Chapter 1 Algebra and Equations

Section 1.1 The Real Numbers

1. True. This statement is true, since every integer can be written as the ratio of the integer and 1. For example, $5 = \frac{5}{1}$.

2. False. For example, 5 is a real number, and $5 = \frac{10}{2}$ which is not an irrational number.

3. Answers vary with the calculator, but $\frac{2,508,429,787}{798,458,000}$ is the best.

4. $0 + (-7) = -7 + 0$
This illustrates the commutative property of addition.

5. $6(t + 4) = 6t + 6 \cdot 4$
This illustrates the distributive property.

6. $3 + (-3) = (-3) + 3$
This illustrates the commutative property of addition.

7. $-5 + 0 = -5$
This illustrates the identity property of addition.

8. $(-4)\bullet(\frac{-1}{4}) = 1$
This illustrates the multiplicative inverse property.

9. $8 + (12 + 6) = (8 + 12) + 6$
This illustrates the associative property of addition.

10. $1\bullet(-20) = -20$
This illustrates the identity property of multiplication.

11. Answers vary. One possible answer: The sum of a number and its additive inverse is the additive identity. The product of a number and its multiplicative inverse is the multiplicative identity.

12. Answers vary. One possible answer: When using the commutative property, the order of the addends or multipliers is changed, while the grouping of the addends or multipliers is changed when using the associative property.

For Exercises 13–16, let $p = -2$, $q = 3$ and $r = -5$.

13. $-3(p + 5q) = -3[-2 + 5(3)] = -3[-2 + 15]$
$= -3(13) = -39$

14. $2(q - r) = 2(3 + 5) = 2(8) = 16$

15. $\frac{q + r}{q + p} = \frac{3 + (-5)}{3 + (-2)} = \frac{-2}{1} = -2$

16. $\frac{3q}{3p - 2r} = \frac{3(3)}{3(-2) - 2(-5)} = \frac{9}{-6 + 10} = \frac{9}{4}$

17. Let $r = 3.8$.
$\text{APR} = 12r = 12(3.8) = 45.6\%$

18. Let $r = 0.8$.
$\text{APR} = 12r = 12(0.8) = 9.6\%$

19. Let $APR = 11$.
$APR = 12r$
$11 = 12r$
$\frac{11}{12} = r$
$r \approx .9167\%$

20. Let $APR = 13.2$.
$APR = 12r$
$13.2 = 12r$
$\frac{13.2}{12} = r$
$r = 1.1\%$

21. $3 - 4 \cdot 5 + 5 = 3 - 20 + 5 = -17 + 5 = -12$

22. $8 - (-4)^2 - (-12)$
Take powers first.
$8 - 16 - (-12)$
Then add and subtract in order from left to right.
$8 - 16 + 12 = -8 + 12 = 4$

23. $(4 - 5) \cdot 6 + 6 = -1 \cdot 6 + 6 = -6 + 6 = 0$

24. $\dfrac{2(3-7)+4(8)}{4(-3)+(-3)(-2)}$

Work above and below fraction bar. Do multiplications and work inside parentheses.

$=\dfrac{2(-4)+32}{-12+6}=\dfrac{-8+32}{-12+6}=\dfrac{24}{-6}=-4$

25. $8-4^2-(-12)$

Take powers first.

$8-16-(-12)$

Then add and subtract in order from left to right.

$8-16+12=-8+12=4$

26. $-(3-5)-\left[2-\left(3^2-13\right)\right]$

Take powers first.

$-(3-5)-[2-(9-13)]$

Work inside brackets and parentheses.

$-(-2)-[2-(-4)]=2-[2+4]$

$=2-6=-4$

27. $\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{\left(-\sqrt{16}\right)}}{\sqrt{64}-1}$

Work above and below fraction bar. Take roots.

$\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{(-4)}}{8-1}$

Do multiplications and divisions.

$\dfrac{-6-\frac{3}{2}+\frac{1}{2}}{8-1}$

Add and subtract.

$\dfrac{-\frac{12}{2}-\frac{3}{2}+\frac{1}{2}}{7}=\dfrac{-\frac{14}{2}}{7}=\dfrac{-7}{7}=-1$

28. $\dfrac{6^2-3\sqrt{25}}{\sqrt{6^2+13}}$

Take powers and roots.

$\dfrac{36-3(5)}{\sqrt{36+13}}=\dfrac{36-15}{\sqrt{49}}=\dfrac{21}{7}=3$

29. $\dfrac{2040}{523}, \dfrac{189}{37}, \sqrt{27}, \dfrac{4587}{691}, 6.735, \sqrt{47}$

30. $\dfrac{187}{63}, 2.9884, \sqrt{\sqrt{85}}, \pi, \sqrt{10}, \dfrac{385}{117}$

31. 12 is less than 18.5.

$12<18.5$

32. –2 is greater than –20.

$-2>-20$

33. x is greater than or equal to 5.7.

$x\ge 5.7$

34. y is less than or equal to –5.

$y\le -5$

35. z is at most 7.5.

$z\le 7.5$

36. w is negative.

$w<0$

37. $-6<-2$

38. $3/4=.75$

39. $3.14<\pi$

40. $1/3>.33$

41. a lies to the right of b or is equal to b.

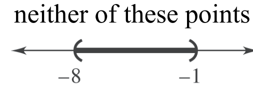
42. $b+c=a$

43. $c<a<b$

44. a lies to the right of 0

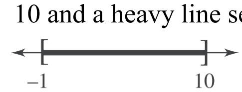
45. $(-8,-1)$

This represents all real numbers between –8 and –1, not including –8 and –1. Draw parentheses at –8 and –1 and a heavy line segment between them. The parentheses at –8 and –1 show that neither of these points belongs to the graph.



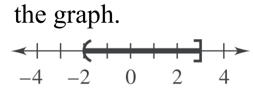
46. $[-1, 10]$

This represents all real numbers between –1 and 10, including –1 and 10. Draw brackets at –1 and 10 and a heavy line segment between them.



47. $(-2,3]$

This represents all real numbers x such that $-2<x\le 3$. Draw a heavy line segment from –2 to 3. Use a parenthesis at –2 since it is not part of the graph. Use a bracket at 3 since it is part of the graph.

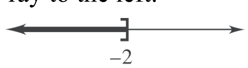


48. $[-2, 2)$
This represents all real numbers between –2 and 2, including –2, not including 2.
Draw a bracket at –2, a parenthesis at 2, and a heavy line segment between them.

49. $(-2, \infty)$

This represents all real numbers x such that $x > -2$. Start at –2 and draw a heavy line segment to the right. Use a parenthesis at –2 since it is not part of the graph.

50. $(-\infty, -2]$
This represents all real numbers less than or equal to –2. Draw a bracket at –2 and a heavy ray to the left.



51. $|-9| - |-12| = 9 - (12) = -3$

52. $|8| - |-4| = 8 - (4) = 4$

53. $-|-4| - |-1 - 14| = -(4) - |-15|$
$= -(4) - 15 = -19$

54. $-|6| - |-12 - 4| = -(6) - |-16| = -6 - (16) = -22$

55. $|5| \underline{\quad} |-5|$
$5 \underline{\quad} 5$
$5 = 5$

56. $-|-4| \underline{\quad} |4|$
$-4 \underline{\quad} 4$
$-4 < 4$

57. $|10 - 3| \underline{\quad} |3 - 10|$
$|7| \underline{\quad} |-7|$
$7 \underline{\quad} 7$
$7 = 7$

58. $|6 - (-4)| \underline{\quad} |-4 - 6|$
$|10| \underline{\quad} |-10|$
$10 \underline{\quad} 10$
$10 = 10$

59. $|-2 + 8| \underline{\quad} |2 - 8|$
$|6| \underline{\quad} |-6|$
$6 \underline{\quad} 6$
$6 = 6$

60. $|3| \cdot |-5| \underline{\quad} |3(-5)|$
$|3| \cdot |-5| \underline{\quad} |-15|$
$3 \cdot 5 \underline{\quad} 15$
$15 = 15$

61. $|3 - 5| \underline{\quad} |3| - |5|$
$|-2| \underline{\quad} 3 - 5$
$2 \underline{\quad} -2$
$2 > -2$

62. $|-5 + 1| \underline{\quad} |-5| + |1|$
$|-4| \underline{\quad} 5 + 1$
$4 \underline{\quad} 6$
$4 < 6$

63. When $a < 7$, $a - 7$ is negative.
So $|a - 7| = -(a - 7) = 7 - a$.

64. When $b \geq c$, $b - c$ is positive.
So $|b - c| = b - c$.

Answers will vary for exercises 65–67. Sample answers are given.

65. No, it is not always true that $|a + b| = |a| + |b|$. For example, let $a = 1$ and $b = -1$. Then,
$|a + b| = |1 + (-1)| = |0| = 0$, but
$|a| + |b| = |1| + |(-1)| = 1 + 1 = 2$.

66. Yes, if a and b are any two real numbers, it is always true that $|a - b| = |b - a|$. In general, $a - b = -(b - a)$. When we take the absolute value of each side, we get
$|a - b| = |-(b - a)| = |b - a|$.

67. $|2-b| = |2+b|$ only when $b = 0$. Then each side of the equation is equal to 2. If b is any other value, subtracting it from 2 and adding it to 2 will produce two different values.

68. For females: $|x - 63.5| \le 8.4$; for males: $|x - 68.9| \le 9.3$

69. 1; 2007

70. 8; 2003, 2004, 2005, 2008, 2009, 2010, 2011, 2012

71. 9; 2003, 2004, 2005, 2006, 2008, 2009, 2010, 2011, 2012

72. 2; 2008, 2010

73. 7; 2003, 2004, 2005, 2006, 2007, 2009, 2011

74. 9; 2003, 2004, 2005, 2006, 2007, 2009, 2010, 2011, 2012

75. $|3.4 - (-46.5)| = |49.9| = 49.9$

76. $|4.4 - (-10.8)| = |15.2| = 15.2$

77. $|0.6 - (4.4)| = |-3.8| = 3.8$

78. $|36.5 - (0.6)| = |35.9| = 35.9$

79. $|-10.8 - (-46.5)| = |35.7| = 35.7$

80. $|4.4 - (3.4)| = |1.0| = 1.0$

81. 5; 2005, 2006, 2008, 2010, 2011

82. 3; 2008, 2010, 2011

83. 5; 2007, 2008, 2009, 2010, 2011

84. 4; 2005, 2006, 2007, 2009

Section 1.2 Polynomials

1. $11.2^6 \approx 1{,}973{,}822.685$

2. $(-6.54)^{11} \approx -936{,}171{,}103.1$

3. $\left(-\frac{18}{7}\right)^6 \approx 289.0991339$

4. $\left(\frac{5}{9}\right)^7 \approx .0163339967$

5. -3^2 is negative, whereas $(-3)^2$ is positive. Both -3^3 and $(-3)^3$ are negative.

6. To multiply 4^3 and 4^5, add the exponents since the bases are the same. The product of 4^3 and 3^4 cannot be found in the same way since the bases are different. To evaluate the product, first do the powers, and then multiply the results.

7. $4^2 \cdot 4^3 = 4^{2+3} = 4^5$

8. $(-4)^4 \cdot (-4)^6 = (-4)^{4+6} = (-4)^{10}$

9. $(-6)^2 \cdot (-6)^5 = (-6)^{2+5} = (-6)^7$

10. $(2z)^5 \cdot (2z)^6 = (2z)^{5+6} = (2z)^{11}$

11. $\left[(5u)^4\right]^7 = (5u)^{4\cdot 7} = (5u)^{28}$

12.
$$(6y)^3 \cdot \left[(6y)^5\right]^4 = (6y)^3 \cdot (6y)^{20} = (6y)^{23}$$

13. degree 4; coefficients: 6.2, –5, 4, –3, 3.7; constant term 3.7.

14. degree 7; coefficients: 6, 4, 0, 0, –1, 0, 1, 0; constant term 0.

15. Since the highest power of x is 3, the degree is 3.

16. Since the highest power of x is 5, the degree is 5.

17.
$$\begin{aligned}&(3x^3 + 2x^2 - 5x) + (-4x^3 - x^2 - 8x)\\ &= (3x^3 - 4x^3) + (2x^2 - x^2) + (-5x - 8x)\\ &= -x^3 + x^2 - 13x\end{aligned}$$

18.
$$\begin{aligned}&(-2p^3 - 5p + 7) + (-4p^2 + 8p + 2)\\ &= -2p^3 - 4p^2 + (-5p + 8p) + (7 + 2)\\ &= -2p^3 - 4p^2 + 3p + 9\end{aligned}$$

19. $\left(-4y^2-3y+8\right)-\left(2y^2-6y+2\right)$
$=\left(-4y^2-3y+8\right)+\left(-2y^2+6y-2\right)$
$=-4y^2-3y+8-2y^2+6y-2$
$=\left(-4y^2-2y^2\right)+(-3y+6y)+(8-2)$
$=-6y^2+3y+6$

20. $\left(7b^2+2b-5\right)-\left(3b^2+2b-6\right)$
$=\left(7b^2+2b-5\right)+\left(-3b^2-2b+6\right)$
$=\left(7b^2-3b^2\right)+(2b-2b)+(-5+6)$
$=4b^2+1$

21. $\left(2x^3+2x^2+4x-3\right)-\left(2x^3+8x^2+1\right)$
$=\left(2x^3+2x^2+4x-3\right)+\left(-2x^3-8x^2-1\right)$
$=2x^3+2x^2+4x-3-2x^3-8x^2-1$
$=\left(2x^3-2x^3\right)+\left(2x^2-8x^2\right)+(4x)+(-3-1)$
$=-6x^2+4x-4$

22. $\left(3y^3+9y^2-11y+8\right)-\left(-4y^2+10y-6\right)$
$=\left(3y^3+9y^2-11y+8\right)+\left(4y^2-10y+6\right)$
$=3y^3+\left(9y^2+4y^2\right)+(-11y-10y)+(8+6)$
$=3y^3+13y^2-21y+14$

23. $-9m\left(2m^2+6m-1\right)$
$=(-9m)\left(2m^2\right)+(-9m)(6m)+(-9m)(-1)$
$=-18m^3-54m^2+9m$

24. $2a\left(4a^2-6a+8\right)$
$=2a\left(4a^2\right)+2a(-6a)+2a(8)$
$=8a^3-12a^2+16a$

25. $(3z+5)\left(4z^2-2z+1\right)$
$=(3z)\left(4z^2-2z+1\right)+(5)\left(4z^2-2z+1\right)$
$=12z^3-6z^2+3z+20z^2-10z+5$
$=12z^3+14z^2-7z+5$

26. $(2k+3)\left(4k^3-3k^2+k\right)$
$=2k\left(4k^3-3k^2+k\right)+3\left(4k^3-3k^2+k\right)$
$=8k^4-6k^3+2k^2+12k^3-9k^2+3k$
$=8k^4+6k^3-7k^2+3k$

27. $(6k-1)(2k+3)$
$=(6k)(2k+3)+(-1)(2k+3)$
$=12k^2+18k-2k-3$
$=12k^2+16k-3$

28. $(8r+3)(r-1)$
Use FOIL.
$=8r^2-8r+3r-3$
$=8r^2-5r-3$

29. $(3y+5)(2y+1)$
Use FOIL.
$=6y^2+3y+10y+5$
$=6y^2+13y+5$

30. $(5r-3s)(5r-4s)$
$=25r^2-20rs-15rs+12s^2$
$=25r^2-35rs+12s^2$

31. $(9k+q)(2k-q)$
$=18k^2-9kq+2kq-q^2$
$=18k^2-7kq-q^2$

32. $(.012x-.17)(.3x+.54)$
$=(.012x)(.3x)+(.012x)(.54)$
$+(-.17)(.3x)+(-.17)(.54)$
$=.0036x^2+.00648x-.051x-.0918$
$=.0036x^2-.04452x-.0918$

33. $(6.2m-3.4)(.7m+1.3)$
$=4.34m^2+8.06m-2.38m-4.42$
$=4.34m^2+5.68m-4.42$

34. $2p-3[4p-(8p+1)]$
$=2p-3(4p-8p-1)$
$=2p-3(-4p-1)$
$=2p+12p+3$
$=14p+3$

35. $5k-[k+(-3+5k)]$
$=5k-[6k-3]$
$=5k-6k+3$
$=-k+3$

36. $(3x-1)(x+2)-(2x+5)^2$
$$=\left(3x^2+5x-2\right)-\left(4x^2+20x+25\right)$$
$$=3x^2+5x-2-4x^2-20x-25$$
$$=\left(3x^2-4x^2\right)+(5x-20x)+(-2-25)$$
$$=-x^2-15x-27$$

37. $R = 5\,(1000x) = 5000x$
$C = 200{,}000 + 1800x$
$P = (5000x) - (200{,}000 + 1800x)$
$= 3200x - 200{,}000$

38. $R = 8.50(1000x) = 8500x$
$C = 225{,}000 + 4200x$
$P = (8500x) - (225{,}000 + 4200x)$
$= 4300x - 225{,}000$

39. $R = 9.75(1000x) = 9750x$
$$C = 260,000 + (-3x^2 + 3480x - 325)$$
$$= -3x^2 + 3480x + 259,675$$
$$P = (9750x) - (-3x^2 + 3480x + 259,675)$$
$$= 3x^2 + 6270x - 259,675$$

40. $R = 23.50(1000x) = 23{,}500x$
$$C = 145,000 + (-4.2x^2 + 3220x - 425)$$
$$= -4.2x^2 + 3220x + 144,575$$
$$P = (23,500x) - (-4.2x^2 + 3220x + 144,575)$$
$$= 4.2x^2 + 20,280x - 144,575.$$

41. **a.** According to the bar graph, the net earnings in 2001 were $265,000,000.

b. Let $x = 3$.
$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(3)^4 + 50.0(3)^3 - 576(3)^2 + 2731(3) - 4027$$
$$= 212.12$$
According to the polynomial, the net earnings in 200 were approximately $212,000,000.

42. **a.** According to the bar graph, the net earnings in 2007 were $673,000,000.

b. Let $x = 7$.
$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(7)^4 + 50.0(7)^3 - 576(7)^2 + 2731(7) - 4027$$
$$= 462.52$$
According to the polynomial, the net earnings in 2007 were approximately $462,520,000.

43. **a.** According to the bar graph, the net earnings in 2010 were $948,000,000.

b. Let $x = 10$.
$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(10)^4 + 50.0(10)^3 - 576(10)^2 + 2731(10) - 4027$$
$$= 883$$
According to the polynomial, the net earnings in 2010 were approximately $883,000,000.

44. **a.** According to the bar graph, the net earnings in 2012 were $1,385,000,000.

b. Let $x = 12$.
$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(12)^4 + 50.0(12)^3 - 576(12)^2 + 2731(12) - 4027$$
$$= 1511.72$$
According to the polynomial, the net earnings in 20012 were approximately $1,511,720,000.

45. Let $x = 13$.
$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(13)^4 + 50.0(13)^3 - 576(13)^2 + 2731(13) - 4027$$
$$= 1711.72$$
According to the polynomial, the net earnings in 2013 will be approximately $1,711,720,000.

46. Let $x = 14$.

$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(14)^4 + 50.0(14)^3 - 576(14)^2 + 2731(14) - 4027$$
$$= 1655.32$$

According to the polynomial, the net earnings in 2014 will be approximately $1,655,320,000.

47. Let $x = 15$.

$$-1.48x^4 + 50.0x^3 - 576x^2 + 2731x - 4027$$
$$= -1.48(15)^4 + 50.0(15)^3 - 576(15)^2 + 2731(15) - 4027$$
$$= 1163$$

According to the polynomial, the net earnings in 2015 will be approximately $1,163,000,000.

48. The figures for 2013 – 2015 seem high, but plausible. To see how accurate these conclusions are, search Starbucks.com for later annual reports.

For exercises 49–52, we use the polynomial $-.0057x^4 + .157x^3 - 1.43x^2 + 5.14x + 6.3$.

49. Let $x = 4$.

$$-.0057(4)^4 + .157(4)^3 - 1.43(4)^2 + 5.14(4) + 6.3$$
$$= 12.5688$$

Thus, there were approximately 12.6% below the poverty line in 2004. The statement is false.

50. Let $x = 10$.

$$-.0057(10)^4 + .157(10)^3 - 1.43(10)^2 + 5.14(10) + 6.3$$
$$= 14.7$$

Thus, there were approximately 14.7% below the poverty line in 2010. The statement is true.

51. Let $x = 3$.

$$-.0057(3)^4 + .157(3)^3 - 1.43(3)^2 + 5.14(3) + 6.3$$
$$= 12.6273$$

Let $x = 6$.

$$-.0057(6)^4 + .157(6)^3 - 1.43(6)^2 + 5.14(6) + 6.3$$
$$= 12.1848$$

Thus, there were 12.6% below the poverty line in 2003 and 12.2% below the poverty line in 2006. The statement is true.

