1.15

The proof is by induction on n, the upper limit of the sum. The base case is $n = 0$. Then $\sum_{i=1}^{0} i^2 = 0$, and $\frac{2n^3+3n^2+n}{6} = 0$. So the equation holds for the base case. For $n > 0$, assume the formula holds for $n - 1$.

$$\begin{aligned}\sum_{i=1}^{n} i^2 &= \sum_{i=1}^{n-1} i^2 + n^2 = \frac{2(n-1)^3 + 3(n-1)^2 + n - 1}{6} + n^2 \\ &= \frac{2n^3 - 6n^2 + 6n - 2 + 3n^2 - 6n + 3 + n - 1}{6} + n^2 \\ &= \frac{2n^3 - 3n^2 + n}{6} + \frac{6n^2}{6} = \frac{2n^3 + 3n^2 + n}{6}\end{aligned}$$

1.18

Consider any two reals $w < z$. We need to show that $f(w) \leq f(z)$; that is, $f(z) - f(w) \geq 0$. Since $f(x)$ is differentiable, it is continuous. We call upon the *Mean Value Theorem* (sometimes called the *Theorem of the Mean*), which can be found in any college calculus text. By this theorem there is some point y, such that $w < y < z$, for which

$$f'(y) = \frac{(f(z) - f(w))}{(z - w)}.$$

By the hypothesis of the lemma, $f'(y) \geq 0$. Also, $(z - w) > 0$. Therefore, $f(z) - f(w) \geq 0$.

1.20

Let $\equiv$ abbreviate the phrase, "is logically equivalent to". We use the identity $\neg\neg A \equiv A$ as needed.

$$\begin{aligned}\neg(\forall x(A(x) \Rightarrow B(x))) &\equiv \exists x \neg(A(x) \Rightarrow B(x)) && \text{(by Eq. 1.24)} \\ &\equiv \exists x \neg(\neg A(x) \vee B(x)) && \text{(by Eq. 1.21)} \\ &\equiv \exists x (A(x) \wedge \neg B(x)) && \text{(by DeMorgan's law, Eq. 1.23).}\end{aligned}$$

Section 1.4: *Analyzing Algorithms and Problems*

1.22

The total number of operations in the worst case is $4n + 2$; they are:

Comparisons involving K:	n
Comparisons involving **`index`**:	$n+1$
Additions:	n
Assignments to **`index`**:	$n+1$

1.23

a)

```
if (a < b)
     if (b < c)
          median = b;
     else if (a < c)
          median = c;
     else
          median = a;
else if (a < c)
     median = a;
else if (b < c)
     median = c;
else
     median = b;
```