52. Let $x = 9$.

$$-.0057(9)^4 + .157(9)^3 - 1.43(9)^2 + 5.14(9) + 6.3$$
$$= 13.7853$$

Let $x = 8$.

$$-.0057(8)^4 + .157(8)^3 - 1.43(8)^2 + 5.14(8) + 6.3$$
$$= 12.9368$$

Thus, there were 13.8% below the poverty line in 2009 and 12.9% below the poverty line in 2008. The statement is false.

For exercises 53–56, we use the polynomial $1 - .0058x - .00076x^2$.

53. Let $x = 10$.

$$1 - .0058x - .00076x^2$$
$$= 1 - .0058(10) - .00076(10)^2 = .866$$

54. Let $x = 15$.

$$1 - .0058x - .00076x^2$$
$$= 1 - .0058(15) - .00076(15)^2 = .742$$

55. Let $x = 22$.

$$1 - .0058x - .00076x^2$$
$$= 1 - .0058(22) - .00076(22)^2 = .505$$

56. Let $x = 30$.

$$1 - .0058x - .00076x^2$$
$$= 1 - .0058(30) - .00076(30)^2 = .142$$

For exercises 57 and 58, use $V = \frac{1}{3}h\left(a^2 + ab + b^2\right)$.

57. **a.** Calculate the volume of the Great Pyramid when $h = 200$ feet, $b = 756$ feet and $a = 314$ feet.

$$V = \frac{1}{3}(200)\left(314^2 + (314)(756) + 756^2\right)$$
$$\approx 60,501,067 \text{ cubic feet}$$

b. When $a = b$, the shape becomes a rectangular box with a square base, with volume b^2h.

c. If we let $a = b$, then $\frac{1}{3}h\left(a^2 + ab + b^2\right)$ becomes $\frac{1}{3}h\left(b^2 + b(b) + b^2\right)$ which simplifies to hb^2. Yes, the Egyptian formula gives the same result.

58. **a.** $V = \frac{1}{3}h\left(b^2 + (0)(b) + (0)^2\right)$

$= \frac{1}{3}h\left(b^2\right) = \frac{1}{3}hb^2$

b. For the Great Pyramid, b = 756 feet and h = 481 feet.

$V = \frac{1}{3}(481)(756)^2 \approx 91.6$ million cubic feet

The Great Pyramid is slightly smaller than the Superdome.

c. The Great Pyramid covers

$b^2 = 756^2 = 571{,}536$ square feet.

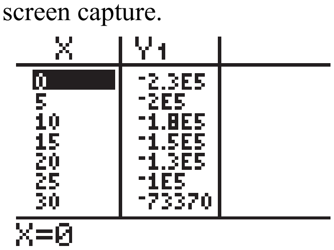
$\frac{571{,}536 \text{ ft}^2}{43{,}560 \text{ ft}^2/\text{acre}} \approx 13.1$ acre

59. **a.** Some or all of the terms may drop out of the sum, so the degree of the sum could be 0, 1, 2, or 3 or no degree (if one polynomial is the negative of the other).

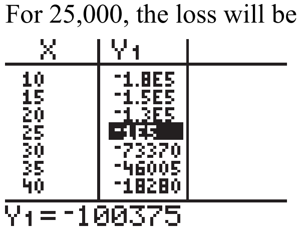
b. Some or all of the terms may drop out of the difference, so the degree of the difference could be 0, 1, 2, or 3 or no degree (if they are equal).

c. Multiplying a degree 3 polynomial by a degree 3 polynomial results in a degree 6 polynomial.

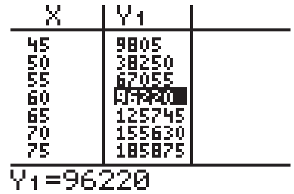
60. $P = 7.2x^2 + 5005x - 230{,}000$. Here is part of the screen capture.



For 25,000, the loss will be $100,375;



For 60,000, there profit will be $96,220.

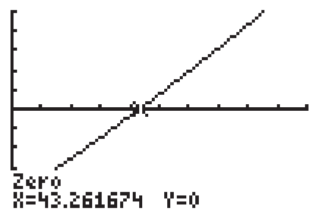


There is a loss at the beginning because of large fixed costs. When more items are made, these costs become a smaller portion of the total costs.

61. In order for the company to make a profit,

$P = 7.2x^2 + 5005x - 230{,}000 > 0$

Graph the function and locate a zero.



[0, 100] by [–250000, 250000]

The zero is at $x \approx 43.3$. Therefore, between 40,000 and 45,000 calculators must be sold for the company to make a profit.

62. Let x = 100 (in thousands)

$7.2(100)^2 + 5005(100) - 230{,}000 = 342{,}500$

Let x = 150 (in thousands)

$7.2(150)^2 + 5005(150) - 230{,}000 = 682{,}750$

The profit for selling 100,000 calculators is $342,500 and for selling 150,000 calculators is $682,750.

Section 1.3 Factoring

1. $12x^2 - 24x = 12x \cdot x - 12x \cdot 2 = 12x(x-2)$

2. $5y - 65xy = 5y(1) - 5y(13x) = 5y(1-13x)$

3. $r^3 - 5r^2 + r = r\left(r^2\right) - r(5r) + r(1)$

$= r\left(r^2 - 5r + 1\right)$

4. $t^3 + 3t^2 + 8t = t\left(t^2\right) + t(3t) + t(8)$

$= t\left(t^2 + 3t + 8\right)$

5. $6z^3 - 12z^2 + 18z$
$= 6z(z^2) - 6z(2z) + 6z(3)$
$= 6z(z^2 - 2z + 3)$

6. $5x^3 + 55x^2 + 10x$
$= 5x(x^2) + 5x(11x) + 5x(2)$
$= 5x(x^2 + 11x + 2)$

7. $3(2y-1)^2 + 7(2y-1)^3$
$= (2y-1)^2(3) + (2y-1)^2 \cdot 7(2y-1)$
$= (2y-1)^2[3 + 7(2y-1)]$
$= (2y-1)^2(3 + 14y - 7)$
$= (2y-1)^2(14y - 4)$
$= 2(2y-1)^2(7y-2)$

8. $(3x+7)^5 - 4(3x+7)^3$
$= (3x+7)^3(3x+7)^2 - (3x+7)^3(4)$
$= (3x+7)^3\left[(3x+7)^2 - 4\right]$
$= (3x+7)^3(9x^2 + 42x + 49 - 4)$
$= (3x+7)^3(9x^2 + 42x + 45)$

9. $3(x+5)^4 + (x+5)^6$
$= (x+5)^4 \cdot 3 + (x+5)^4(x+5)^2$
$= (x+5)^4\left[3 + (x+5)^2\right]$
$= (x+5)^4(3 + x^2 + 10x + 25)$
$= (x+5)^4(x^2 + 10x + 28)$

10. $3(x+6)^2 + 6(x+6)^4$
$= 3(x+6)^2(1) + 3(x+6)^2\left[2(x+6)^2\right]$
$= 3(x+6)^2\left[1 + 2(x+6)^2\right]$
$= 3(x+6)^2\left[1 + 2(x^2 + 12x + 36)\right]$
$= 3(x+6)^2(1 + 2x^2 + 24x + 72)$
$= 3(x+6)^2(2x^2 + 24x + 73)$

11. $x^2 + 5x + 4 = (x+1)(x+4)$

12. $u^2 + 7u + 6 = (u+1)(u+6)$

13. $x^2 + 7x + 12 = (x+3)(x+4)$

14. $y^2 + 8y + 12 = (y+2)(y+6)$

15. $x^2 + x - 6 = (x+3)(x-2)$

16. $x^2 + 4x - 5 = (x+5)(x-1)$

17. $x^2 + 2x - 3 = (x+3)(x-1)$

18. $y^2 + y - 12 = (y+4)(y-3)$

19. $x^2 - 3x - 4 = (x+1)(x-4)$

20. $u^2 - 2u - 8 = (u+2)(u-4)$

21. $z^2 - 9z + 14 = (z-2)(z-7)$

22. $w^2 - 6w - 16 = (w+2)(w-8)$

23. $z^2 + 10z + 24 = (z+4)(z+6)$

24. $r^2 + 16r + 60 = (r+6)(r+10)$

25. $2x^2 - 9x + 4 = (2x-1)(x-4)$

26. $3w^2 - 8w + 4 = (3w-2)(w-2)$

27. $15p^2 - 23p + 4 = (3p-4)(5p-1)$

28. $8x^2 - 14x + 3 = (4x-1)(2x-3)$

29. $4z^2 - 16z + 15 = (2z-5)(2z-3)$

30. $12y^2 - 29y + 15 = (3y-5)(4y-3)$

31. $6x^2 - 5x - 4 = (2x+1)(3x-4)$

32. $12z^2 + z - 1 = (4z-1)(3z+1)$

33. $10y^2 + 21y - 10 = (5y-2)(2y+5)$

34. $15u^2 + 4u - 4 = (5u-2)(3u+2)$

35. $6x^2 + 5x - 4 = (2x-1)(3x+4)$

36. $12y^2 + 7y - 10 = (3y-2)(4y+5)$

37. $3a^2 + 2a - 5 = (3a+5)(a-1)$

38. $6a^2 - 48a - 120 = 6\left(a^2 - 8a - 20\right)$
$= 6(a-10)(a+2)$

39. $x^2 - 81 = x^2 - (9)^2 = (x+9)(x-9)$

40. $x^2 + 17xy + 72y^2 = (x+8y)(x+9y)$.

41. $9p^2 - 12p + 4 = (3p)^2 - 2(3p)(2) + 2^2$
$= (3p-2)^2$

42. $3r^2 - r - 2 = (3r+2)(r-1)$.

43. $r^2 + 3rt - 10t^2 = (r-2t)(r+5t)$.

44. $2a^2 + ab - 6b^2 = (2a-3b)(a+2b)$.

45. $m^2 - 8mn + 16n^2 = (m)^2 - 2(m)(4n) + (4n)^2$
$= (m-4n)^2$

46. $8k^2 - 16k - 10 = 2\left(4k^2 - 8k - 5\right)$
$= 2(2k+1)(2k-5)$

47. $4u^2 + 12u + 9 = (2u+3)^2$

48. $9p^2 - 16 = (3p)^2 - 4^2 = (3p-4)(3p+4)$

49. $25p^2 - 10p + 4$
This polynomial cannot be factored

50. $10x^2 - 17x + 3 = (5x-1)(2x-3)$

51. $4r^2 - 9v^2 = (2r+3v)(2r-3v)$

52. $x^2 + 3xy - 28y^2 = (x+7y)(x-4y)$

53. $x^2 + 4xy + 4y^2 = (x+2y)^2$

54. $16u^2 + 12u - 18 = 2\left(8u^2 + 6u - 9\right)$
$= 2(4u-3)(2u+3)$

55. $3a^2 - 13a - 30 = (3a+5)(a-6)$.

56. $3k^2 + 2k - 8 = (3k-4)(k+2)$

57. $21m^2 + 13mn + 2n^2 = (7m+2n)(3m+n)$

58. $81y^2 - 100 = (9y+10)(9y-10)$

59. $y^2 - 4yz - 21z^2 = (y-7z)(y+3z)$

60. $49a^2 + 9$
This polynomial cannot be factored.

61. $121x^2 - 64 = (11x+8)(11x-8)$

62. $4z^2 + 56zy + 196y^2$
$= 4\left(z^2 + 14zy + 49y^2\right)$
$= 4\left[z^2 + 2(z)(7y) + (7y)^2\right] = 4(z+7y)^2$

63. $a^3 - 64 = a^3 - (4)^3 = (a-4)\left(a^2 + 4a + 16\right)$

64. $b^3 + 216 = b^3 + 6^3 = (b+6)\left(b^2 - 6b + 36\right)$

65. $8r^3 - 27s^3$
$= (2r)^3 - (3s)^3$
$= (2r-3s)\left[(2r)^2 + (2r)(3s) + (3s)^2\right]$
$= (2r-3s)\left(4r^2 + 6rs + 9s^2\right)$

66. $1000p^3 + 27q^3$
$= (10p)^3 + (3q)^3$
$= (10p+3q)\left(100p^2 - 30pq + 9q^2\right)$

67. $64m^3 + 125$
$= (4m)^3 + (5)^3$
$= (4m+5)\left[(4m)^2 - (4m)(5) + (5)^2\right]$
$= (4m+5)\left(16m^2 - 20m + 25\right)$

68. $216y^3 - 343$
$= (6y)^3 - (7)^3$
$= (6y - 7)\left(36y^2 + 42y + 49\right)$

69. $1000y^3 - z^3$
$= (10y)^3 - (z)^3$
$= (10y - z)\left[(10y)^2 + (10y)(z) + (z)^2\right]$
$= (10y - z)\left(100y^2 + 10yz + z^2\right)$

70. $125p^3 + 8q^3 = (5p)^3 + (2q)^3$
$= (5p + 2q)\left(25p^2 - 10pq + 4q^2\right)$

71. $x^4 + 5x^2 + 6 = \left(x^2 + 2\right)\left(x^2 + 3\right)$

72. $y^4 + 7y^2 + 10 = \left(y^2 + 2\right)\left(y^2 + 5\right)$

73. $b^4 - b^2 = b^2\left(b^2 - 1\right) = b^2(b + 1)(b - 1)$

74. $z^4 - 3z^2 - 4 = \left(z^2 - 4\right)\left(z^2 + 1\right)$
$= (z + 2)(z - 2)\left(z^2 + 1\right)$

75. $x^4 - x^2 - 12 = \left(x^2 - 4\right)\left(x^2 + 3\right)$
$= (x + 2)(x - 2)\left(x^2 + 3\right)$

76. $4x^4 + 27x^2 - 81 = \left(4x^2 - 9\right)\left(x^2 + 9\right)$
$= (2x + 3)(2x - 3)\left(x^2 + 9\right)$

77. $16a^4 - 81b^4 = \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right)$
$= (2a + 3b)(2a - 3b)\left(4a^2 + 9b^2\right)$

78. $x^6 - y^6 = \left(x^2\right)^3 - \left(y^2\right)^3$
$= \left(x^2 - y^2\right)\left(x^4 + x^2y^2 + y^4\right)$
$= (x + y)(x - y)\left(x^2 + xy + y^2\right)\cdot$
$\left(x^2 - xy + y^2\right)$

79. $x^8 + 8x^2 = x^2\left(x^6 + 8\right) = x^2\left(\left(x^2\right)^3 + 2^3\right)$
$= x^2\left(x^2 + 2\right)\left(x^4 - 2x^2 + 4\right)$

80. $x^9 - 64x^3 = x^3\left(x^6 - 64\right) = x^3\left(x^6 - 2^6\right)$
$= x^3\left[\left(x^3\right)^2 - \left(2^3\right)^2\right]$
$= x^3\left(x^3 - 2^3\right)\left(x^3 + 2^3\right)$
$= x^3(x - 2)\left(x^2 + 2x + 4\right)\cdot$
$(x + 2)\left(x^2 - 2x + 4\right)$

81. $6x^4 - 3x^2 - 3 = \left(2x^2 + 1\right)\left(3x^2 - 3\right)$ is not the correct complete factorization because $3x^2 - 3$ contains a common factor of 3. This common factor should be factored out as the first step. This will reveal a difference of two squares, which requires further factorization. The correct factorization is
$6x^4 - 3x^2 - 3 = 3\left(2x^4 - x^2 - 1\right)$
$= 3\left(2x^2 + 1\right)\left(x^2 - 1\right)$
$= 3\left(2x^2 + 1\right)(x + 1)(x - 1)$

82. The sum of two squares can be factored when the terms have a common factor. An example is
$(3x)^2 + 3^2 = 9x^2 + 9 = 9(x^2 + 1)$

83. $(x + 2)^3 = (x + 2)(x + 2)^2$
$= (x + 2)(x^2 + 4x + 4)$
$= x^3 + 4x^2 + 2x^2 + 8x + 4x + 8$
$= x^3 + 6x^2 + 12x + 8,$
which is not equal to $x^3 + 8$. The correct factorization is $x^3 + 8 = (x + 2)(x^2 - 2x + 4).$

84. Factoring and multiplication are inverse operations. If we factor a polynomial and then multiply the factors, we get the original polynomial. For example, we can factor $x^2 - x - 6$ to get $(x - 3)(x + 2)$. Then if we multiply the factors, we get
$(x - 3)(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$

Section 1.4 Rational Expressions

1. $\frac{8x^2}{56x} = \frac{x \cdot 8x}{7 \cdot 8x} = \frac{x}{7}$

2. $\frac{27m}{81m^3} = \frac{27m}{27m \cdot 3m^2} = \frac{1}{3m^2}$

3. $\frac{25p^2}{35p^3} = \frac{5 \cdot 5p^2}{7p \cdot 5p^2} = \frac{5}{7p}$

4. $\frac{18y^4}{24y^2} = \frac{6y^2 \cdot 3y^2}{6y^2 \cdot 4} = \frac{3y^2}{4}$

5. $\frac{5m+15}{4m+12} = \frac{5(m+3)}{4(m+3)} = \frac{5}{4}$

6. $\frac{10z+5}{20z+10} = \frac{5(2x+1)}{5(2x+1) \cdot 2} = \frac{1}{2}$

7. $\frac{4(w-3)}{(w-3)(w+6)} = \frac{4}{w+6}$

8. $\frac{-6(x+2)}{(x+4)(x+2)} = \frac{-6}{x+4}$ or $-\frac{6}{x+4}$

9. $\frac{3y^2-12y}{9y^3} = \frac{3y(y-4)}{3y\left(3y^2\right)}$
$= \frac{y-4}{3y^2}$

10. $\frac{15k^2+45k}{9k^2} = \frac{15k(k+3)}{3k \cdot 3k}$
$= \frac{3k \cdot 5(k+3)}{3k \cdot 3k}$
$= \frac{5(k+3)}{3k}$

11. $\frac{m^2-4m+4}{m^2+m-6} = \frac{(m-2)(m-2)}{(m+3)(m-2)} = \frac{m-2}{m+3}$

12. $\frac{r^2-r-6}{r^2+r-12} = \frac{(r-3)(r+2)}{(r+4)(r-3)} = \frac{r+2}{r+4}$

13. $\frac{x^2+2x-3}{x^2-1} = \frac{(x+3)(x-1)}{(x+1)(x-1)} = \frac{x+3}{x+1}$

14. $\frac{z^2+4z+4}{z^2-4} = \frac{(z+2)^2}{(z+2)(z-2)} = \frac{z+2}{z-2}$

15. $\frac{3a^2}{64} \cdot \frac{8}{2a^3} = \frac{3a^2 \cdot 8}{64 \cdot 2a^3} = \frac{3}{16a}$

16. $\frac{2u^2}{8u^4} \cdot \frac{10u^3}{9u} = \frac{2u^2 \cdot 10u^3}{8u^4 \cdot 9u} = \frac{5}{18}$

17. $\frac{7x}{11} \div \frac{14x^3}{66y} = \frac{7x}{11} \cdot \frac{66y}{14x^3} = \frac{7x \cdot 66y}{11 \cdot 14x^3} = \frac{3y}{x^2}$

18. $\frac{6x^2y}{2x} \div \frac{21xy}{y} = \frac{6x^2y}{2x} \cdot \frac{y}{21xy} = \frac{y}{7}$

19. $\frac{2a+b}{3c} \cdot \frac{15}{4(2a+b)} = \frac{(2a+b) \cdot 15}{(2a+b) \cdot 12c} = \frac{15}{12c} = \frac{5}{4c}$

20. $\frac{4(x+2)}{w} \cdot \frac{3w^2}{8(x+2)} = \frac{4w^2(x+2) \cdot 3}{4w(x+2) \cdot 2} = \frac{3w}{2}$

21. $\frac{15p-3}{6} \div \frac{10p-2}{3} = \frac{15p-3}{6} \cdot \frac{3}{10p-2}$
$= \frac{3(5p-1) \cdot 3}{3 \cdot 2 \cdot 2 \cdot (5p-1)}$
$= \frac{3(5p-1) \cdot 3}{3(5p-1) \cdot 2 \cdot 2} = \frac{3}{4}$

22. $\frac{2k+8}{6} \div \frac{3k+12}{3} = \frac{2k+8}{6} \cdot \frac{3}{3k+12}$
$= \frac{2(k+4)}{6} \cdot \frac{3}{3(k+4)}$
$= \frac{6(k+4)}{18(k+4)} = \frac{6}{18} = \frac{1}{3}$

23. $\frac{9y-18}{6y+12} \cdot \frac{3y+6}{15y-30} = \frac{9(y-2)}{6(y+2)} \cdot \frac{3(y+2)}{15(y-2)}$
$= \frac{27(y-2)(y+2)}{90(y+2)(y-2)} = \frac{27}{90} = \frac{3}{10}$

24. $\frac{12r+24}{36r-36} \div \frac{6r+12}{8r-8} = \frac{12(r+2)}{36(r-1)} \div \frac{6(r+2)}{8(r-1)}$
$= \frac{r+2}{3(r-1)} \div \frac{3(r+2)}{4(r-1)}$
$= \frac{r+2}{3(r-1)} \cdot \frac{4(r-1)}{3(r+2)} = \frac{4}{9}$

25. $\frac{4a+12}{2a-10} \div \frac{a^2-9}{a^2-a-20} = \frac{4a+12}{2a-10} \cdot \frac{a^2-a-20}{a^2-9}$
$= \frac{4(a+3)}{2(a-5)} \cdot \frac{(a-5)(a+4)}{(a+3)(a-3)}$
$= \frac{4(a+3)(a-5)(a+4)}{2(a-5)(a+3)(a-3)}$
$= \frac{2(a+4)}{a-3}$

26. $\frac{6r-18}{9r^2+6r-24} \cdot \frac{12r-16}{4r-12}$
$= \frac{6(r-3)}{3\left(3r^2+2r-8\right)} \cdot \frac{4(3r-4)}{4(r-3)} = \frac{2(3r-4)}{\left(3r^2+2r-8\right)}$
$= \frac{2(3r-4)}{(3r-4)(r+2)} = \frac{2}{r+2}$

27. $\frac{k^2-k-6}{k^2+k-12} \cdot \frac{k^2+3k-4}{k^2+2k-3}$
$= \frac{(k-3)(k+2)}{(k+4)(k-3)} \cdot \frac{(k+4)(k-1)}{(k+3)(k-1)}$
$= \frac{(k-3)(k+2)(k+4)(k-1)}{(k+4)(k-3)(k+3)(k-1)} = \frac{k+2}{k+3}$

28. $\frac{n^2-n-6}{n^2-2n-8} \div \frac{n^2-9}{n^2+7n+12}$
$= \frac{(n-3)(n+2)}{(n-4)(n+2)} \div \frac{(n-3)(n+3)}{(n+3)(n+4)}$
$= \frac{n-3}{n-4} \div \frac{n-3}{n+4} = \frac{n-3}{n-4} \cdot \frac{n+4}{n-3} = \frac{n+4}{n-4}$

Answers will vary for exercises 29 and 30. Sample answers are given.

29. To find the least common denominator for two fractions, factor each denominator into prime factors, multiply all unique prime factors raising each factor to the highest frequency it occurred.

30. To add three rational expressions, first factor each denominator completely. Then, find the least common denominator and rewrite each expression with that denominator. Next, add the numerators and place over the common denominator. Finally, simplify the resulting expression and write it in lowest terms.

31. The common denominator is $35z$.
$\frac{2}{7z} - \frac{1}{5z} = \frac{2 \cdot 5}{7z \cdot 5} - \frac{1 \cdot 7}{5z \cdot 7} = \frac{10}{35z} - \frac{7}{35z} = \frac{3}{35z}$

32. The common denominator is $12z$.
$\frac{4}{3z} - \frac{5}{4z} = \frac{4 \cdot 4}{3z \cdot 4} - \frac{5 \cdot 3}{4z \cdot 3} = \frac{16}{12z} - \frac{15}{12z} = \frac{1}{12z}$

33. $\frac{r+2}{3} - \frac{r-2}{3} = \frac{(r+2)-(r-2)}{3}$
$= \frac{r+2-r+2}{3} = \frac{4}{3}$

34. $\frac{3y-1}{8} - \frac{3y+1}{8} = \frac{(3y-1)-(3y+1)}{8} = \frac{-2}{8} = -\frac{1}{4}$

35. The common denominator is $5x$.
$\frac{4}{x} + \frac{1}{5} = \frac{4 \cdot 5}{x \cdot 5} + \frac{1 \cdot x}{5 \cdot x} = \frac{20}{5x} + \frac{x}{5x} = \frac{20+x}{5x}$

36. The common denominator is $4r$.
$\frac{6}{r} - \frac{3}{4} = \frac{6 \cdot 4}{r \cdot 4} - \frac{3 \cdot r}{4 \cdot r} = \frac{24}{4r} - \frac{3r}{4r}$
$= \frac{24-3r}{4r} = \frac{3(8-r)}{4r}$

37. The common denominator is $m(m-1)$.
$\frac{1}{m-1} + \frac{2}{m} = \frac{m \cdot 1}{m \cdot (m-1)} + \frac{(m-1) \cdot 2}{(m-1) \cdot m}$
$= \frac{m}{m(m-1)} + \frac{2(m-1)}{m(m-1)}$
$= \frac{m+2(m-1)}{m(m-1)} = \frac{m+2m-2}{m(m-1)}$
$= \frac{3m-2}{m(m-1)}$

38. The common denominator is $(y+2)y$.
$\frac{8}{y+2} - \frac{3}{y} = \frac{8y}{(y+2)y} - \frac{3(y+2)}{y(y+2)} = \frac{8y-3(y+2)}{(y+2)y}$
$= \frac{8y-3y-6}{(y+2)y} = \frac{5y-6}{(y+2)y}$ or $\frac{5y-6}{y(y+2)}$

39. The common denominator is $5(b+2)$.
$\frac{7}{b+2} + \frac{2}{5(b+2)} = \frac{7 \cdot 5}{(b+2) \cdot 5} + \frac{2}{5(b+2)}$
$= \frac{35+2}{5(b+2)} = \frac{37}{5(b+2)}$

40. The common denominator is $3(k+1)$.
$\frac{4}{3(k+1)} + \frac{3}{k+1} = \frac{4}{3(k+1)} + \frac{3 \cdot 3}{3(k+1)}$
$= \frac{4+9}{3(k+1)} = \frac{13}{3(k+1)}$

41. The common denominator is $20(k-2)$.

$$\frac{2}{5(k-2)}+\frac{5}{4(k-2)}=\frac{8}{20(k-2)}+\frac{25}{20(k-2)}$$
$$=\frac{8+25}{20(k-2)}=\frac{33}{20(k-2)}$$

42. The common denominator is $6(p+4)$.

$$\frac{11}{3(p+4)}-\frac{5}{6(p+4)}=\frac{22}{6(p+4)}-\frac{5}{6(p+4)}$$
$$=\frac{22-5}{6(p+4)}=\frac{17}{6(p+4)}$$

43. First factor the denominators in order to find the common denominator.

$x^2-4x+3=(x-3)(x-1)$

$x^2-x-6=(x-3)(x+2)$

The common denominator is $(x-3)(x-1)(x+2)$.

$$\frac{2}{x^2-4x+3}+\frac{5}{x^2-x-6}$$
$$=\frac{2}{(x-3)(x-1)}+\frac{5}{(x-3)(x+2)}$$
$$=\frac{2(x+2)}{(x-3)(x-1)(x+2)}+\frac{5(x-1)}{(x-3)(x+2)(x-1)}$$
$$=\frac{2(x+2)+5(x-1)}{(x-3)(x+2)(x-1)}=\frac{2x+4+5x-5}{(x-3)(x-1)(x+2)}$$
$$=\frac{7x-1}{(x-3)(x-1)(x+2)}$$

44. First factor the denominators in order to find the common denominator.

$m^2-3m-10=(m-5)(m+2)$

$m^2-m-20=(m-5)(m+4)$

The common denominator is $(m-5)(m+2)(m+4)$.

$$\frac{3}{m^2-3m-10}+\frac{7}{m^2-m-20}$$
$$=\frac{3}{(m-5)(m+2)}+\frac{7}{(m-5)(m+4)}$$
$$=\frac{3(m+4)}{(m-5)(m+2)(m+4)}+\frac{7(m+2)}{(m-5)(m+4)(m+2)}$$
$$=\frac{3m+12+7m+14}{(m-5)(m+2)(m+4)}=\frac{10m+26}{(m-5)(m+2)(m+4)}$$

45. First factor the denominators in order to find the common denominator.

$y^2+7y+12=(y+3)(y+4)$

$y^2+5y+6=(y+3)(y+2)$

The common denominator is $(y+4)(y+3)(y+2)$.

$$\frac{2y}{y^2+7y+12}-\frac{y}{y^2+5y+6}$$
$$=\frac{2y}{(y+4)(y+3)}-\frac{y}{(y+3)(y+2)}$$
$$=\frac{2y(y+2)}{(y+4)(y+3)(y+2)}-\frac{y(y+4)}{(y+4)(y+3)(y+2)}$$
$$=\frac{2y(y+2)-y(y+4)}{(y+4)(y+3)(y+2)}=\frac{2y^2+4y-y^2-4y}{(y+4)(y+3)(y+2)}$$
$$=\frac{y^2}{(y+4)(y+3)(y+2)}$$

46. First factor the denominators in order to find the common denominator.

$r^2-10r+16=(r-8)(r-2)$

$r^2+2r-8=(r+4)(r-2)$

The common denominator is $(r-8)(r-2)(r+4)$.

$$\frac{-r}{r^2-10r+16}-\frac{3r}{r^2+2r-8}$$
$$=\frac{-r}{(r-8)(r-2)}-\frac{3r}{(r+4)(r-2)}$$
$$=\frac{-r(r+4)}{(r-8)(r-2)(r+4)}-\frac{3r(r-8)}{(r+4)(r-2)(r-8)}$$
$$=\frac{-r^2-4r-\left(3r^2-24r\right)}{(r-8)(r-2)(r+4)}=\frac{-r^2-4r-3r^2+24r}{(r-8)(r-2)(r+4)}$$
$$=\frac{-4r^2+20r}{(r-8)(r-2)(r+4)}$$

47. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

Multiply both numerator and denominator of this complex fraction by the common denominator, x.

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}=\frac{x\left(1+\frac{1}{x}\right)}{x\left(1-\frac{1}{x}\right)}=\frac{x\cdot 1+x\left(\frac{1}{x}\right)}{x\cdot 1-x\left(\frac{1}{x}\right)}=\frac{x+1}{x-1}$$

48. $\dfrac{2-\frac{2}{y}}{2+\frac{2}{y}}$

Multiply both numerator and denominator by the common denominator, y.

$$\frac{2-\frac{2}{y}}{2+\frac{2}{y}}=\frac{y\left(2-\frac{2}{y}\right)}{y\left(2+\frac{2}{y}\right)}=\frac{2y-2}{2y+2}=\frac{2(y-1)}{2(y+1)}=\frac{y-1}{y+1}$$

49. $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

The common denominator in the numerator is $x(x+h)$

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\frac{\frac{x-x-h}{x(x+h)}}{h}=\frac{\frac{-h}{x(x+h)}}{h}$$
$$=\frac{-h}{x(x+h)}\div h=\frac{-h}{x(x+h)}\cdot\frac{1}{h}$$
$$=\frac{-1}{x(x+h)} \text{ or } -\frac{1}{x(x+h)}$$

50. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

The common denominator of the numerator is $(x+h)^2x^2$.

$$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2}{(x+h)^2x^2}-\frac{(x+h)^2}{(x+h)^2x^2}}{\frac{h}{1}}$$
$$=\frac{x^2-(x+h)^2}{(x+h)^2x^2}\cdot\frac{1}{h}$$
$$=\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\cdot\frac{1}{h}$$
$$=\frac{-2xh-h^2}{(x+h)^2x^2h}=\frac{h(-2x-h)}{(x+h)^2x^2h}$$
$$=\frac{-2x-h}{(x+h)^2x^2}$$

51. The length of each side of the dartboard is $2x$, so the area of the dartboard is $4x^2$. The area of the shaded region is πx^2.

a. The probability that a dart will land in the shaded region is $\dfrac{\pi x^2}{4x^2}$.

b. $\dfrac{\pi x^2}{4x^2}=\dfrac{\pi}{4}$

52. The radius of the dartboard is $x + 2x + 3x = 6x$, so the area of the dartboard is $\pi(6x)^2=36\pi x^2$. The area of the shaded region is πx^2.

a. The probability that a dart will land in the shaded region is $\dfrac{\pi x^2}{36\pi x^2}$.

b. $\dfrac{\pi x^2}{36\pi x^2}=\dfrac{1}{36}$

53. The length of each side of the dartboard is $5x$, so the area of the dartboard is $25x^2$. The area of the shaded region is x^2.

a. The probability that a dart will land in the shaded region is $\dfrac{x^2}{25x^2}$.

b. $\dfrac{x^2}{25x^2}=\dfrac{1}{25}$

54. The length of each side of the dartboard is $3x$, so the area of the dartboard is $9x^2$. The area of the shaded region is $\frac{1}{2}x^2$.

a. The probability that a dart will land in the shaded region is $\dfrac{\frac{1}{2}x^2}{9x^2}=\dfrac{x^2}{18x^2}$.

b. $\dfrac{x^2}{18x^2}=\dfrac{1}{18}$

55. Average cost = total cost C divided by the number of calculators produced.

$$\frac{-7.2x^2+6995x+230{,}000}{1000x}$$

56. Let $x = 20$ (in thousands).

$$\frac{-7.2(20)^2+6995(20)+230{,}000}{1000(20)}=\$18.35$$

Let $x = 50$ (in thousands).

$$\frac{-7.2(50)^2+6995(50)+230{,}000}{1000(50)}=\$11.24$$

Let $x = 125$ (in thousands).

$$\frac{-7.2(125)^2+6995(125)+230{,}000}{1000(125)}=\$7.94$$

57. Let $x = 10$. Then
$$\frac{.314(10)^2 - 1.399(10) + 15.0}{10+1} \approx 2.95$$
The ad cost approximately \$2.95 million in 2010

58. Let $x = 12$. Then
$$\frac{.314(12)^2 - 1.399(12) + 15.0}{12+1} \approx 3.34$$
The ad cost approximately \$3.34 million in 2012

59. Let $x = 18$. Then
$$\frac{.314(18)^2 - 1.399(18) + 15.0}{18+1} \approx 4.82$$
The cost of an ad will not reach \$5 million in 2018.

60. Let $x = 23$. Then
$$\frac{.314(23)^2 - 1.399(23) + 15.0}{23+1} \approx 6.21$$
The cost of an ad will reach \$6 million in 2023.

61. Let $x = 11$. Then
$$\frac{.265(11)^2 + 1.47(11) + 3.63}{11+2} \approx 3.99$$
The hourly insurance cost in 2011 was \$3.99.

62. Let $x = 12$. Then
$$\frac{.265(12)^2 + 1.47(12) + 3.63}{12+2} \approx 4.25$$
The hourly insurance cost in 2012 was \$4.25.

63. Let $x = 15$. Then
$$\frac{.265(15)^2 + 1.47(15) + 3.63}{15+2} \approx 5.02$$
The hourly insurance cost in 2015 will be \$5.02.
The annual cost will be 5.02(2100) = \$10,537.68

64. Yes; the annual cost was already more than \$10,000 in the year 2015.

Section 1.5 Exponents and Radicals

1. $\frac{7^5}{7^3} = 7^{5-3} = 7^2 = 49$

2. $\frac{(-6)^{14}}{(-6)^6} = (-6)^8 = 1,679,616$

3. $(4c)^2 = 4^2c^2 = 16c^2$

4. $(-2x)^4 = (-2)^4 x^4 = 16x^4$

5. $\left(\frac{2}{x}\right)^5 = \frac{2^5}{x^5} = \frac{32}{x^5}$

6. $\left(\frac{5}{xy}\right)^3 = \frac{5^3}{x^3y^3} = \frac{125}{x^3y^3}$

7. $(3u^2)^3(2u^3)^2 = (27u^6)(4u^6) = 108u^{12}$

8. $\frac{(5v^2)^3}{(2v)^4} = \frac{125v^6}{16v^4} = \frac{125v^2}{16}$

9. $7^{-1} = \frac{1}{7^1} = \frac{1}{7}$

10. $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$

11. $-6^{-5} = -\frac{1}{6^5} = -\frac{1}{7776}$

12. $(-x)^{-4} = \frac{1}{(-x)^4} = \frac{1}{x^4}$

13. $(-y)^{-3} = \frac{1}{(-y)^3} = -\frac{1}{y^3}$

14. $\left(\frac{1}{6}\right)^{-2} = \left(\frac{6}{1}\right)^2 = 6^2 = 36$

15. $\left(\frac{4}{3}\right)^{-2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

16. $\left(\frac{x}{y^2}\right)^{-2} = \left(\frac{y^2}{x}\right)^2 = \frac{y^4}{x^2}$

17. $\left(\frac{a}{b^3}\right)^{-1} = \left(\frac{b^3}{a}\right)^1 = \frac{b^3}{a}$

18. $-2^{-4} = -\frac{1}{2^4} = -\frac{1}{16}$, but

$(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$

19. $49^{1/2} = 7$ because $7^2 = 49$.

20. $8^{1/3} = 2$ because $2^3 = 8$.

21. $(5.71)^{1/4} = (5.71)^{.25} \approx 1.55$ Use a calculator.

22. $12^{5/2} = \left(12^{1/2}\right)^5 \approx 498.83$

23. $-64^{2/3} = -\left(64^{1/3}\right)^2 = -(4)^2 = -16$

24. $-64^{3/2} = -\left[\left(64^{1/2}\right)^3\right] = -\left(8^3\right) = -512$

25. $\left(\frac{8}{27}\right)^{-4/3} = \left(\frac{27^{1/3}}{8^{1/3}}\right)^4 = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$

26. $\left(\frac{27}{64}\right)^{-1/3} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}$

27. $\frac{5^{-3}}{4^{-2}} = \frac{4^2}{5^3} = \frac{16}{125}$

28. $\frac{7^{-4}}{7^{-3}} = 7^{-4} \cdot 7^3 = 7^{-1} = \frac{1}{7}$

29. $4^{-3} \cdot 4^6 = 4^3 = 64$

30. $9^{-9} \cdot 9^{10} = 9^1 = 9$

31. $\frac{4^{10} \cdot 4^{-6}}{4^{-4}} = 4^{10} \cdot 4^{-6} \cdot 4^4 = 4^8 = 65{,}536$

32. $\frac{5^{-4} \cdot 5^6}{5^{-1}} = 5^{-4} \cdot 5^6 \cdot 5^1 = 5^3 = 125$

33. $\frac{z^6 \cdot z^2}{z^5} = \frac{z^8}{z^5} = z^{8-5} = z^3$

34. $\frac{k^6 \cdot k^9}{k^{12}} = \frac{k^{15}}{k^{12}} = k^{15-12} = k^3$

35. $\frac{3^{-1}\left(p^{-2}\right)^3}{3p^{-7}} = \frac{3^{-1}p^{-6}}{3^1 p^{-7}} = 3^{-1-1}p^{-6-(-7)}$

$= 3^{-2}p^1 = \frac{1}{3^2} \cdot p = \frac{p}{9}$

36. $\frac{\left(5x^3\right)^{-2}}{x^4} = \frac{5^{-2}\left(x^3\right)^{-2}}{x^4} = \frac{5^{-2}x^{-6}}{x^4}$

$= 5^{-2}x^{-10} = \frac{1}{25x^{10}}$

37. $\left(q^{-5}r^3\right)^{-1} = q^5 r^{-3} = q^5 \cdot \frac{1}{r^3} = \frac{q^5}{r^3}$

38. $\left(2y^2z^{-2}\right)^{-3} = 2^{-3}\left(y^2\right)^{-3}\left(z^{-2}\right)^{-3}$

$= 2^{-3}y^{-6}z^6 = \frac{z^6}{2^3 y^6} = \frac{z^6}{8y^6}$

39. $\left(2p^{-1}\right)^3 \cdot \left(5p^2\right)^{-2} = 2^3\left(p^{-1}\right)^3 (5)^{-2}\left(p^2\right)^{-2}$

$= 2^3\left(p^{-3}\right)\left(\frac{1}{5^2}\right)\left(p^{-4}\right)$

$= 2^3\left(\frac{1}{p^3}\right)\left(\frac{1}{5^2}\right)\left(\frac{1}{p^4}\right)$

$= \frac{8}{25p^7}$

40. $\left(4^{-1}x^3\right)^{-2} \cdot \left(3x^{-3}\right)^4$

$= \left(4^{-1}\right)^{-2} \cdot \left(x^3\right)^{-2} \cdot (3)^4 \cdot \left(x^{-3}\right)^4$

$= 4^2 \cdot x^{-6} \cdot 3^4 \cdot x^{-12} = 1296x^{-18} = \frac{1296}{x^{18}}$

41. $(2p)^{1/2} \cdot \left(2p^3\right)^{1/3} = 2^{1/2}p^{1/2} \cdot 2^{1/3} \cdot \left(p^3\right)^{1/3}$

$= 2^{1/2}p^{1/2} \cdot 2^{1/3} \cdot p^1$

$= 2^{5/6}p^{3/2}$

42. $\left(5k^2\right)^{3/2} \cdot \left(5k^{1/3}\right)^{3/4} = 5^{3/2}\left(k^2\right)^{3/2} \cdot 5^{3/4} \cdot \left(k^{1/3}\right)^{3/4}$

$= 5^{\frac{3}{2}+\frac{3}{4}}k^3k^{1/4} = 5^{9/4}k^{13/4}$

43. $p^{2/3}\left(2p^{1/3}+5p\right)=p^{2/3}\left(2p^{1/3}\right)+p^{2/3}(5p)$
$=2p+5p^{5/3}$

44. $3x^{3/2}\left(2x^{-3/2}+x^{3/2}\right)=3x^{3/2}\cdot 2x^{-3/2}+3x^{3/2}\cdot x^{3/2}$
$=6x^{0}+3x^{6/2}=6+3x^{3}$

45. $\dfrac{\left(x^2\right)^{1/3}\left(y^2\right)^{2/3}}{3x^{2/3}y^2}=\dfrac{(x)^{2/3}(y)^{4/3}}{3x^{2/3}y^2}$
$=\dfrac{1}{3y^{2-4/3}}=\dfrac{1}{3y^{2/3}}$

46. $\dfrac{\left(c^{1/2}\right)^3\left(d^3\right)^{1/2}}{\left(c^3\right)^{1/4}\left(d^{1/4}\right)^3}=\dfrac{\left(c^{3/2}\right)\left(d^{3/2}\right)}{\left(c^{3/4}\right)\left(d^{3/4}\right)}$
$=c^{(3/2)-(3/4)}d^{(3/2)-(3/4)}$
$=c^{3/4}d^{3/4}$

47. $\dfrac{(7a)^2(5b)^{3/2}}{(5a)^{3/2}(7b)^4}=\dfrac{7^2a^25^{3/2}b^{3/2}}{5^{3/2}a^{3/2}7^4b^4}=\dfrac{a^{2-\frac{3}{2}}}{7^2b^{4-\frac{3}{2}}}$
$=\dfrac{a^{1/2}}{49b^{5/2}}$

48. $\dfrac{(4x)^{1/2}\sqrt{xy}}{x^{3/2}y^2}=\dfrac{(4x)^{1/2}(xy)^{1/2}}{x^{3/2}y^2}=\dfrac{4^{1/2}x^{1/2}x^{1/2}y^{1/2}}{x^{3/2}y^2}$
$=2xy^{1/2}x^{-3/2}y^{-2}=2x^{-1/2}y^{-3/2}$
$=\dfrac{2}{x^{1/2}y^{3/2}}$

49. $x^{1/2}\left(x^{2/3}-x^{4/3}\right)=x^{1/2}x^{2/3}-x^{1/2}x^{4/3}$
$=x^{7/6}-x^{11/6}$

50. $x^{1/2}\left(3x^{3/2}+2x^{-1/2}\right)=3x^{1/2}x^{3/2}+2x^{1/2}x^{-1/2}$
$=3x^2+2$

51. $\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right)=\left(x^{1/2}\right)^2-\left(y^{1/2}\right)^2$
$=x-y$

52. $\left(x^{1/3}+y^{1/2}\right)\left(2x^{1/3}-y^{3/2}\right)$
$=x^{1/3}\cdot 2x^{1/3}-x^{1/3}y^{3/2}+2x^{1/3}y^{1/2}$
$-y^{1/2}y^{3/2}$
$=2x^{2/3}+2x^{1/3}y^{1/2}-x^{1/3}y^{3/2}-y^2$

53. $(-3x)^{1/3}=\sqrt[3]{-3x}$, (f)

54. $-3x^{1/3}=-3\sqrt[3]{x}$, (b)

55. $(-3x)^{-1/3}=\dfrac{1}{(-3x)^{1/3}}=\dfrac{1}{\sqrt[3]{-3x}}$, (h)

56. $-3x^{-1/3}=\dfrac{-3}{x^{1/3}}=\dfrac{-3}{\sqrt[3]{x}}$, (d)

57. $(3x)^{1/3}=\sqrt[3]{3x}$, (g)

58. $3x^{-1/3}=\dfrac{3}{x^{1/3}}=\dfrac{3}{\sqrt[3]{x}}$, (a)

59. $(3x)^{-1/3}=\dfrac{1}{(3x)^{1/3}}=\dfrac{1}{\sqrt[3]{3x}}$, (c)

60. $3x^{1/3}=3\sqrt[3]{x}$, (e)

61. $\sqrt[3]{125}=125^{1/3}=5$

62. $\sqrt[6]{64}=64^{1/6}=2$

63. $\sqrt[4]{625}=625^{1/4}=5$

64. $\sqrt[7]{-128}=(-128)^{1/7}=-2$

65. $\sqrt{63}\cdot\sqrt{7}=3\sqrt{7}\cdot\sqrt{7}=3\cdot 7=21$

66. $\sqrt[3]{81}\cdot\sqrt[3]{9}=\sqrt[3]{729}=9$

67. $\sqrt{81-4}=\sqrt{77}$

68. $\sqrt{49-16}=\sqrt{33}$

69. $\sqrt{5}\sqrt{15}=\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\sqrt{3}=5\sqrt{3}$

70. $\sqrt{8}\sqrt{96}=\sqrt{8}\sqrt{8\cdot 12}=\sqrt{8}\sqrt{8}\sqrt{12}=8\sqrt{4\cdot 3}$
$=8\sqrt{4}\sqrt{3}=8\cdot 2\sqrt{3}=16\sqrt{3}$

71. $\sqrt{50}-\sqrt{72}=5\sqrt{2}-6\sqrt{2}=-\sqrt{2}$

72 $\sqrt{75}+\sqrt{192}=5\sqrt{3}+8\sqrt{3}=13\sqrt{3}$

73. $5\sqrt{20}-\sqrt{45}+2\sqrt{80}$
$=5\cdot 2\sqrt{5}-3\sqrt{5}+2\cdot 4\sqrt{5}$
$=10\sqrt{5}-3\sqrt{5}+8\sqrt{5}=15\sqrt{5}$

74. $\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)=\left(\sqrt{3}\right)^2-2^2=3-4=-1$

75. $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$
$=5-2=3$

76. $\sqrt[3]{4}\cdot\sqrt[3]{4}=4$. A correct statement would be $\sqrt[3]{4}\cdot\sqrt[3]{4}=\sqrt[3]{16}$.

77. $\frac{3}{1-\sqrt{2}}=\frac{3}{1-\sqrt{2}}\cdot\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{3\left(1+\sqrt{2}\right)}{(1)^2-\left(\sqrt{2}\right)^2}$
$=\frac{3\left(1+\sqrt{2}\right)}{1-2}=\frac{3\left(1+\sqrt{2}\right)}{-1}$
$=-3\left(1+\sqrt{2}\right)=-3-3\sqrt{2}$

78. $\frac{2}{1+\sqrt{5}}=\frac{2}{1+\sqrt{5}}\cdot\frac{1-\sqrt{5}}{1-\sqrt{5}}=\frac{2\left(1-\sqrt{5}\right)}{1-5}$
$=\frac{2\left(1-\sqrt{5}\right)}{-4}=\frac{1-\sqrt{5}}{-2}\cdot\frac{-1}{-1}=\frac{\sqrt{5}-1}{2}$

79. $\frac{9-\sqrt{3}}{3-\sqrt{3}}=\frac{9-\sqrt{3}}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}}=\frac{27+9\sqrt{3}-3\sqrt{3}-3}{3^2-\left(\sqrt{3}\right)^2}$
$=\frac{24+6\sqrt{3}}{9-3}=\frac{24+6\sqrt{3}}{6}=4+\sqrt{3}$

80. $\frac{\sqrt{3}-1}{\sqrt{3}-2}=\frac{\sqrt{3}-1}{\sqrt{3}-2}\cdot\frac{\sqrt{3}+2}{\sqrt{3}+2}=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}+2\right)}{3-4}$
$=\frac{3+2\sqrt{3}-\sqrt{3}-2}{-1}=\frac{1+\sqrt{3}}{-1}=-1-\sqrt{3}$

81. $\frac{3-\sqrt{2}}{3+\sqrt{2}}=\frac{3-\sqrt{2}}{3+\sqrt{2}}\cdot\frac{3+\sqrt{2}}{3+\sqrt{2}}$
$=\frac{9-2}{9+6\sqrt{2}+2}=\frac{7}{11+6\sqrt{2}}$

82. $\frac{1+\sqrt{7}}{2-\sqrt{3}}=\frac{1+\sqrt{7}}{2-\sqrt{3}}\cdot\frac{1-\sqrt{7}}{1-\sqrt{7}}$
$=\frac{1-7}{2-2\sqrt{7}-\sqrt{3}+\sqrt{3}\sqrt{7}}$
$=\frac{-6}{2-2\sqrt{7}-\sqrt{3}+\sqrt{21}}$

83. $x=\sqrt{\frac{kM}{f}}$

Note that because x represents the number of units to order, the value of x should be rounded to the nearest integer.

a. $k=\$1, f=\$500, M=100{,}000$
$x=\sqrt{\frac{1\cdot 100{,}000}{500}}=\sqrt{200}\approx 14.1$
The number of units to order is 14.

b. $k=\$3, f=\$7, M=16{,}700$
$x=\sqrt{\frac{3\cdot 16{,}700}{7}}\approx 84.6$
The number of units to order is 85.

c. $k=\$1, f=\$5, M=16{,}800$
$x=\sqrt{\frac{1\cdot 16{,}800}{5}}=\sqrt{3360}\approx 58.0$
The number of units to order is 58.

84. $h=12.3T^{1/3}$
If $T=216$, find h.
$h=12.3(216)^{1/3}=73.8$
A height of 73.8 in. corresponds to a threshold weight of 216 lb.

For exercises 85–88, we use the model
revenue $=8.19x^{0.096}$, $x\ge 1$, $x=1$ corresponds to 2001.

85. Let $x=10$. Then $8.19(10)^{0.096}\approx 10.2$
The domestic revenue for 2010 were about \$10,200,000,000.

86. Let $x=13$. Then $8.19(13)^{0.096}\approx 10.5$
The domestic revenue for 2013 will be about \$10,500,000,000.

87. Let $x=15$. Then $8.19(15)^{0.096}\approx 10.6$
The domestic revenue for 2015 will be about \$10,600,000,000.

88. Let $x=18$. Then $8.19(18)^{0.096}\approx 10.8$
The domestic revenue for 2018 will be about \$10,800,000,000.

For exercises 89–92, we use the model death rate = $262.5x^{-.156}$, $x \ge 1$, $x = 1$ corresponds to 2001.

89. Let $x = 11$. Then $262.5(11)^{-.156} \approx 180.6$
The death rate associated with heart disease in 2011 was approximately 180.6.

90. Let $x = 13$. Then $262.5(13)^{-.156} \approx 175.9$
The death rate associated with heart disease in 2013 will be approximately 175.9.

91. Let $x = 17$. Then $262.5(17)^{-.156} \approx 168.7$
The death rate associated with heart disease in 2017 will be approximately 168.7.

92. Let $x = 20$. Then $262.5(20)^{-.156} \approx 164.5$
The death rate associated with heart disease in 2020 will be approximately 164.5.

For exercises 93–96, we use the model Pell Grant Aid $= 3.96x^{0.239}$; $x \ge 1$, $x = 1$corresponds to 2001.

93. Let $x = 5$. Then $3.96(5)^{0.239} \approx 5.8$
According to the model, there were approximately 5,800,000 students receiving Pell Grants in 2005.

94. Let $x = 10$. Then $3.96(10)^{0.239} \approx 6.9$
According to the model, there were approximately 6,900,000 students receiving Pell Grants in 2010.

95. Let $x = 13$. Then $3.96(13)^{0.239} \approx 7.3$
According to the model, there will be approximately 7,300,000 students receiving Pell Grants in 2013.

96. Let $x = 18$. Then $3.96(18)^{0.239} \approx 7.9$
According to the model, there will be approximately 7,900,000 students receiving Pell Grants in 2018.

For exercises 97–100, we use the model Annual CT scans $= 3.5x^{1.04}$, $x \ge 5$, $x = 5$ corresponds to 1995.

97. Let $x = 8$. Then $3.5(8)^{1.04} \approx 30.4$
The number of annual CT scans for 1998 were about 30,400,000.

98. Let $x = 15$. Then $3.5(15)^{1.04} \approx 58.5$
The number of annual CT scans for 2005 were about 58,500,000.

99. Let $x = 22$. Then $3.5(22)^{1.04} \approx 87.1$
The number of annual CT scans for 2012 were about 87,100,000.

100. Let $x = 23$. Then $3.5(23)^{1.04} \approx 91.3$
The number of annual CT scans for 2013 will be about 91,300,000

Section 1.6 First-Degree Equations

1.
$$\begin{aligned} 3x + 8 &= 20 \\ 3x + 8 - 8 &= 20 - 8 \\ 3x &= 12 \\ \frac{1}{3}(3x) &= \frac{1}{3}(12) \\ x &= 4 \end{aligned}$$

2. $4 - 5y = 19$
$$\begin{aligned} 4 - 5y + (-4) &= 19 + (-4) \\ -5y &= 15 \\ -\frac{1}{5}(-5y) &= -\frac{1}{5}(15) \\ y &= -3 \end{aligned}$$

3.
$$\begin{aligned} .6k - .3 &= .5k + .4 \\ .6k - .5k - .3 &= .5k - .5k + .4 \\ .1k - .3 &= .4 \\ .1k - .3 + .3 &= .4 + .3 \\ .1k &= .7 \\ \frac{.1k}{.1} &= \frac{.7}{.1} \Rightarrow k = 7 \end{aligned}$$

4.
$$\begin{aligned} 2.5 + 5.04m &= 8.5 - .06m \\ 2.5 + 5.04m + .06m &= 8.5 - .06m + .06m \\ 2.5 + 5.1m &= 8.5 \\ 2.5 + 5.1m + (-2.5) &= 8.5 + (-2.5) \\ 5.1m &= 6.0 \\ \frac{5.1m}{5.1} &= \frac{6.0}{5.1} \\ m &= \frac{6.0}{5.1} \Rightarrow m \approx 1.18 \end{aligned}$$

5.
$$\begin{aligned} 2a-1 &= 4(a+1)+7a+5 \\ 2a-1 &= 4a+4+7a+5 \\ 2a-1 &= 11a+9 \\ 2a-2a-1 &= 11a-2a+9 \\ -1 &= 9a+9 \\ -1-9 &= 9a+9-9 \\ -10 &= 9a \\ \frac{-10}{9} &= \frac{9a}{9} \Rightarrow -\frac{10}{9} = a \end{aligned}$$

6.
$$\begin{aligned} 3(k-2)-6 &= 4k-(3k-1) \\ 3k-6-6 &= 4k-3k+1 \\ 3k-12 &= k+1 \\ 3k-12+(-k) &= k+1+(-k) \\ 2k-12 &= 1 \\ 2k-12+12 &= 1+12 \\ 2k &= 13 \\ \frac{2k}{2} &= \frac{13}{2} \Rightarrow k = \frac{13}{2} \end{aligned}$$

7.
$$\begin{aligned} 2[x-(3+2x)+9] &= 3x-8 \\ 2(x-3-2x+9) &= 3x-8 \\ 2(-x+6) &= 3x-8 \\ -2x+12 &= 3x-8 \\ 12 &= 5x-8 \\ 20 &= 5x \Rightarrow 4 = x \end{aligned}$$

8.
$$\begin{aligned} -2\left[4(k+2)-3(k+1)\right] &= 14+2k \\ -2(4k+8-3k-3) &= 14+2k \\ -2(k+5) &= 14+2k \\ -2k-10 &= 14+2k \\ -2k-10+2k &= 14+2k+2k \\ -10 &= 14+4k \\ -10-14 &= 14+4k-14 \\ -24 &= 4k \\ \frac{-24}{4} &= \frac{4k}{4} \Rightarrow -6 = k \end{aligned}$$

9. $\frac{3x}{5}-\frac{4}{5}(x+1) = 2-\frac{3}{10}(3x-4)$

Multiply both sides by the common denominator, 10.

$$10\left(\frac{3x}{5}\right)-10\left(\frac{4}{5}\right)(x+1)$$
$$= (10)(2)-(10)\left(\frac{3}{10}\right)(3x-4)$$
$$\begin{aligned} 2(3x)-8(x+1) &= 20-3(3x-4) \\ 6x-8x-8 &= 20-9x+12 \\ -2x-8 &= 32-9x \\ -2x+9x &= 32+8 \\ 7x &= 40 \\ \frac{1}{7}(7x) &= \frac{1}{7}(40) \Rightarrow x = \frac{40}{7} \end{aligned}$$

10.
$$\begin{aligned} \frac{4}{3}(x-2)-\frac{1}{2} &= 2\left(\frac{3}{4}x-1\right) \\ \frac{4}{3}x-\frac{8}{3}-\frac{1}{2} &= \frac{3}{2}x-2 \\ \frac{4}{3}x-\frac{19}{6} &= \frac{3}{2}x-2 \\ \frac{4}{3}x-\frac{19}{6}-\frac{4}{3}x &= \frac{3}{2}x-2-\frac{4}{3}x \\ -\frac{19}{6} &= \frac{1}{6}x-2 \\ -\frac{19}{6}+2 &= \frac{1}{6}x-2+2 \\ -\frac{7}{6} &= \frac{1}{6}x \\ 6\left(-\frac{7}{6}\right) &= 6\left(\frac{1}{6}\right)x \Rightarrow -7 = x \end{aligned}$$

11.
$$\begin{aligned} \frac{5y}{6}-8 &= 5-\frac{2y}{3} \\ 6\left(\frac{5y}{6}-8\right) &= 6\left(5-\frac{2y}{3}\right) \\ 6\left(\frac{5y}{6}\right)-6(8) &= 6(5)-6\left(\frac{2y}{3}\right) \\ 5y-48 &= 30-4y \\ 9y-48 &= 30 \\ 9y &= 78 \\ y &= \frac{78}{9} = \frac{26}{3} \end{aligned}$$

12. $\frac{x}{2}-3=\frac{3x}{5}+1$

Multiply both sides by the common denominator, 10, to eliminate the fractions.

$$10\left(\frac{x}{2}-3\right)=10\left(\frac{3x}{5}+1\right)$$
$$5x-30=6x+10$$
$$5x-30-5x=6x+10-5x$$
$$-30=x+10$$
$$-30-10=x+10-10$$
$$-40=x$$

13. $$\frac{m}{2}-\frac{1}{m}=\frac{6m+5}{12}$$
$$12m\left(\frac{m}{2}-\frac{1}{m}\right)=12m\left(\frac{6m+5}{12}\right)$$
$$(12m)\left(\frac{m}{2}\right)-(12m)\left(\frac{1}{m}\right)=m(6m)+m(5)$$
$$6m^2-12=6m^2+5m$$
$$-12=5m$$
$$\frac{1}{5}(-12)=\frac{1}{5}(5m)\Rightarrow -\frac{12}{5}=m$$

14. $-\frac{3k}{2}+\frac{9k-5}{6}=\frac{11k+8}{k}$

Multiply both sides by the common denominator, 6*k* to eliminate the fractions.

$$6k\left(-\frac{3k}{2}+\frac{9k-5}{6}\right)=6k\left(\frac{11k+8}{k}\right)$$
$$6k\left(-\frac{3k}{2}\right)+6k\left(\frac{9k-5}{6}\right)=6k\left(\frac{11k}{k}\right)+6k\left(\frac{8}{k}\right)$$
$$-9k^2+k(9k-5)=6(11k)+6(8)$$
$$-9k^2+9k^2-5k=66k+48$$
$$-5k=66k+48$$
$$-5k-66k=66k+48-66k$$
$$-71k=48$$
$$\frac{-71k}{-71}=\frac{48}{-71}\Rightarrow k=-\frac{48}{71}$$

15. $\frac{4}{x-3}-\frac{8}{2x+5}+\frac{3}{x-3}=0$

$$\frac{4}{x-3}+\frac{3}{x-3}-\frac{8}{2x+5}=0$$
$$\frac{7}{x-3}-\frac{8}{2x+5}=0$$

Multiply each side by the common denominator, $(x-3)(2x+5)$.

$$(x-3)(2x+5)\left(\frac{7}{x-3}\right)-(x-3)(2x+5)\left(\frac{8}{2x+5}\right)=(x-3)(2x+5)(0)$$
$$7(2x+5)-8(x-3)=0$$
$$14x+35-8x+24=0$$
$$6x+59=0$$
$$6x=-59\Rightarrow x=-\frac{59}{6}$$

16. $$\frac{5}{2p+3}-\frac{3}{p-2}=\frac{4}{2p+3}$$
$$\frac{5}{2p+3}-\frac{3}{p-2}-\frac{5}{2p+3}=\frac{4}{2p+3}-\frac{5}{2p+3}$$
$$-\frac{3}{p-2}=-\frac{1}{2p+3}$$

Multiply both sides by the common denominator, $(2p+3)(p-2)$.

$$\left(-\frac{3}{p-2}\right)(p-2)(2p+3)=\left(-\frac{1}{2p+3}\right)(p-2)(2p+3)$$
$$-3(2p+3)=-1(p-2)$$
$$-6p-9=-p+2$$
$$-5p=11\Rightarrow p=-\frac{11}{5}$$

17. $\frac{3}{2m+4}=\frac{1}{m+2}-2$

$$\frac{3}{2(m+2)}=\frac{1}{m+2}-2$$
$$2(m+2)\left(\frac{3}{2(m+2)}\right)=2(m+2)\left(\frac{1}{m+2}\right)-2(m+2)(2)$$
$$3=2-4(m+2)$$
$$3=2-4m-8$$
$$3=-6-4m\Rightarrow 9=-4m\Rightarrow m=-\frac{9}{4}$$

18. $\dfrac{8}{3k-9}-\dfrac{5}{k-3}=4$

Multiply both sides by the common denominator, $3k-9$.

$$(3k-9)\left[\frac{8}{3k-9}-\frac{5}{k-3}\right]=(3k-9)4$$
$$(3k-9)\left(\frac{8}{3k-9}\right)+3(k-3)\left(-\frac{5}{k-3}\right)=12k-36$$
$$8-15=12k-36$$
$$-7=12k-36$$
$$29=12k\Rightarrow\frac{29}{12}=k$$

19.
$$9.06x+3.59(8x-5)=12.07x+.5612$$
$$9.06x+28.72x-17.95=12.07x+.5612$$
$$9.06x+28.72x-12.07x=17.95+.5612$$
$$25.71x=18.5112$$
$$x=\frac{18.5112}{25.71}=.72$$

20.
$$-5.74(3.1-2.7p)=1.09p+5.2588$$
$$-17.794+15.498p=1.09p+5.2588$$
$$15.498p-1.09p=5.2588+17.794$$
$$14.408p=23.0528$$
$$p=\frac{23.0528}{14.408}=1.6$$

21. $\dfrac{2.63r-8.99}{1.25}-\dfrac{3.90r-1.77}{2.45}=r$

Multiply by the common denominator (1.25)(2.45) to eliminate the fractions.

$$(2.45)(2.63r-8.99)-(1.25)(3.90r-1.77)$$
$$=(2.45)(1.25)r$$
$$6.4435r-22.0255-4.875r+2.2125=3.0625r$$
$$1.5685r-19.813=3.0625r$$
$$-19.813=1.494r$$
$$-\frac{19.813}{1.494}=\frac{1.494r}{1.494}$$
$$r\approx-13.26$$

22. $\dfrac{8.19m+2.55}{4.34}-\dfrac{8.17m-9.94}{1.04}=4m$

$$(1.04)(8.19m+2.55)-(4.34)(8.17m-9.94)$$
$$=4m(1.04)(4.34)$$
$$8.5176m+2.652-35.4578m+43.1396$$
$$=18.0544m$$
$$-26.9402m+45.7916=18.0544m$$
$$45.7916=44.9946m$$
$$m=\frac{45.7916}{44.9946}\Rightarrow m\approx1.02$$

23.
$$4(a+x)=b-a+2x$$
$$4a+4x=b-a+2x$$
$$4a=b-a-2x$$
$$5a-b=-2x$$
$$\frac{5a-b}{-2}=\frac{-2x}{-2}$$
$$-\frac{5a-b}{2}=x\text{ or }x=\frac{b-5a}{2}$$

24.
$$(3a-b)-bx=a(x-2)$$
$$3a-b-bx=ax-2a$$
$$3a-b=ax-2a+bx$$
$$5a-b=ax+bx$$
$$5a-b=(a+b)x\Rightarrow\frac{5a-b}{a+b}=x$$

25.
$$5(b-x)=2b+ax$$
$$5b-5x=2b+ax$$
$$5b=2b+ax+5x$$
$$3b=ax+5x$$
$$3b=(a+5)x$$
$$\frac{3b}{a+5}=\frac{(a+5)x}{a+5}\Rightarrow\frac{3b}{a+5}=x$$

26. $bx-2b=2a-ax$

Isolate terms with x on the left.

$$bx+ax=2a+2b$$
$$ax+bx=2a+2b$$
$$(a+b)x=2(a+b)$$
$$x=\frac{2(a+b)}{a+b}\Rightarrow x=2$$

27. $PV=k$ for V

$$\frac{1}{P}(PV)=\frac{1}{P}(k)\Rightarrow V=\frac{k}{P}$$

28. $i = prt$ for p

$\frac{i}{rt} = p$

29. $V = V_0 + gt$ for g

$V - V_0 = gt$

$\frac{V - V_0}{t} = \frac{gt}{t} \Rightarrow \frac{V - V_0}{t} = g$

30. $S = S_0 + gt^2 + k$

$S - S_0 - k = gt^2$

$\frac{S - S_0 - k}{t^2} = \frac{gt^2}{t^2} \Rightarrow \frac{S - S_0 - k}{t^2} = g$

31. $A = \frac{1}{2}(B + b)h$ for B

$A = \frac{1}{2}Bh + \frac{1}{2}bh$

$2A = Bh + bh$ Multiply by 2.

$2A - bh = Bh$

$\frac{2A - bh}{h} = \frac{Bh}{h}$ Multiply by $\frac{1}{h}$.

$\frac{2A - bh}{h} = \frac{2A}{h} - b = B$

32. $C = \frac{5}{9}(F - 32)$ for F

$\frac{9}{5}C = F - 32 \Rightarrow \frac{9}{5}C + 32 = F$

33. $|2h - 1| = 5$

$2h - 1 = 5$ or $2h - 1 = -5$

$2h = 6$ or $2h = -4$

$h = 3$ or $h = -2$

34. $|4m - 3| = 12$

$4m - 3 = 12$ or $4m - 3 = -12$

$4m = 15$ or $4m = -9$

$m = \frac{15}{4}$ or $m = -\frac{9}{4}$

35. $|6 + 2p| = 10$

$6 + 2p = 10$ or $6 + 2p = -10$

$2p = 4$ or $2p = -16$

$p = 2$ or $p = -8$

36. $|-5x + 7| = 15$

$-5x + 7 = 15$ or $-5x + 7 = -15$

$-5x = 8$ or $-5x = -22$

$x = -\frac{8}{5}$ or $x = \frac{22}{5}$

37. $\left|\frac{5}{r - 3}\right| = 10$

$\frac{5}{r - 3} = 10$ or $\frac{5}{r - 3} = -10$

$5 = 10(r - 3)$ or $5 = -10(r - 3)$

$5 = 10r - 30$ or $5 = -10r + 30$

$35 = 10r$ or $-25 = -10r$

$\frac{35}{10} = \frac{7}{2} = r$ or $\frac{-25}{-10} = \frac{5}{2} = r$

38. $\left|\frac{3}{2h - 1}\right| = 4$

$\frac{3}{2h - 1} = 4$ or $\frac{3}{2h - 1} = -4$

$3 = 4(2h - 1)$ or $3 = -4(2h - 1)$

$3 = 8h - 4$ or $3 = -8h + 4$

$7 = 8h$ or $-1 = -8h$

$\frac{7}{8} = h$ or $\frac{1}{8} = h$

39. $1.250 = \frac{x}{8} \Rightarrow x = 10$

The stroke lasted 10 hours.

40. $2.4 = \frac{x}{8} \Rightarrow x = 19.2$

The stroke lasted 19.2 hours.

41. $-5 = \frac{5}{9}(F - 32)$

$-5\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$-9 = F - 32 \Rightarrow 23 = F$

The temperature –5°C = 23°F.

42. $-15 = \frac{5}{9}(F - 32)$

$-15\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$-27 = F - 32 \Rightarrow 5 = F$

The temperature –15°C = 5°F.

43. $22 = \frac{5}{9}(F - 32)$

$22\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$39.6 = F - 32 \Rightarrow 71.6 = F$

The temperature 22°C = 71.6°F.

44. $36 = \frac{5}{9}(F - 32)$

$36\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$

$64.8 = F - 32 \Rightarrow 96.8 = F$

The temperature 36°C = 96.8°F.

45. $y = 1.16x + 1.76$

Substitute 13.36 for y.

$13.36 = 1.16x + 1.76$

$11.6 = 1.16x \Rightarrow 10 = x$

Therefore, the federal deficit will be $13.36 trillion in 2010.

46. $y = 1.16x + 1.76$

Substitute 16.84 for y.

$16.84 = 1.16x + 1.76$

$15.08 = 1.16x \Rightarrow 13 = x$

Therefore, the federal deficit will be $16.84 trillion in 2013.

47. $y = 1.16x + 1.76$

Substitute 19.16 for y.

$19.16 = 1.16x + 1.76$

$17.4 = 1.16x \Rightarrow 15 = x$

Therefore, the federal deficit will be $19.16 trillion in 2015.

48. $y = 1.16x + 1.76$

Substitute 24.96 for y.

$24.96 = 1.16x + 1.76$

$23.2 = 1.16x \Rightarrow 20 = x$

Therefore, the federal deficit will be $24.96 trillion in 2020.

49. $E = .118x + 1.45$

Substitute $2.63 in for E.

$2.63 = .118x + 1.45$

$1.18 = .118x \Rightarrow 10 = x$

The health care expenditures were $2.63 trillion in 2010.

50. $E = .118x + 1.45$

Substitute $2.866 in for E.

$2.866 = .118x + 1.45$

$1.416 = .118x \Rightarrow 12 = x$

The health care expenditures were $2.866 trillion in 2012.

51. $E = .118x + 1.45$

Substitute $3.338 in for E.

$3.338 = .118x + 1.45$

$1.888 = .118x \Rightarrow 16 = x$

The health care expenditures will be $3.338 trillion in 2016.

52. $E = .118x + 1.45$

Substitute $3.574 in for E.

$3.574 = .118x + 1.45$

$2.124 = .118x \Rightarrow 18 = x$

The health care expenditures will be $3.574 trillion in 2018.

53. $.09(x - 2004) = 12y - 1.44$

Substitute .18 for y and solve for x.

$.09(x - 2004) = 12(.18) - 1.44$

$.09x - 180.36 = .72$

$.09x = 181.08$

$x = 2012$

18.0% of workers were covered in 2012.

54. $.09(x - 2004) = 12y - 1.44$

Substitute .195 for y and solve for x.

$.09(x - 2004) = 12(.195) - 1.44$

$.09x - 180.36 = .9$

$.09x = 181.26$

$x = 2014$

19.5% of workers will be covered in 2014.

55. $.09(x - 2004) = 12y - 1.44$

Substitute .21 for y and solve for x.

$.09(x - 2004) = 12(.21) - 1.44$

$.09x - 180.36 = 1.08$

$.09x = 181.44$

$x = 2016$

21% of workers will be covered in 2016.

56. $.09(x-2004)=12y-1.44$

Substitute .2325 for y and solve for x.

$$.09(x-2004)=12(.2325)-1.44$$
$$.09x-180.36=1.35$$
$$.09x=181.71$$
$$x=2019$$

23.25% of workers will be covered in 2019.

57. $A=4.35x-12$

Substitute 20,777 for A and solve for x.

$$20777=4.35x-12$$
$$20789=4.35x \Rightarrow x\approx 4779$$

California had approximately 4,779,000 tax returns filed in 2013.

58. $A=4.35x-12$

Substitute 13,732 for A and solve for x.

$$13732=4.35x-12$$
$$13744=4.35x \Rightarrow x\approx 3160$$

New York had approximately 3,160,000 tax returns filed in 2013.

59. $A=4.35x-12$

Substitute 13,360 for A and solve for x.

$$13360=4.35x-12$$
$$13372=4.35x \Rightarrow x\approx 3074$$

Texas had approximately 3,074,000 tax returns filed in 2013.

60. $A=4.35x-12$

Substitute 9596 for A and solve for x.

$$9596=4.35x-12$$
$$9608=4.35x \Rightarrow x\approx 2209$$

Florida had approximately 2,209,000 tax returns filed in 2013.

61. $f=800$, $n=18$, $q=36$

$$u=f\cdot\frac{n(n+1)}{q(q+1)}=800\cdot\frac{18(19)}{36(37)}$$
$$=800\cdot\frac{342}{1332}\approx 205.41$$

The amount of unearned interest is \$205.41.

62. $f=1400$, $q=48$, $n=12$

$$u=f\cdot\frac{n(n+1)}{q(q+1)}=1400\cdot\frac{12(12+1)}{48(48+1)}\approx \$92.86$$

The amount of unearned interest is \$92.86.

63. Let x = the number invested at 5%.
Then 52,000 – x = the amount invested at 4%.
Since the total interest is \$2290, we have

$$.05x+.04(52,000-x)=2290$$
$$.05x+2080-.04x=2290$$
$$.01x+2080=2290$$
$$.01x+2080-2080=2290-2080$$
$$.01x=210$$
$$\frac{.01x}{.01x}=\frac{210}{.01}$$
$$x=21,000$$

Joe invested \$21,000 at 5%.

64. Let x represent the amount invested at 4%. Then 20,000 – x is the amount invested at 6%. Since the total interest is \$1040, we have

$$.04x+.06(20,000-x)=1040$$
$$.04x+1200-.06x=1040$$
$$-.02x+1200=1040$$
$$-.02x=-160$$
$$x=8000$$

She invested \$8000 at 4%.

65. Let x = price of first plot.
Then 120,000 – x = price of second plot.
.15x = profit from first plot
–.10(120,000 – x) = loss from second plot.

$$.15x-.10(120,000-x)=5500$$
$$.15x-12,000+.10x=5500$$
$$.25x=17,500$$
$$x=70,000$$

Maria paid \$70,000 for the first plot and 120,000 – 70,000, or \$50,000 for the second plot.

66. Let x represent the amount invested at 4%. \$20,000 invested at 5% (or .05) plus x dollars invested at 4% (or .04) must equal 4.8% (or .048) of the total investment (or 20,000 + x). Solve this equation.

$$.05(20,000)+.04x=.048(20,000+x)$$
$$1000+.04x=960+.048x$$
$$1000=960+.008x$$
$$40=.008x \Rightarrow x=5000$$

\$5000 should be invested at 4%.

67. Let x = average rate of growth of Tumblr.com. Then 450,000 + x = average rate of growth of Pinterest.com.
$63x$ = visitors to Tumblr.com
$30(450{,}000 + x)$ = visitors to Pinterest.com.

$$63x = 30(450,000 + x)$$
$$63x = 13,500,000 + 30x$$
$$33x = 13,500,000$$
$$x = 409,091$$

Since x represents the average rate of growth of Tumblr.com, there was an average growth of 409,091 visitors.

68. The average rate of growth of Pinterest.com was 450,000 + x or 859,091 visitors.

69. The number of visitors to Tumblr.com was $63x$ = 63(409091) = 25,772,733.

70. The number of visitors to Pinterest.com was $30(450{,}000 + x)$ = 30(450,000+409,091) = 25,772,730 (Note: Due to rounding error, the values in 69 and 70 are different)

71. Let x = the number of liters of 94 octane gas; 200 = the number of liters of 99 octane gas; 200 + x = the number of liters of 97 octane gas.

$$94x + 99(200) = 97(200 + x)$$
$$94x + 19,800 = 19,400 + 97x$$
$$400 = 3x$$
$$\frac{400}{3} = x$$

Thus, $\frac{400}{3}$ liters of 94 octane gas are needed.

72. Let x be the amount of 92 octane gasoline. Then $12 - x$ is the amount of 98 octane gasoline. A mixture of the two must yield 12 L of 96 octane gasoline, so

$$92x + 98(12 - x) = 96(12)$$
$$92x + 1176 - 98x = 1152$$
$$-6x = -24$$
$$x = 4$$

$12 - x = 12 - 4 = 8$.
Mix 4 L of 92 octane gasoline with 8 L of 98 octane gasoline.

73. Let x = number of miles driven

$$55 + .22x = 78$$
$$.22x = 23$$
$$x = 105$$

You must drive 105 miles in a day for the costs to be equal.

74. Let x = the amount of fluid drained and replaced with 100% antifreeze. The given information can be used to fill out a table in the following way.

Quantity	% Antifreeze	Amount of Antifreeze
$8.5 - x$	35%	$.35(8.5 - x)$
x	100%	x
8.5	65%	.65(8.5)

$$.35(8.5 - x) + x = .65(8.5)$$
$$2.975 - .35x + x = 5.525$$
$$.65x = 2.55$$
$$x \approx 3.9 \text{ quarts}$$

75. $y = 10(x - 75) + 100$
Substitute 180 in for y.

$$180 = 10(x - 75) + 100$$
$$80 = 10(x - 75)$$
$$8 = x - 75 \Rightarrow 83 = x$$

Paul was driving 83 mph.

76. $y = 10(x - 75) + 100$
Substitute 120 in for y.

$$120 = 10(x - 75) + 100$$
$$20 = 10(x - 75)$$
$$2 = x - 75 \Rightarrow 77 = x$$

Sarah was driving 77 mph.

77. Let x = the number of gallons of premium gas. The number of gallons of regular gas = $15.5 - x$

$$3.80x + 3.10(15.5 - x) = 50$$
$$3.80x + 48.05 - 3.10x = 50$$
$$.7x = 1.95 \Rightarrow x = 2.8$$
$$15.5 - x = 15.5 - 2.8 = 12.7$$

Jack should get 2.8 gallons of premium gas and 12.7 gallons of regular gas.

78. Let x = the number of gallons of premium gas.
The number of gallons of regular gas = $15.5 - x$

$3.80x + 3.10(15.5 - x) = 53$

$3.80x + 48.05 - 3.10x = 53$

$.7x = 4.95 \Rightarrow x = 7.1$

$15.5 - x = 15.5 - 7.1 = 8.4$

Jack should get 7.1 gallons of premium gas and 8.4 gallons of regular gas.

Section 1.7 Quadratic Equations

1. $(x + 4)(x - 14) = 0$

$x + 4 = 0$ or $x - 14 = 0$

$x = -4$ or $x = 14$

The solutions are –4 and 14.

2. $(p - 16)(p - 5) = 0$

$p - 16 = 0$ or $p - 5 = 0$

$p = 16$ or $p = 5$

The solutions are 16 and 5.

3. $x(x + 6) = 0$

$x = 0$ or $x + 6 = 0$

$x = -6$

The solutions are 0 and –6.

4. $x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ or $x - 2 = 0$

$x = 2$

The solutions are 0 and 2.

5. $2z^2 = 4z$

$2z^2 - 2z = 0$

$2z(z - 2) = 0$

$2z = 0$ or $z - 2 = 0$

$z = 0$ or $z = 2$

The solutions are 0 and 2.

6. $x^2 - 64 = 0$

$(x - 8)(x + 8) = 0$

$x - 8 = 0$ or $x + 8 = 0$

$x = 8$ or $x = -8$

The solutions are 8 and –8.

7. $y^2 + 15y + 56 = 0$

$(y + 7)(y + 8) = 0$

$y + 7 = 0$ or $y + 8 = 0$

$y = -7$ or $y = -8$

The solutions are –7 and –8.

8. $k^2 - 4k - 5 = 0$

$(k + 1)(k - 5) = 0$

$k + 1 = 0$ or $k - 5 = 0$

$k = -1$ or $k = 5$

The solutions are –1 and 5.

9. $2x^2 = 7x - 3$

$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$2x - 1 = 0$ or $x - 3 = 0$

$x = \frac{1}{2}$ or $x = 3$

The solutions are $\frac{1}{2}$ and 3.

10. $2 = 15z^2 + z$

$0 = 15z^2 + z - 2$

$0 = (3z - 1)(5z + 2)$

$3z - 1 = 0$ or $5z + 2 = 0$

$z = \frac{1}{3}$ or $z = -\frac{2}{5}$

The solutions are $\frac{1}{3}$ and $-\frac{2}{5}$.

11. $6r^2 + r = 1$

$6r^2 + r - 1 = 0$

$(3r - 1)(2r + 1) = 0$

$3r - 1 = 0$ or $2r + 1 = 0$

$r = \frac{1}{3}$ or $r = -\frac{1}{2}$

The solutions are $\frac{1}{3}$ and $-\frac{1}{2}$.

12. $3y^2 = 16y - 5$

$3y^2 - 16y + 5 = 0$

$(3y - 1)(y - 5) = 0$

$3y - 1 = 0$ or $y - 5 = 0$

$y = \frac{1}{3}$ or $y = 5$

The solutions are $\frac{1}{3}$ and 5.

13. $2m^2 + 20 = 13m$

$2m^2 - 13m + 20 = 0$

$(2m - 5)(m - 4) = 0$

$2m - 5 = 0$ or $m - 4 = 0$

$m = \frac{5}{2}$ or $m = 4$

The solutions are $\frac{5}{2}$ and 4.

14. $6a^2 + 17a + 12 = 0$

$(2a + 3)(3a + 4) = 0$

$2a + 3 = 0$ or $3a + 4 = 0$

$a = -\frac{3}{2}$ or $a = -\frac{4}{3}$

The solutions are $-\frac{3}{2}$ and $-\frac{4}{3}$.

15. $m(m + 7) = -10$

$m^2 + 7m + 10 = 0$

$(m + 5)(m + 2) = 0$

$m + 5 = 0$ or $m + 2 = 0$

$m = -5$ or $m = -2$

The solutions are –5 and –2.

16. $z(2z + 7) = 4$

$2z^2 + 7z - 4 = 0$

$(2z - 1)(z + 4) = 0$

$2z - 1 = 0$ or $z + 4 = 0$

$z = \frac{1}{2}$ or $z = -4$

The solutions are $\frac{1}{2}$ and –4.

17. $9x^2 - 16 = 0$

$(3x + 4)(3x - 4) = 0$

$3x + 4 = 0$ or $3x - 4 = 0$

$3x = -4$ $\quad 3x = 4$

$x - \frac{4}{3}$ or $x = \frac{4}{3}$

The solutions are $-\frac{4}{3}$ and $\frac{4}{3}$.

18. $36y^2 - 49 = 0$

$(6y - 7)(6y + 7) = 0$

$6y - 7 = 0$ or $6y + 7 = 0$

$y = \frac{7}{6}$ or $y = -\frac{7}{6}$

The solutions are $\frac{7}{6}$ and $-\frac{7}{6}$.

19. $16x^2 - 16x = 0$

$16x(x - 1) = 0$

$16x = 0$ or $x - 1 = 0$

$x = 0$ or $x = 1$

The solutions are 0 and 1.

20. $12y^2 - 48y = 0$

$12y(y - 4) = 0$

$y = 0$ or $y - 4 = 0$

$y = 0$ or $y = 4$

The solutions are 0 and 4.

21. $(r - 2)^2 = 7$

$r - 2 = \sqrt{7}$ or $r - 2 = -\sqrt{7}$

$r = 2 + \sqrt{7}$ or $r = 2 - \sqrt{7}$

We abbreviate the solutions as $2 \pm \sqrt{7}$.

22. $(b + 4)^2 = 27$

$b + 4 = \sqrt{27}$ or $b + 4 = -\sqrt{27}$

$b + 4 = 3\sqrt{3}$ or $b + 4 = -3\sqrt{3}$

$b = -4 + 3\sqrt{3}$ or $b = -4 - 3\sqrt{3}$

We abbreviate the solutions as $-4 \pm 3\sqrt{3}$.

23. $(4x-1)^2 = 20$

Use the square root property.

$4x-1=\sqrt{20}$ or $4x-1=-\sqrt{20}$

$4x-1=2\sqrt{5}$ or $4x-1=-2\sqrt{5}$

$4x=1+2\sqrt{5}$ or $4x=1-2\sqrt{5}$

The solutions are $\frac{1\pm 2\sqrt{5}}{4}$.

24. $(3t+5)^2 = 11$

$3t+5=\sqrt{11}$ or $3t+5=-\sqrt{11}$

$3t=-5+\sqrt{11}$ or $3t=-5-\sqrt{11}$

$t=\frac{-5+\sqrt{11}}{3}$ or $t=\frac{-5\pm\sqrt{11}}{3}$

The solutions are $\frac{-5\pm\sqrt{11}}{3}$.

25. $2x^2+7x+1=0$

Use the quadratic formula with $a = 2$, $b = 7$, and $c = 1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-7\pm\sqrt{7^2-4(2)(1)}}{2(2)}$$
$$=\frac{-7\pm\sqrt{49-8}}{4}=\frac{-7\pm\sqrt{41}}{4}$$

The solutions are $\frac{-7+\sqrt{41}}{4}$ and $\frac{-7-\sqrt{41}}{4}$, which are approximately –.1492 and –3.3508.

26. $3x^2-x-7=0$

Use the quadratic formula with $a = 3$, $b = -1$, and $c = -7$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-7)}}{2(3)}$$
$$=\frac{1\pm\sqrt{1+84}}{6}=\frac{1\pm\sqrt{85}}{6}$$

The solutions $\frac{1\pm\sqrt{85}}{6}$ are approximately 1.7033 and –1.3699.

27. $4k^2+2k=1$

Rewrite the equation in standard form.

$4k^2+2k-1=0$

Use the quadratic formula with $a = 4$, $b = 2$, and $c = -1$.

$$k=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}$$
$$=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm 2\sqrt{5}}{8}=\frac{2\left(-1\pm\sqrt{5}\right)}{2\cdot 4}$$
$$k=\frac{-1\pm\sqrt{5}}{4}$$

The solutions are $\frac{-1+\sqrt{5}}{4}$ and $\frac{-1-\sqrt{5}}{4}$, which are approximately .309 and –.809.

28. $r^2=3r+5$

$r^2-3r-5=0$

Use the quadratic formula with $a = 1$, $b = -3$, and $c = -5$.

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$r=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-5)}}{2(1)}$$
$$=\frac{3\pm\sqrt{9+20}}{2}=\frac{3\pm\sqrt{29}}{2}$$

The solutions $\frac{3\pm\sqrt{29}}{2}$ are approximately 4.1926 and –1.1926.

29. $5y^2+5y=2$

$5y^2+5y-2=0$

$a = 5$, $b = 5$, $c = -2$

$$y=\frac{-5\pm\sqrt{5^2-4(5)(-2)}}{2(5)}=\frac{-5\pm\sqrt{25+40}}{10}$$
$$=\frac{-5\pm\sqrt{65}}{10}=\frac{-5\pm\sqrt{65}}{10}$$

The solutions are $\frac{-5+\sqrt{65}}{10}$ and $\frac{-5-\sqrt{65}}{10}$, which are approximately .3062 and –1.3062.

30. $2z^2 + 3 = 8z$

$2z^2 - 8z + 3 = 0$

$a = 2$, $b = -8$, and $c = 3$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{64 - 24}}{4} = \frac{8 \pm \sqrt{40}}{4} = \frac{8 \pm 2\sqrt{10}}{4}$$
$$= \frac{2\left(4 \pm \sqrt{10}\right)}{4} = \frac{4 \pm \sqrt{10}}{2}$$

The solutions $\frac{4 \pm \sqrt{10}}{2}$, are approximately 3.5811 and .4189.

31. $6x^2 + 6x + 4 = 0$

$a = 6$, $b = 6$, $c = 4$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(6)(4)}}{2(6)} = \frac{-6 \pm \sqrt{36 - 96}}{12}$$
$$= \frac{-6 \pm \sqrt{-60}}{12}$$

Because $\sqrt{-60}$ is not a real number, the given equation has no real number solutions.

32. $3a^2 - 2a + 2 = 0$

$a = 3$, $b = -2$, and $c = 2$.

$$a = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(2)}}{2(3)}$$
$$= \frac{2 \pm \sqrt{4 - 24}}{6} = \frac{2 \pm \sqrt{-20}}{6}$$

Since $\sqrt{-20}$ is not a real number, the given equation has no real number solutions.

33. $2r^2 + 3r - 5 = 0$

$a = 2$, $b = 3$, $c = -5$

$$r = \frac{-(3) \pm \sqrt{9 - 4(2)(-5)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 40}}{4}$$
$$= \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$$
$$r = \frac{-3 + 7}{4} = \frac{4}{4} = 1 \text{ or } r = \frac{-3 - 7}{4} = \frac{-10}{4} = \frac{-5}{2}$$

The solutions are $-\frac{5}{2}$ and 1.

34. $8x^2 = 8x - 3$

$8x^2 - 8x + 3 = 0$

$a = 8$, $b = -8$, and $c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(3)}}{2(8)}$$
$$= \frac{8 \pm \sqrt{64 - 96}}{16} = \frac{8 \pm \sqrt{-32}}{16}$$

Because $\sqrt{-32}$ is not a real number, there are no real number solutions.

35. $2x^2 - 7x + 30 = 0$

$a = 2$, $b = -7$, $c = 30$

$$x = \frac{-(-7) \pm \sqrt{49 - 4(2)(30)}}{2(2)} = \frac{7 \pm \sqrt{-191}}{4}$$

Since $\sqrt{-191}$ is not a real number, there are no real solutions.

36. $3k^2 + k = 6$

$3k^2 + k - 6 = 0$

$a = 3$, $b = 1$, and $c = -6$.

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$k = \frac{-1 \pm \sqrt{1^2 - 4(3)(-6)}}{2(3)} = \frac{-1 \pm \sqrt{1 + 72}}{6}$$
$$k = \frac{-1 \pm \sqrt{73}}{6}$$

The solutions $\frac{-1 \pm \sqrt{73}}{6}$ are approximately 1.2573 and –1.5907.

37. $1 + \frac{7}{2a} = \frac{15}{2a^2}$

To eliminate fractions, multiply both sides by the common denominator, $2a^2$.

$2a^2 + 7a = 15 \Rightarrow 2a^2 + 7a - 15 = 0$

$a = 2$, $b = 7$, $c = -15$

$$a = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)} = \frac{-7 \pm \sqrt{49 + 120}}{4}$$
$$= \frac{-7 \pm \sqrt{169}}{4} \Rightarrow a = \frac{-7 + 13}{4} = \frac{6}{4} = \frac{3}{2} \text{ or}$$
$$a = \frac{-7 - 13}{4} = \frac{-20}{4} = -5$$

The solutions are $\frac{3}{2}$ and –5.

38. $5-\frac{4}{k}-\frac{1}{k^2}=0$

Multiply both sides by k^2.

$5k^2-4k-1=0$

$a=5, b=-4, c=-1$

$$k=\frac{-(-4)\pm\sqrt{(-4)^2-4(5)(-1)}}{2(5)}$$

$$=\frac{4\pm\sqrt{16+20}}{10}=\frac{4\pm\sqrt{36}}{10}=\frac{4\pm 6}{10}$$

$$k=\frac{4+6}{10}=\frac{10}{10}=1 \text{ or } k=\frac{4-6}{10}=\frac{-2}{10}=-\frac{1}{5}$$

The solutions are $-\frac{1}{5}$ and 1.

39. $25t^2+49=70t$

$25t^2-70t+49=0$

$$\begin{aligned}b^2-4ac&=(-70)^2-4(25)(49)\\&=4900-4900\\&=0\end{aligned}$$

The discriminant is 0.
There is one real solution to the equation.

40. $9z^2-12z=1$

$9z^2-12z-1=0$

$$\begin{aligned}b^2-4ac&=(-12)^2-4(9)(-1)\\&=144+36\\&=180\end{aligned}$$

The discriminant is positive.
There are two real solutions to the equation.

41. $13x^2+24x-5=0$

$$\begin{aligned}b^2-4ac&=(24)^2-4(13)(-5)\\&=576+260=836\end{aligned}$$

The discriminant is positive.
There are two real solutions to the equation.

42. $20x^2+19x+5=0$

$$\begin{aligned}b^2-4ac&=(19)^2-4(20)(5)\\&=361-400=-39\end{aligned}$$

The discriminant is negative.
There are no real solutions to the equation.

For Exercises 43–46 use the quadratic formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

43. $4.42x^2-10.14x+3.79=0$

$$x=\frac{-(-10.14)\pm\sqrt{(-10.14)^2-4(4.42)(3.79)}}{2(4.42)}$$

$$\approx\frac{10.14\pm 5.9843}{8.84}\approx .4701 \text{ or } 1.8240$$

44. $3x^2-82.74x+570.4923=0$

$$x=\frac{-(-82.74)\pm\sqrt{(-82.74)^2-4(3)(570.4923)}}{2(3)}$$

$$=\frac{82.74}{6}=13.79$$

45. $7.63x^2+2.79x=5.32$

$7.63x^2+2.79x-5.32=0$

$$x=\frac{-2.79\pm\sqrt{(2.79)^2-4(7.63)(-5.32)}}{2(7.63)}$$

$$\approx\frac{-2.79\pm 13.0442}{15.26}\approx -1.0376 \text{ or } .6720$$

46. $8.06x^2+25.8726x=25.047256$

$8.06x^2+25.8726x-25.047256=0$

$$x=\frac{-25.8726\pm\sqrt{(25.8726)^2-4(8.06)(-25.047256)}}{2(8.06)}$$

$$\approx\frac{-25.8726\pm 38.4307}{16.12}\approx -3.9890 \text{ or } .7790$$

47. **a.** Let $R = 450$ ft.

$450=.5x^2\Rightarrow 900=x^2\Rightarrow 30=x$

The maximum taxiing speed is 30 mph.

b. Let $R = 615$ ft.

$615=.5x^2\Rightarrow 1230=x^2\Rightarrow 35\approx x$

The maximum taxiing speed is about 35 mph.

c. Let $R = 970$ ft.

$970=.5x^2\Rightarrow 1940=x^2\Rightarrow 44\approx x$

The maximum taxiing speed is about 44 mph.

48. $E = .011x^2 + 10.7$

a. Let $E = 14.7$

$$14.7 = .011x^2 + 10.7$$
$$.011x^2 = 4$$
$$x^2 = 363.63 \Rightarrow x \approx \pm 19.07$$

The negative solution is not applicable. The enrollment is 14,700,000 approxiamtely nineteen years after 1990 or in 2009.

b. Let $E = 17.6$

$$17.6 = .011x^2 + 10.7$$
$$.011x^2 = 6.9$$
$$x^2 = 627.27 \Rightarrow x \approx \pm 25.05$$

The negative solution is not applicable. The enrollment is 17,600,000 approximately twenty-five years after 1990 or in 2015.

49. a. Let $F = 12.6$.

$$12.6 = -.079x^2 + .46x + 13.3$$
$$0 = -.079x^2 + .46x + .7$$

Store $\sqrt{b^2 - 4ac} = \sqrt{(.46)^2 - 4(-.079)(.7)}$ $\approx .6579$ in your calculator.

By the quadratic formula, $x \approx 7.1$ or $x \approx -1.25$. The negative solution is not applicable. There were 12,600 traffic fatalities in 2007.

b. Let $F = 11$.

$$11 = -.079x^2 + .46x + 13.3$$
$$0 = -.079x^2 + .46x + 2.3$$

Store $\sqrt{b^2 - 4ac} = \sqrt{(.46)^2 - 4(-.079)(2.3)}$ $\approx .9687$ in your calculator.

By the quadratic formula, $x \approx 9.04$ or $x \approx -3.22$. The negative solution is not applicable. There were 11,000 traffic fatalities in 2009.

50. a.

$$A = .237x^2 - 3.96x + 28.2$$
$$A = .237(8)^2 - 3.96(8) + 28.2$$
$$A \approx 11.7$$

The total assets in 2008 were about 11,700,000,000,000.

b.

$$A = .237x^2 - 3.96x + 28.2$$
$$12.3 = .237x^2 - 3.96x + 28.2$$
$$0 = .237x^2 - 3.96x + 15.9$$
$$x = \frac{3.96 \pm \sqrt{(-3.96)^2 - 4(.237)(15.9)}}{2(.237)}$$
$$\approx 6.71 \text{ or } 10$$

The total assets were \$12.3 trillion in 2006 and 2010. Since we are looking after 2008, the answer is the year 2010.

51. a.

$$A = .169x^2 - 2.85x + 19.6$$
$$A = .169(9)^2 - 2.85(9) + 19.6$$
$$A = 7.639$$

The total assets in 2008 were 7,639,000,000,000.

b.

$$A = .169x^2 - 2.85x + 19.6$$
$$7.9 = .169x^2 - 2.85x + 19.6$$
$$0 = .169x^2 - 2.85x + 11.7$$
$$x = \frac{2.85 \pm \sqrt{(-2.85)^2 - 4(.169)(11.7)}}{2(.169)}$$
$$\approx 7.07 \text{ or } 9.8$$

The total assets were \$7.9 trillion in 2007 and 2009. Since we are looking before 2008, the answer is the yer 2007.

52. $A = .877x^2 - 9.33x + 23.4$

a. Let $A = 17.8$.

$$17.8 = .877x^2 - 9.33x + 23.4$$
$$0 = .877x^2 - 9.33x + 5.6$$

Using the quadratic formula, we have

$$x = \frac{9.33 \pm \sqrt{(-9.33)^2 - 4(.877)(5.6)}}{2(.877)}$$
$$= \frac{9.33 \pm \sqrt{67.4041}}{2(.877)} \approx 0.64 \text{ or } 10$$

0.64 is not applicable because the formula is defined for $6 \le x \le 12$. Thus, net income were about \$17.8 billion in 2010.

b. Let $R = 37.7$.

$37.7 = .877x^2 - 9.33x + 23.4$

$0 = .877x^2 - 9.33x - 14.3$

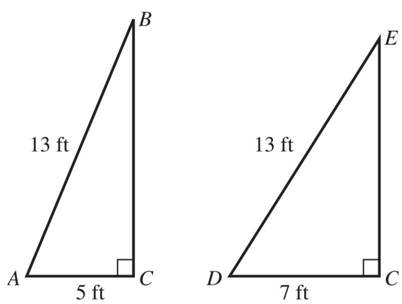
Using the quadratic formula, we have

$$x = \frac{9.33 \pm \sqrt{(-9.33)^2 - 4(.877)(-14.3)}}{2(.877)}$$

$$= \frac{9.33 \pm \sqrt{137.2133}}{2(.877)} \approx -1.36 \text{ or } 12.0$$

The negative solution is not applicable because the formula is defined for $6 \le x \le 12$. Thus, net income were about \$37.7 billion in 2012.

53. Triangle ABC represents the original position of the ladder, while triangle DEC represents the position of the ladder after it was moved.



Use the Pythagorean theorem to find the distance from the top of the ladder to the ground.

In triangle ABC,

$13^2 = 5^2 + BC^2 \Rightarrow 169 - 25 = BC^2 \Rightarrow$

$144 = BC^2 \Rightarrow 12 = BC$

Thus, the top of the ladder was originally 12 feet from the ground.

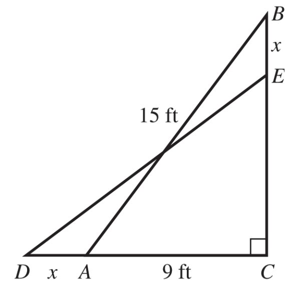
In triangle DEC,

$13^2 = 7^2 + EC^2 \Rightarrow 169 - 49 = EC^2 \Rightarrow$

$120 = EC^2 \Rightarrow EC = \sqrt{120}$

The top of the ladder was $\sqrt{120}$ feet from the ground after the ladder was moved. Therefore, the ladder moved down $12 - \sqrt{120} \approx 1.046$ feet.

54. Triangle ABC shows the original position of the ladder against the wall, and triangle DEC is the position of the ladder after it was moved.



In triangle ABC,

$15^2 = BC^2 + 9^2 \Rightarrow BC^2 = 15^2 - 9^2 = 144 \Rightarrow$

$BC = 12$

In triangle DEC

$15^2 = (12 - x)^2 + (9 + x)^2 \Rightarrow$

$225 = 144 - 24x + x^2 + 81 + 18x + x^2 \Rightarrow$

$0 = -6x + 2x^2 \Rightarrow 2x(x - 3) = 0 \Rightarrow$

$2x = 0 \Rightarrow x = 0$ or $x - 3 = 0 \Rightarrow x = 3$

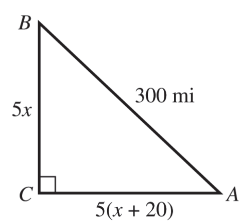
The bottom of the ladder should be pulled 3 feet away from the wall.

55. a. The eastbound train travels at a speed of $x + 20$.

b. The northbound train travels a distance of $5x$ in 5 hours.

The eastbound train travels a distance of $5(x + 20) = 5x + 100$ in 5 hours.

c.



By the Pythagorean theorem,

$(5x)^2 + (5x + 100)^2 = 300^2$

d. Expand and combine like terms.

$$25x^2+25x^2+1000x+10{,}000=90{,}000$$
$$50x^2+1000x-80{,}000=0$$

Factor out the common factor, 50, and divide both sides by 50.

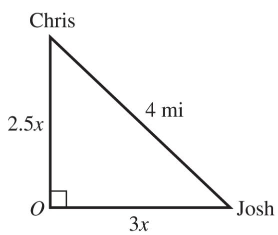
$$50(x^2+20x-1600)=0$$
$$x^2+20x-1600=0$$

Now use the quadratic formula to solve for x.

$$x=\frac{-20\pm\sqrt{20^2-4(1)(-1600)}}{2(1)}$$
$$\approx -51.23 \text{ or } 31.23$$

Since x cannot be negative, the speed of the northbound train is $x \approx 31.23$ mph, and the speed of the eastbound train is $x + 20 \approx 51.23$ mph.

56. Let x represent the length of time they will be able to talk to each other. Since $d = rt$, Chris' distance is $2.5x$ and Josh's distance is $3x$.



This is a right triangle, so

$$(2.5x)^2+(3x)^2=4^2 \Rightarrow 6.25x^2+9x^2=16 \Rightarrow$$
$$15.25x^2=16 \Rightarrow x^2=\frac{16}{15.25} \Rightarrow$$
$$x=\pm\sqrt{\frac{16}{15.25}} \approx \pm 1.024$$

Since time must be nonnegative, they will be able to talk to each other for about 1.024 hr or approximately 61 min.

57. a. Let x represent the length. Then, $\frac{300-2x}{2}$ or $150 - x$ represents the width.

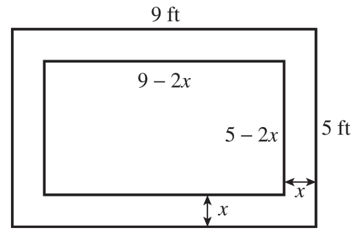
b. Use the formula for the area of a rectangle.
$LW = A \Rightarrow x(150-x)=5000$

c. $150x - x^2 = 5000$
Write this quadratic equation in standard form and solve by factoring.

$$0=x^2-150x+5000$$
$$x^2-150x+5000=0$$
$$(x-50)(x-100)=0$$
$$x-50=0 \quad \text{or} \quad x-100=0$$
$$x=50 \quad \text{or} \quad x=100$$

Choose $x = 100$ because the length is the larger dimension. The length is 100 m and the width is $150 - 100 = 50$ m.

58. Let x represent the width of the border.

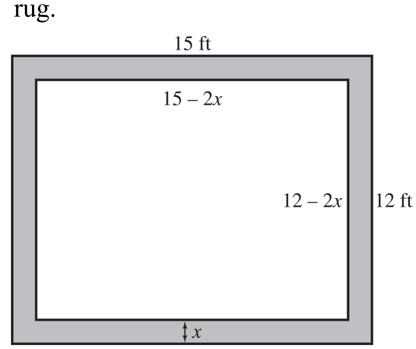


The area of the flower bed is $9 \cdot 5 = 45$. The area of the center is $(9-2x)(5-2x)$. Therefore,

$$45-(9-2x)(5-2x)=24$$
$$45-\left(45-28x+4x^2\right)=24$$
$$45-45+28x-4x^2=24$$
$$0=4x^2-28x+24$$
$$0=4\left(x^2-7x+6\right)$$
$$0=4(x-1)(x-6)$$
$$x-1=0 \quad \text{or} \quad x-6=0$$
$$x=1 \quad \text{or} \quad x=6$$

The solution $x = 6$ is impossible since both $9 - 2x$ and $5 - 2x$ would be negative. Therefore, the width of the border is 1 ft.

59. Let x = the width of the uniform strip around the rug.



The dimensions of the rug are $15 - 2x$ and $12 - 2x$. The area, 108, is the length times the width.

(*continued on next page*)

(*continued from page 35*)

Solve the equation.

$$(15-2x)(12-2x)=108$$
$$180-54x+4x^2=108$$
$$4x^2-54x+72=0$$
$$2x^2-27x+36=0$$
$$(x-12)(2x-3)=0$$
$$x-12=0 \quad \text{or} \quad 2x-3=0$$
$$x=12 \quad \text{or} \quad x=\frac{3}{2}$$

Discard $x = 12$ since both $12 - 2x$ and $15 - 2x$ would be negative.

If $x=\frac{3}{2}$, then

$$15-2x=15-2\left(\frac{3}{2}\right)=12$$

and $12-2x=12-2\left(\frac{3}{2}\right)=9$.

The dimensions of the rug should be 9 ft by 12 ft.

60. Let x = Haroud's speed. Then $x + 92$ = Franchitti's speed. From the formula $d = rt$, we have Haroud's time = $\frac{500}{x}$ and Franchitti's time = $\frac{500}{x+92}$. Then, we have

$$\frac{500}{x+92}=\frac{500}{x}-3.72 \Rightarrow$$
$$500x=500(x+92)-3.72x(x+92) \Rightarrow$$
$$500x=500x+46{,}000-3.72x^2-342.24x \Rightarrow$$
$$0=-3.72x^2-342.24x+46{,}000$$

Use the quadratic formula to solve for x.

$$x=\frac{-(-342.24)\pm\sqrt{(-342.24)^2-4(-3.72)(46{,}000)}}{2(-3.72)}$$
$$\approx -166.3 \text{ or } 74.3$$

The negative value is not applicable. Haroud's speed was 74.3 mph and Franchitti's speed was 74.3 + 92 = 166.3 mph.

For exercises 61–66, use the formula $h=-16t^2+v_0t+h_0$, where h_0 is the height of the object when $t = 0$, and v_0 is the initial velocity at time $t = 0$.

61. $v_0=0,\ h_0=625,\ h=0$

$$0=-16t^2+(0)t+625 \Rightarrow 16t^2-625=0 \Rightarrow$$
$$(4t-25)(4t+25)=0 \Rightarrow t=\frac{25}{4}=6.25 \text{ or}$$
$$t=-\frac{25}{4}=-6.25$$

The negative solution is not applicable. It takes 6.25 seconds for the baseball to reach the ground.

62. When the ball has fallen 196 feet, it is 625 – 196 = 429 feet above the ground.

$v_0=0,\ h_0=625,\ h=429$

$$429=-16t^2+(0)t+625 \Rightarrow 16t^2-196=0 \Rightarrow$$
$$(4t-14)(4t+14)=0 \Rightarrow t=\frac{14}{4}=3.5 \text{ or}$$
$$t=-\frac{14}{4}=-3.5$$

The negative solution is not applicable. It takes 3.5 seconds for the ball to fall 196 feet.

63. a. $v_0=0,\ h_0=200,\ h=0$

$$0=-16t^2-(0)t+200 \Rightarrow 16t^2=200 \Rightarrow$$
$$t^2=\frac{200}{16} \Rightarrow t=\pm\frac{\sqrt{200}}{4} \approx \pm 3.54$$

The negative solution is not applicable. It will take about 3.54 seconds for the rock to reach the ground if it is dropped.

b. $v_0=-40,\ h_0=200,\ h=0$

$$0=-16t^2-40t+200$$

Using the quadratic formula, we have

$$t=\frac{-(-40)\pm\sqrt{(-40)^2-4(-16)(200)}}{2(-16)}$$
$$=-5 \text{ or } 2.5$$

The negative solution is not applicable. It will take about 2.5 seconds for the rock to reach the ground if it is thrown with an initial velocity of 40 ft/sec.

c. $v_0=-40,\ h_0=200,\ t=2$

$$h=-16(2)^2-40(2)+200=56$$

After 2 seconds, the rock is 56 feet above the ground. This means it has fallen 200 – 56 = 144 feet.

64. a. $v_0 = 800,\ h_0 = 0,\ h = 3200$

$3200 = -16t^2 + 800t + 0 \Rightarrow$
$16t^2 - 800t + 3200 = 0 \Rightarrow$
$t^2 - 50t + 200 = 0$

Using the quadratic formula, we have

$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(200)}}{2(1)} \Rightarrow$$

$t \approx 4.38$ or $t \approx 45.62$

The rocket rises to 3200 feet after about 4.38 seconds. It also reaches 3200 feet as it falls back to the ground after about 45.62 seconds. Because we are asked how long to rise to 3200 feet, the answer will be about 4.38 sec.

b. $v_0 = 800,\ h_0 = 0,\ h = 0$

$0 = -16t^2 + 800t + 0 \Rightarrow$
$16t^2 - 800t = 0 \Rightarrow 16t(t-50) = 0 \Rightarrow$
$t = 0$ or $t = 50$

The rocket hits the ground after 50 seconds.

65. a. $v_0 = 64,\ h_0 = 0,\ h = 64$

$64 = -16t^2 + 64t + 0 \Rightarrow$
$16t^2 - 64t + 64 = 0 \Rightarrow$
$t^2 - 4t + 4 = 0 \Rightarrow (t-2)^2 = 0 \Rightarrow t = 2$

The ball will reach 64 feet after 2 seconds.

b. $v_0 = 64,\ h_0 = 0,\ h = 39$

$39 = -16t^2 + 64t + 0 \Rightarrow$
$16t^2 - 64t + 39 = 0 \Rightarrow (4t-13)(4t-3) \Rightarrow$
$t = \frac{13}{4} = 3.25$ or $t = \frac{3}{4} = .75$

The ball will reach 39 feet after .75 seconds and after 3.25 seconds.

c. Two answers are possible because the ball reaches the given height twice, once on the way up and once on the way down.

66. a. $v_0 = 100,\ h_0 = 0,\ h = 50$

$50 = -16t^2 + 100t + 0 \Rightarrow$
$16t^2 - 100t + 50 = 0 \Rightarrow$
$8t^2 - 50t + 25 = 0 \Rightarrow t \approx .55$ or 5.7

The ball will reach 50 feet on the way up after approximately 0.55 seconds.

b. $v_0 = 100,\ h_0 = 0,\ h = 35$

$35 = -16t^2 + 100t + 0 \Rightarrow$
$16t^2 - 100t + 35 = 0 \Rightarrow$
$t \approx .37$ or 5.88

The ball will reach 35 feet on the way up after approximately 0.37 seconds.

In exercises 67–72, we discard negative roots since all variables represent positive real numbers.

67. $S = \frac{1}{2}gt^2$ for t

$$2S = gt^2$$
$$\frac{2S}{g} = t^2$$
$$\sqrt{\frac{2S}{g}} \cdot \frac{\sqrt{g}}{\sqrt{g}} = t$$
$$\frac{\sqrt{2Sg}}{g} = t$$

68. Solve for r.

$$a = \pi r^2$$
$$\frac{a}{\pi} = r^2$$
$$\sqrt{\frac{a}{\pi}} \cdot \frac{\sqrt{\pi}}{\sqrt{\pi}} = r \quad (r > 0)$$
$$r = \sqrt{\frac{a}{\pi}} = \frac{\sqrt{\pi a}}{\pi}$$

69. $L = \frac{d^4k}{h^2}$ for h

$$Lh^2 = d^4k$$
$$h^2 = \frac{d^4k}{L}$$
$$h = \sqrt{\frac{d^4k}{L}} \cdot \frac{\sqrt{L}}{\sqrt{L}} = \frac{\sqrt{d^4kL}}{L}$$
$$h = \frac{d^2\sqrt{kL}}{L}$$

70. Solve for v.

$$F = \frac{kMv^2}{r}$$

$$\frac{Fr}{kM} = v^2$$

$$\sqrt{\frac{Fr}{kM}} \cdot \frac{\sqrt{kM}}{\sqrt{kM}} = v \ (v > 0)$$

$$\sqrt{\frac{Fr}{kM}} = \frac{\sqrt{FrkM}}{kM} = v$$

71.

$$P = \frac{E^2R}{(r+R)^2} \text{ for } R$$

$$P(r+R)^2 = E^2R$$

$$P\left(r^2 + 2rR + R^2\right) = E^2R$$

$$Pr^2 + 2PrR + PR^2 = E^2R$$

$$PR^2 + \left(2Pr - E^2\right)R + Pr^2 = 0$$

Solve for R by using the quadratic formula with $a = P,\ b = 2\text{Pr} - E^2$, and $c = \text{Pr}^2$.

$$R = \frac{-\left(2\text{Pr} - E^2\right) \pm \sqrt{\left(2\text{Pr} - E^2\right)^2 - 4P \cdot \text{Pr}^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{4P^2r^2 - 4\,\text{Pr}\,E^2 + E^4 - 4P^2r^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{E^4 - 4\,\text{Pr}\,E^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{E^2\left(E^2 - 4\text{Pr}\right)}}{2P}$$

$$R = \frac{-2\text{Pr} + E^2 \pm E\sqrt{E^2 - 4\text{Pr}}}{2P}$$

72. Solve for r.

$S = 2\pi rh + 2\pi r^2$

Write as a quadratic equation in r.

$(2\pi)r^2 + (2\pi h)r - S = 0$

Solve for r using the quadratic formula with $a = 2\pi$, $b = 2\pi h$, and $c = -S$.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)}$$

$$= \frac{-2\pi h \pm \sqrt{4\pi^2h^2 + 8\pi S}}{4\pi}$$

$$= \frac{-2\pi h \pm 2\sqrt{\pi^2h^2 + 2\pi S}}{4\pi}$$

$$= \frac{2\left(-\pi h \pm \sqrt{\pi^2h^2 + 2\pi S}\right)}{4\pi}$$

$$r = \frac{-\pi h \pm \sqrt{\pi^2h^2 + 2\pi S}}{2\pi}$$

73. **a.** Let $x = z^2$.

$x^2 - 2x = 15$

b. $x^2 - 2x = 15$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ or $x = -3$

c. Let $z^2 = 5$.

$z = \pm\sqrt{5}$

74. $6p^4 = p^2 + 2 \Rightarrow 6p^4 - p^2 - 2 = 0$

Let $u = p^2$. Then $u^2 = p^4$.

$6u^2 - u - 2 = 0 \Rightarrow (2u+1)(3u-2) = 0$

$2u + 1 = 0$ or $3u - 2 = 0$

$u = -\frac{1}{2}$ or $u = \frac{2}{3}$

Since $u = p^2$,

$p^2 = -\frac{1}{2}$ or $p^2 = \frac{2}{3}$

$p = \pm\sqrt{-\frac{1}{2}}$ or $p = \pm\sqrt{\frac{2}{3}}$

Not real or $p = \pm\frac{\sqrt{6}}{3}$

The solutions are $\pm\frac{\sqrt{6}}{3}$.

75. $2q^4 + 3q^2 - 9 = 0$

Let $u = q^2$; then $u^2 = q^4$.

$2u^2 + 3u - 9 = 0 \Rightarrow (2u-3)(u+3) = 0$

$2u - 3 = 0$ or $u + 3 = 0$

$u = \frac{3}{2}$ or $u = -3$

Since $u = q^2$,

$q^2 = \frac{3}{2}$ or $q^2 = -3$

$q = \pm\sqrt{\frac{3}{2}}$ or $q = \pm\sqrt{-3}$ (not real)

$q = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{6}}{2}$

The solutions are $\pm\frac{\sqrt{6}}{2}$.

76. $4a^4 = 2 - 7a^2 \Rightarrow 4a^4 + 7a^2 - 2 = 0$

Let $x = a^2$. Then $x^2 = a^4$.

$4x^2 + 7x - 2 = 0$

$(4x-1)(x+2) = 0$

$4x - 1 = 0$ or $x + 2 = 0$

$x = \frac{1}{4}$ or $x = -2$

Since $x = a^2$,

$a^2 = \frac{1}{4}$ or $a^2 = -2$

$a = \pm\frac{1}{2}$ or $a = \pm\sqrt{-2}$

Not real

The solutions are $\pm\frac{1}{2}$.

77. $z^4 - 3z^2 - 1 = 0$

Let $x = z^2$; then $x^2 = z^4$.

$x^2 - 3x - 1 = 0$

By the quadratic formula, $x = \frac{3 \pm \sqrt{13}}{2}$.

$\frac{3+\sqrt{13}}{2} = z^2$

$\pm\sqrt{\frac{3+\sqrt{13}}{2}} = z$

78. $2r^4 - r^2 - 5 = 0$

Let $x = r^2$; then $x^2 = r^4$.

$2x^2 - x - 5 = 0$

By the quadratic formula, $x = \frac{1 \pm \sqrt{41}}{4}$

$\frac{1+\sqrt{41}}{4} = r^2$

$r = \pm\frac{\sqrt{1+\sqrt{41}}}{2}$

Chapter 1 Review Exercises

1. 0 and 6 are whole numbers.

2. –12, –6, $-\sqrt{4}$, 0, and 6 are integers.

3. –12, –6, $-\frac{9}{10}$, $-\sqrt{4}$, 0, $\frac{1}{8}$, and 6 are rational numbers.

4. $-\sqrt{7}, \frac{\pi}{4}, \sqrt{11}$ are irrational numbers.

5. 9[(–3)4] = 9[4(–3)]
Commutative property of multiplication

6. 7(4 + 5) = (4 + 5)7
Commutative property of multiplication

7. $6(x + y - 3) = 6x + 6y + 6(-3)$
Distributive property

8. 11 + (5 + 3) = (11 + 5) + 3
Associative property of addition

9. x is at least 9.
$x \geq 9$

10. x is negative.
$x < 0$

11. $|6-4| = 2,\ -|-2| = -2,\ |8+1| = |9| = 9,$
$-|3-(-2)| = -|3+2| = -|5| = -5$
Since –5, –2, 2, 9 are in order, then
$-|3-(-2)|,\ -|-2|,\ |6-4|,\ |8+1|$ are in order.

12. $-\left|\sqrt{16}\right| = -4,\ -\sqrt{8},\ \sqrt{7},\ \left|-\sqrt{12}\right| = \sqrt{12}$

13. $7 - |-8| = 7 - 8 = -1$

14. $|-3| - |-9+6| = 3 - |-3| = 3 - 3 = 0$

15. $x \geq -3$
Start at –3 and draw a ray to the right. Use a bracket at –3 to show that –3 is a part of the graph.

16. $-4 < x \leq 6$
Put a parenthesis at –4 and a bracket at 6. Draw a line segment between these two endpoints.

17. $\dfrac{-9+(-6)(-3)\div 9}{6-(-3)} = \dfrac{-9+18\div 9}{6+3} = \dfrac{-9+2}{9} = -\dfrac{7}{9}$

18. $\dfrac{20\div 4\cdot 2\div 5-1}{-9-(-3)-12\div 3} = \dfrac{5\cdot 2\div 5-1}{-9-(-3)-4}$
$= \dfrac{10\div 5-1}{-9+3-4} = \dfrac{2-1}{-6-4} = -\dfrac{1}{10}$

19. $\left(3x^4 - x^2 + 5x\right) - \left(-x^4 + 3x^2 - 6x\right)$
$= 3x^4 - x^2 + 5x + x^4 - 3x^2 + 6x$
$= \left(3x^4 + x^4\right) + \left(-x^2 - 3x^2\right) + (5x + 6x)$
$= 4x^4 - 4x^2 + 11x$

20. $\left(-8y^3 + 8y^2 - 5y\right) - \left(2y^3 + 4y^2 - 10\right)$
$= -8y^3 + 8y^2 - 5y - 2y^3 - 4y^2 + 10$
$= -8y^3 - 2y^3 + 8y^2 - 4y^2 - 5y + 10$
$= -10y^3 + 4y^2 - 5y + 10$

21. $(5k - 2h)(5k + 2h) = (5k)^2 - (2h)^2 = 25k^2 - 4h^2$

22. $(2r - 5y)(2r + 5y) = (2r)^2 - (5y)^2$
$= 4r^2 - 25y^2$

23. $(3x + 4y)^2 = (3x)^2 + 2(3x)(4y) + \left(4y^2\right)$
$= 9x^2 + 24xy + 16y^2$

24. $(2a - 5b)^2 = (2a)^2 - 2(2a)(5b) + (5b)^2$
$= 4a^2 - 20ab + 25b^2$

25. $2kh^2 - 4kh + 5k = k\left(2h^2 - 4h + 5\right)$

26. $2m^2n^2 + 6mn^2 + 16n^2 = 2n^2\left(m^2 + 3m + 8\right)$

27. $5a^4 + 12a^3 + 4a^2 = a^2\left(5a^2 + 12a + 4\right)$
$= a^2(5a + 2)(a + 2)$

28. $24x^3 + 4x^2 - 4x = 4x\left(6x^2 + x - 1\right)$
$= 4x(3x - 1)(2x + 1)$

29. $144p^2 - 169q^2 = (12p)^2 - (13q)^2$
$= (12p - 13q)(12p + 13q)$

30. $81z^2 - 25x^2 = (9z)^2 - (5x)^2$
$= (9z + 5x)(9z - 5x)$

31. $27y^3 - 1 = (3y)^3 - 1^3$
$= (3y - 1)\left[(3y)^2 + (3y)(1) + 1^2\right]$
$= (3y - 1)\left(9y^2 + 3y + 1\right)$

32. $125a^3 + 216 = (5a)^3 + (6)^3$
$= (5a+6)\left[(5a)^2 - 5a(6) + 6^2\right]$
$= (5a+6)\left(25a^2 - 30a + 36\right)$

33. $\frac{3x}{5} \cdot \frac{45x}{12} = \frac{3x \cdot 45x}{5 \cdot 12} = \frac{3 \cdot 5 \cdot 3 \cdot 3x^2}{4 \cdot 5 \cdot 3} = \frac{9x^2}{4}$

34. $\frac{5k^2}{24} - \frac{70k}{36} = \frac{5k^2 \cdot 3}{24 \cdot 3} - \frac{70k \cdot 2}{36 \cdot 2} = \frac{15k^2}{72} - \frac{140k}{72}$
$= \frac{15k^2 - 140k}{72} = \frac{5k(3k-28)}{72}$

35. $\frac{c^2 - 3c + 2}{2c(c-1)} \div \frac{c-2}{8c} = \frac{(c-1)(c-2)}{2c(c-1)} \cdot \frac{8c}{(c-2)}$
$= \frac{8c(c-1)(c-2)}{2c(c-1)(c-2)} = \frac{8}{2} = 4$

36. $\frac{p^3 - 2p^2 - 8p}{3p\left(p^2 - 16\right)} \div \frac{p^2 + 4p + 4}{9p^2}$
$= \frac{p\left(p^2 - 2p - 8\right)}{3p(p+4)(p-4)} \cdot \frac{9p^2}{(p+2)(p+2)}$
$= \frac{p(p-4)(p+2) \cdot 9p^2}{3p(p+4)(p-4)(p+2)(p+2)}$
$= \frac{3p(p-4)(p+2) \cdot 3p^2}{3p(p-4)(p+2) \cdot (p+4)(p+2)}$
$= \frac{3p^2}{(p+4)(p+2)}$

37. $\frac{2m^2 - 4m + 2}{m^2 - 1} \div \frac{6m + 18}{m^2 + 2m - 3}$
$= \frac{2\left(m^2 - 2m + 1\right)}{(m+1)(m-1)} \cdot \frac{m^2 + 2m - 3}{6m + 18}$
$= \frac{2(m-1)^2}{(m+1)(m-1)} \cdot \frac{(m+3)(m-1)}{6(m+3)}$
$= \frac{2(m-1)(m+3) \cdot (m-1)^2}{2(m-1)(m+3) \cdot 3(m+1)} = \frac{(m-1)^2}{3(m+1)}$

38. $\frac{x^2 + 6x + 5}{4\left(x^2 + 1\right)} \cdot \frac{2x(x+1)}{x^2 - 25}$
$= \frac{(x+5)(x+1) \cdot 2x(x+1)}{4\left(x^2+1\right) \cdot (x+5)(x-5)}$
$= \frac{2(x+5) \cdot x(x+1)^2}{2(x+5) \cdot 2\left(x^2+1\right)(x-5)}$
$= \frac{x(x+1)^2}{2\left(x^2+1\right)(x-5)}$

39. $5^{-3} = \frac{1}{5^3}$ or $\frac{1}{125}$

40. $10^{-2} = \frac{1}{10^2}$ or $\frac{1}{100}$

41. $-8^0 = -\left(8^0\right) = -1$

42. $\left(-\frac{5}{6}\right)^{-2} = \left(-\frac{6}{5}\right)^2 = \frac{36}{25}$

43. $4^6 \cdot 4^{-3} = 4^{6+(-3)} = 4^3$

44. $7^{-5} \cdot 7^{-2} = 7^{-5+(-2)} = 7^{-7} = \frac{1}{7^7}$

45. $\frac{8^{-5}}{8^{-4}} = 8^{-5-(-4)} = 8^{-5+4} = 8^{-1} = \frac{1}{8}$

46. $\frac{6^{-3}}{6^4} = 6^{-3-4} = 6^{-7} = \frac{1}{6^7}$

47. $5^{-1} + 2^{-1} = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$

48. $5^{-2} + 5^{-1} = \frac{1}{5^2} + \frac{1}{5} = \frac{1}{25} + \frac{1}{5} = \frac{6}{25}$

49. $\frac{5^{1/3}5^{1/2}}{5^{3/2}} = 5^{1/3+1/2-3/2} = 5^{-2/3} = \frac{1}{5^{2/3}}$

50. $\frac{2^{3/4} \cdot 2^{-1/2}}{2^{1/4}} = \frac{2^{1/4}}{2^{1/4}} = 1$

51. $\left(3a^2\right)^{1/2} \cdot \left(3^2 a\right)^{3/2} = 3^{1/2} a \cdot 3^3 a^{3/2} = 3^{7/2} a^{5/2}$

52. $(4p)^{2/3} \cdot \left(2p^3\right)^{3/2} = 4^{2/3} p^{2/3} \cdot 2^{3/2} \cdot p^{9/2}$
$= \left(2^2\right)^{2/3} p^{2/3} \cdot 2^{3/2} p^{9/2}$
$= 2^{4/3} \cdot 2^{3/2} p^{2/3} p^{9/2}$
$= 2^{17/6} p^{31/6}$

53. $\sqrt[3]{27} = 3$

54. $\sqrt[6]{-64}$ is not a real number.

55. $\sqrt[3]{54p^3q^5} = \sqrt[3]{27 \cdot 2p^3q^3q^2}$
$= \sqrt[3]{27p^3q^3} \cdot \sqrt[3]{2q^2} = 3pq\sqrt[3]{2q^2}$

56. $\sqrt[4]{64a^5b^3} = \sqrt[4]{16a^4} \cdot \sqrt[4]{4ab^3} = 2a\sqrt[4]{4ab^3}$

57. $3\sqrt{3} - 12\sqrt{12} = 3\sqrt{3} - 12\sqrt{4 \cdot 3} = 3\sqrt{3} - 12 \cdot 2\sqrt{3}$
$= 3\sqrt{3} - 24\sqrt{3} = -21\sqrt{3}$

58. $8\sqrt{7} + 2\sqrt{63} = 8\sqrt{7} + 2\sqrt{9 \cdot 7}$
$= 8\sqrt{7} + 6\sqrt{7} = 14\sqrt{7}$

59. $\dfrac{\sqrt{3}}{1+\sqrt{2}} = \dfrac{\sqrt{3}\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} = \dfrac{\sqrt{3}-\sqrt{6}}{1-2}$
$= \dfrac{\sqrt{3}-\sqrt{6}}{-1} = \sqrt{6} - \sqrt{3}$

60. $\dfrac{4+\sqrt{2}}{4-\sqrt{5}} = \dfrac{\left(4+\sqrt{2}\right)}{\left(4-\sqrt{5}\right)} \cdot \dfrac{\left(4+\sqrt{5}\right)}{\left(4+\sqrt{5}\right)}$
$= \dfrac{16 + 4\sqrt{2} + 4\sqrt{5} + \sqrt{10}}{16 - \left(\sqrt{5}\right)^2}$
$= \dfrac{16 + 4\sqrt{2} + 4\sqrt{5} + \sqrt{10}}{11}$

61. $3x - 4\left(x-2\right) = 2x + 9$
$3x - 4x + 8 = 2x + 9$
$-x + 8 = 2x + 9$
$-1 = 3x \Rightarrow x = -\dfrac{1}{3}$

62. $4y + 9 = -3(1-2y) + 5$
$4y + 9 = -3 + 6y + 5$
$4y + 9 = 2 + 6y$
$-2y = -7 \Rightarrow y = \dfrac{7}{2}$

63. $\dfrac{2m}{m-3} = \dfrac{6}{m-3} + 4$
$2m = 6 + 4(m-3)$
$2m = 6 + 4m - 12$
$2m = 4m - 6$
$6 = 2m \Rightarrow 3 = m$

Because $m = 3$ would make the denominators of the fractions equal to 0, making the fractions undefined, the given equation has no solution.

64. $\dfrac{15}{k+5} = 4 - \dfrac{3k}{k+5}$
Multiply both sides of the equation by the common denominator $k + 5$.
$15 = 4\left(k+5\right) - 3k$
$15 = 4k + 20 - 3k \Rightarrow k = -5$
If $k = -5$, the fractions would be undefined, so the given equation has no solution.

65. $8ax - 3 = 2x$
$8ax - 2x = 3$
$x\left(8a-2\right) = 3$
$x = \dfrac{3}{8a-2}$

66. $b^2x - 2x = 4b^2$
$\left(b^2 - 2\right)x = 4b^2$
$x = \dfrac{4b^2}{b^2-2}$

67. $\left|\dfrac{2-y}{5}\right| = 8$
$\dfrac{2-y}{5} = 8$ or $\dfrac{2-y}{5} = -8$
$5\left(\dfrac{2-y}{5}\right) = 5(8)$ or $5\left(\dfrac{2-y}{5}\right) = -5(-8)$
$2 - y = 40$ or $2 - y = -40$
$-y = 38$ or $-y = -42$
$y = -38$ or $y = 42$
The solutions are –38 and 42.

68. $|4k+1| = |6k-3|$

$4k+1 = 6k-3$ or $4k+1 = -(6k-3)$

$4k+1 = 6k-3$ or $4k+1 = -6k+3$

$-2k = -4$ or $10k = 2$

$k = 2$ or $k = \frac{1}{5}$

The solutions are 2 and $\frac{1}{5}$.

69. $(b+7)^2 = 5$

Use the square root property to solve this quadratic equation.

$b+7 = \sqrt{5}$ or $b+7 = -\sqrt{5}$

$b = -7+\sqrt{5}$ or $b = -7-\sqrt{5}$

The solutions are $-7+\sqrt{5}$ and $-7-\sqrt{5}$, which we abbreviate as $-7\pm\sqrt{5}$.

70. $(2p+1)^2 = 7$

Solve by the square root property.

$2p+1 = \sqrt{7}$ or $2p+1 = -\sqrt{7}$

$2p = -1+\sqrt{7}$ or $2p = -1-\sqrt{7}$

$p = \frac{-1+\sqrt{7}}{2}$ or $p = \frac{-1-\sqrt{7}}{2}$

The solutions are $\frac{-1\pm\sqrt{7}}{2}$.

71. $2p^2 + 3p = 2$

Write the equation in standard form and solve by factoring.

$2p^2 + 3p - 2 = 0$

$(2p-1)(p+2) = 0$

$2p-1 = 0$ or $p+2 = 0$

$p = \frac{1}{2}$ or $p = -2$

The solutions are $\frac{1}{2}$ and –2.

72. $2y^2 = 15 + y$

Write the equation in standard form and solve by factoring.

$2y^2 - y - 15 = 0$

$(y-3)(2y+5) = 0$

$y = 3$ or $y = -\frac{5}{2}$

The solutions are 3 and $-\frac{5}{2}$.

73. $2q^2 - 11q = 21 \Rightarrow 2q^2 - 11q - 21 = 0 \Rightarrow$

$(2q+3)(q-7) = 0$

$2q+3 = 0$ or $q-7 = 0$

$q = -\frac{3}{2}$ or $q = 7$

The solutions are $-\frac{3}{2}$ and 7.

74. $3x^2 + 2x = 16 \Rightarrow 3x^2 + 2x - 16 = 0 \Rightarrow$

$(3x+8)(x-2) = 0$

$3x+8 = 0$ or $x-2 = 0$

$x = -\frac{8}{3}$ or $x = 2$

The solutions are $-\frac{8}{3}$ and 2.

75. $6k^4 + k^2 = 1 \Rightarrow 6k^4 + k^2 - 1 = 0$

Let $p = k^2$, so $p^2 = k^4$.

$6p^2 + p - 1 = 0$

$(3p-1)(2p+1) = 0$

$3p-1 = 0$ or $2p+1 = 0$

$p = \frac{1}{3}$ or $p = -\frac{1}{2}$

If $p = \frac{1}{3}$, $k^2 = \frac{1}{3} \Rightarrow k = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

If $p = -\frac{1}{2}$, $k^2 = -\frac{1}{2}$ has no real number solution.

The solutions are $\pm\frac{\sqrt{3}}{3}$.

76. $21p^4 = 2 + p^2 \Rightarrow 21p^4 - p^2 - 2 = 0$

Let $u = p^2$; then $u^2 = p^4$.

$$21u^2 - u - 2 = 0$$
$$(3u-1)(7u+2) = 0$$
$$3u - 1 = 0 \quad \text{or} \quad 7u + 2 = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{2}{7}$$
$$p^2 = \frac{1}{3} \quad \text{or} \quad p^2 = -\frac{2}{7}$$

If $x = -\frac{2}{7}$, $p^2 = -\frac{2}{7}$ has no real number solution.

$$p = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$

The solutions are $\pm\frac{\sqrt{3}}{3}$.

77. $p = \dfrac{E^2R}{(r+R)^2}$ for r.

$$p(r+R)^2 = E^2R$$
$$p\left(r^2 + 2rR + R^2\right) = E^2R$$
$$pr^2 + 2rpR + R^2p = E^2R$$
$$pr^2 + 2rpR + R^2p - E^2R = 0$$

Use the quadratic formula to solve for r.

$$r = \frac{-2pR \pm \sqrt{4p^2R^2 - 4p\left(R^2p - E^2R\right)}}{2p}$$
$$r = \frac{-2pR \pm \sqrt{4pE^2R}}{2p} = \frac{-pR \pm E\sqrt{pR}}{p}$$

78. $p = \dfrac{E^2R}{(r+R)^2}$ for E.

$$p(r+R)^2 = E^2R \Rightarrow E^2 = \frac{p(r+R)^2}{R} \Rightarrow$$
$$E = \pm\sqrt{\frac{p(r+R)^2}{R}} = \frac{\pm(r+R)\sqrt{pR}}{R}$$

79. $K = s(s-a)$ for s.

$K = s^2 - as \Rightarrow s^2 - as - K = 0$

Use the quadratic formula.

$$s = \frac{a \pm \sqrt{a^2 - 4(-K)}}{2} = \frac{a \pm \sqrt{a^2 + 4K}}{2}$$

80. $kz^2 - hz - t = 0$ for z.

Use the quadratic formula with $a = k$, $b = -h$, and $c = -t$.

$$z = \frac{-(-h) \pm \sqrt{(-h)^2 - 4(k)(-t)}}{2k}$$
$$= \frac{h \pm \sqrt{h^2 + 4kt}}{2k}$$

81. $|67 - (-44)| = |111| = 111\%$

82. $|-31 - (-45)| = |14| = 14\%$

83. Let x = the original price

$$x - .2x = 895$$
$$.8x = 895$$
$$x = 1118.75$$

The original price of the PC was \$1118.75.

84. Let x = the original price

$$x + .68x = 51$$
$$1.68x = 51$$
$$x = 30.36$$

The original price of the stock was \$30.36.

85. **a**. Let $O = 3.3$

$$3.3 = .2x + 1.5$$
$$1.8 = .2x$$
$$9 = x$$

The amount of outlays reached \$3.3 trillion in the year 2009.

b. Let $O = 3.9$

$$3.9 = .2x + 1.5$$
$$2.4 = .2x$$
$$12 = x$$

The amount of outlays reached \$3.9 trillion in the year 2012.

86. Let $R = 28.4$

$$28.4 = -3.6x + 64.4$$
$$-36 = -3.6x$$
$$10 = x$$

The amount of revenue from the general newspaper industry reached \$28.4 billion in the year 2010.

87. Let $x = 12$

$$V = .89(12)^2 - 17.9(12) + 139.3$$
$$V = 52.66$$

The total sales for the year 2012 was approximately 52, 660, 000.

88. **a**. Let $x = 10$

$$A = -.023(10)^3 + .38(10)^2 + 3.29(10) + 107$$
$$A = 154.9$$

The total sales for the year 2010 was approximately $154, 900, 000, 000.

b. Through the use of graphing calculator, you can find that the function is greater than 140 when x is approximately 7, therefore it will pass $140 billion in the year 2007.

89. **a**. Let $x = 9$

$$F = \frac{.004(9)^2 + 10.3(9) + 5.8}{9+1} = 9.8824$$

The total number of flights that departed in 2009 was approximately 9, 882, 400.

b. Through the use of graphing calculator, you can find that the function is greater than 10 when x is approximately 12, therefore it will pass 10 million flights in the year 2012.

90. **a**. Let $x = 9$

$$M = \frac{.122(9)^2 + 5.48(9) + 6.5}{9+2} \approx 5.97$$

The total amount of sales from manufacturing in 2009 was approximately $5,970, 000, 000.

b. Through the use of graphing calculator, you can find that the function is greater than 6.0 when x is approximately 9, therefore it will pass $6.0 trillion in the year 2009.

91. **a**. Let $x = 10$

$$R = 4.33(10)^{0.66} \approx 19.79$$

The total amount of research of development spending in 2000 was approximately $19,790, 000.

b. Let $x = 20$

$$R = 4.33(20)^{0.66} \approx 31.27$$

The total amount of research of development spending in 2010 was approximately $31,270, 000.

92. **a**. Let $x = 25$

$$E = 91.5(25)^{0.07} \approx 114.6$$

The total amount of employees in the service-providing industry in 2015 will be approximately 114,600, 000.

b. Through the use of graphing calculator, you can find that the function is greater than 115 when x is approximately 26, therefore it will pass 115 million employees in the year 2016.

93. Let x = the single interest rate

$$2000(.12) + 500(.07) = 2500x$$
$$275 = 2500x$$
$$.11 = x$$

Therefore, a single interest rate of 11% will yield the same results.

94. Let x = the amount of beef. Then $30 - x$ = the amount of pork.

$$2.8x + 3.25(30 - x) = 3.10(30)$$
$$2.8x + 97.5 - 3.25x = 93$$
$$-.45x = -4.5$$
$$x = 10$$

Therefore the butcher should use 10 pounds of beef and 30 – 10 = 20 pounds of pork.

95. **a**. Let $x = 19.47$

$$P = 18.2(19.47)^2 - 26.0(19.47) + 789$$
$$P \approx 7{,}182$$

The total number of patents for the state of New York was approximately 7,182.

b. Let $P = 3806$.

$$3806 = 18.2x^2 - 26.0x + 789$$
$$0 = 18.2x^2 - 26.0x - 3017$$

Using the quadratic formula, we have

$$x = \frac{26 \pm \sqrt{(-26)^2 - 4(18.2)(-3017)}}{2(18.2)}$$
$$= \frac{26 \pm \sqrt{220{,}313.6}}{2(18.2)} \approx -12.18 \text{ or } 13.61$$

The negative value is not applicable. Thus, the approximate population of Illinois was 13,610,000.

96. **a**. Let $x = 11$

$$C = .22(11)^2 - 1.9(11) + 23.0$$
$$C \approx 28.72$$

The total number of patents issued in California in 2011 was approximately 28,720.

b. Let $C = 24$.

$$24 = .22x^2 - 1.9x + 23.0$$
$$0 = .22x^2 - 1.9x - 1$$

Using the quadratic formula, we have

$$x = \frac{1.9 \pm \sqrt{(-1.9)^2 - 4(.22)(-1)}}{2(.22)}$$
$$= \frac{1.9 \pm \sqrt{4.49}}{2(.22)} \approx -0.5 \text{ or } 9.14$$

The negative value is not applicable. Thus, the most recent year that saw 24,000 patents in California was 2009.

97. Let x = the width of the walk.

$$\text{The area } = (10 + 2x)(15 + 2x) - 10(15)$$
$$= 150 + 20x + 30x + 4x^2 - 150$$
$$= 4x^2 + 50x$$

To use all of the cement, solve

$$200 = 4x^2 + 50x$$
$$0 = 4x^2 + 50x - 200$$

Using the quadratic formula, we have

$$x = \frac{-50 \pm \sqrt{(50)^2 - 4(4)(-200)}}{2(4)}$$

$$= \frac{-50 \pm \sqrt{5700}}{2(4)} \approx -15.6875 \text{ or } 3.1875$$

The width cannot be negative, so the solution is approximately 3.2 feet.

98. Let x = the length of the yard. Then $160 - x$ = the width of the yard. Area is equal to length times width.

The area $= lw$

$$4000 = (x)(160 - x)$$

$$0 = x^2 - 160x + 4000$$

Using the quadratic formula, we have

$$x = \frac{160 \pm \sqrt{(-160)^2 - 4(1)(4000)}}{2(1)}$$

$$= \frac{160 \pm \sqrt{9600}}{2(1)} \approx 31.01 \text{ or } 128.99$$

The length is longer than the width, so the length is approximately 129 feet and the width is approximately $160 - 129 = 31$ feet.

99. $v_0 = 150,\ h_0 = 0,\ h = 200$

$$200 = -16t^2 + 150t + 0 \Rightarrow$$

$$16t^2 - 150t + 200 = 0 \Rightarrow$$

$$8t^2 - 75t + 100 = 0 \Rightarrow t \approx 1.61 \text{ or } 7.77$$

The ball will reach 200 feet on its downward trip after approximately 7.7 seconds.

100. $v_0 = 55,\ h_0 = 700,\ h = 0$

$$0 = -16t^2 - 55t + 700 \Rightarrow$$

$$16t^2 + 55t - 700 = 0 \Rightarrow t \approx -8.55 \text{ or } 5.12$$

The ball will reach the ground after approximately 5.12 seconds

Case 1 Consumers Often Defy Common Sense

1. The total cost to buy and run this electric hot water tank for x years is $E = 218 + 508x$.

2. The total cost to buy and run this gas hot water tank for x years is $G = 328 + 309x$.

3. Over 10 years, the electric hot water tank costs $218 + 508(10) = 5298$ or \$5298, and the gas hot water tank costs $328 + 309(10) = 3418$ or \$3418. The gas hot water tank costs \$1880 less over 10 years.

4. The total costs for the two hot water tanks will be equal when $218 + 508x = 328 + 309x \Rightarrow$ $199x = 110 \Rightarrow x \approx 0.55$.
   The costs will be equal within the first year.

5. The total cost to buy and run this Maytag refrigerator for x years is $M = 1529.10 + 50x$.

6. The total cost to buy and run this LG refrigerator for x years is $L = 1618.20 + 44x$.

7. Over 10 years, the Maytag refrigerato costs $1529.10 + 50(10) = 2029.10$ or \$2029.10, and the LG refrigerator costs $1618.20 + 44(10) = 2058.20$ or \$2058.20. The LG refrigerator costs \$29.10 more over 10 years.

8. The total costs for the two refrigerators will be equal when $1529.10 + 50x = 1618.20 + 44x \Rightarrow$ $6x = 89.10 \Rightarrow x \approx 14.85$.
   The costs will be equal in the 14th year.